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On intuitionistic fuzzy subsets with diminishing hesitancy values

Peter Vassilev

Department of Mathematical Modeling and Bioinformatics
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
"Acad G. Bonchev" Str., bl. 105, 1113 Sofia, Bulgaria
e-mail: peter.vassilev@gmail.com

Abstract: In the present paper we focus our attention at defining a new way to construct a sequence of intuitionistic fuzzy subsets satisfies a certain condition related to the hesitancy margin. For this purpose we define a generalization of the extended modal operator $F_{\alpha,\beta}$ and establish a sufficient condition that ensures their satisfaction.
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1 Introduction

By an intuitionistic fuzzy set A defined over a discrete or continuous universe set X we understand the following set of ordered triples (see e.g. [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{1}$$

where the mappings $\mu_A, \nu_A : X \to [0, 1]$ are such that

$$\mu_A + \nu_A \leq 1.$$

The mapping μ_A is called a membership function of A, ν_A a non-membership function of A and $\pi_A = 1 - \mu_A - \nu_A$ denotes the hesitancy function of A. If $\pi_A \equiv 0$, then we say that A is a fuzzy set.

Further we require the following definitions

Definition 1.1 (see [2, p.17, Eq (2.1)]). *For any two intuitionistic fuzzy sets A and B defined over the same universe X, we say that A is a subset of B if and only if (iff) for all $x \in X$*

$$A \subseteq B \Leftrightarrow (\mu_A(x) \leq \mu_B(x))\&(\nu_A(x) \geq \nu_B(x)) \tag{2}$$

Definition 1.2 (cf. [2, p.55, Eq (4.6)]). *For any two intuitionistic fuzzy sets A and B defined over the same universe X, we say that A has less hesitancy than B iff for all $x \in X$*

$$A \leq_\pi B \Leftrightarrow \pi_A(x) \leq \pi_B(x).$$

Remark 1.3. *We choose the denotation $\leq_\pi$ instead of the $\sqsubset$ used in the book, in order to be closer to the denotation used by E. Marinov in [3] ($\preceq_\pi$) while stating a subtle difference. Marinov says that $A \preceq_\pi B$ iff for all $x \in X$ it is simultaneously fulfilled:*

$$\mu_A(x) \leq \mu_B(x)\&\nu_A(x) \leq \nu_B(x).$$

It is easy to see that $\preceq_\pi$ implies $\leq_\pi$ but the reverse is not true.

Since further we will be more interested when B has less hesitancy than A we rewrite the equality from Definition 1.2 as:

$$B \leq_\pi A \Leftrightarrow \pi_B(x) \leq \pi_A(x) \tag{3}$$

2 Sequence of intuitionistic fuzzy subsets

Let us consider a sequence of intuitionistic fuzzy sets $\{A_i\}_{i=1}^k$, for some natural k. What conditions should the sets A_i have such that for any any couple A_i, A_{i+1}, $(i < k-1)$ equalities (2) and (3) are fulfilled simultaneously. That is we want:

$$A_1 \subseteq A_2 \subseteq \cdots A_k \ \& \ \pi_{A_1} \geq \pi_{A_2} \cdots \geq \pi_{A_k} \tag{4}$$

Obviously, we must have monotonously increasing membership and decreasing non-membership functions for these sequence of sets with increase which is faster or equal to the rate of decrease.

In other words for all $i \leq k-1$ and for all $x \in X$ we should have:

$$\mu_{A_{i+1}}(x) - \mu_{A_i}(x) = \varepsilon_i(x) \geq \nu_{A_i}(x) - \nu_{A_{i+1}}(x) = \delta_i(x). \tag{5}$$

From (5) we obtain:

$$\Delta\pi_i = \pi_{A_i} - \pi_{A_{i+1}} = \varepsilon_i - \delta i.$$

The last reminds to the way the operator $F_{\alpha,\beta}$ distributes the hesitancy to the degrees of membership and non-membership of an intuitionistic fuzzy set A. Let us recall its definition:

Definition 2.1 (cf. [2, p.77, Eq (5.2)]). *The operator $F_{\alpha,\beta}$ is defined over intuitionistic fuzzy sets as follows:*

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)\rangle | x \in X\},$$

where $0 \leq \alpha, \beta, \alpha + \beta \leq 1$.

Here we have,

$$\pi_A - \pi_{F_{\alpha,\beta}(A)} = (\alpha+\beta)\pi_A(x).$$

This operator, however, works only with positive values of α and β, hence we cannot use it to describe a solution to our problem. On the other hand it has a very convenient for computation form. This leads us to our next step.

3 The generalized operator $\mathcal{F}_{\alpha(x),\beta(x)}$

Here we will consider a generalization of the operator $F_{\alpha,\beta}$, depending on two mappings $\alpha : X \to [-1,1]$ and $\beta : X \to [-1,1]$. We will start with the following definition:

Definition 3.1. *Let $\alpha : X \to [-1,1]$ and $\beta : X \to [-1,1]$ be two mappings such that for all $x \in X$*

$$\alpha(x) + \beta(x) \leq 1.$$

Then the operator $\mathcal{F}_{\alpha(x),\beta(x)}$: IFS $\to$ IFS is defined as

$$\mathcal{F}_{\alpha(x),\beta(x)}(A) = \{\langle x, \mu^*(x), \nu*(x)\rangle | x \in X\}, \tag{6}$$

$$\mu^*(x) = \frac{\mu_A(x) + \alpha(x)\pi_A(x) + |\mu_A(x) + \alpha(x)\pi_A(x)|}{2},$$
$$\nu^*(x) = \frac{\nu_A(x) + \beta(x)\pi_A(x) + |\nu_A(x) + \beta(x)\pi_A(x)|}{2}$$

In order to show that the definition is correct we have to show that for all $x \in X$

$$\mu^*(x) + \nu^*(x) \leq 1.$$

When $\alpha(x) \geq 0$ and $\beta(x) \geq 0$ this is obvious as the operator coincides with $F_{\alpha(x),\beta(x)}$. Let $\alpha(x) \leq 0$, $\beta(x) \leq 0$. We obviously have $0 \leq \mu^* \leq \mu_A$ and $0 \leq \nu^* \leq \nu^A$, hence the above is true. Let us consider the final case i.e. $\alpha(x)\beta(x) < 0$. Let $\alpha(x) > 0$ and $\beta(x) < 0$, then we have $\mu^* \leq 1 - \nu_A$; $\nu^* \leq \nu_A$ i.e. $\mu^* + \nu^* \leq 1$. Completely analogously let $\alpha(x) < 0$ and $\beta(x) > 0$, then we have $\mu^* \leq \mu_A$; $\nu^* \leq 1 - \mu_A$ i.e. $\mu^* + \nu^* \leq 1$.

Now we are ready to formulate our theorem.

Theorem 3.2. *A sufficient condition for the sequence of intuitionistic fuzzy sets $\{A_i\}_{i=1}^k$ to satisfy the relations* (4) *is the existence of mappings $\alpha_i : X \to [0,1]$ and $\beta_i : X \to [-1,0]$ ($\alpha_i(x) + \beta_i(x) > 0$) such that*

$$\mathcal{F}_{\alpha_i(x),\beta_i(x)}(A_i) = A_{i+1},$$

for $i = 1, 2, \ldots, k-1$, with $\mathcal{F}_{\alpha(x),\beta(x)}$ defined by (6).

Proof. Let

$$\mathcal{F}_{\alpha_i(x),\beta_i(x)}(A_i) = A_{i+1}.$$

Since the inclusion follows from the definition of the operator, we only have to show that

$$\pi_{A_i}(x) \geq \pi_{A_{i+1}}(x).$$

But we have

$$\pi_{A_i} = 1 - (\mu_{A_i} + \nu_{A_i}) \geq 1 - (\mu_{A_i}(x) + \nu_{A_i}(\alpha(x) + \beta(x))\pi_{A_i}(x)) = \pi_{A_{i+1}}$$

i.e. (4) is satisfied. □

Remark 3.3. *We note that the condition of the theorem is not necessary since for two IFS A and B which are fuzzy sets we can have both conditions in* (4) *satisfied, and yet the operator $\mathcal{F}_{\alpha_i(x),\beta_i(x)}(A) = A \neq B$. For example:*

$$A = \langle x, 0.7, 0.3 \rangle;\ B = \langle x, 0.8, 0.2 \rangle$$

We obviously have: $A \subseteq B$ and $\pi_A = 0 = \pi_B$.

The importance of Theorem 3.2 lies in the fact that it provides a constructive way of obtaining such a sequence of subsets.

4 Conclusion

We have proposed a way of constructively obtaining sequences of intuitionistic fuzzy subsets with diminishing hesitancy degrees. To this end, a new generalization of the extended modal operator $F_{\alpha,\beta}$ is introduced, thus enabling us to readily implement such techniques in algorithmic form.

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