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# InterCriteria Analysis with weight coefficients of objects or criteria

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**Abstract:** The idea of the InterCriteria Analysis (ICA) was generated by the authors in 2014. For the next (already 9) years, ICA has been an object of intensive research and applications. In the present paper, an extension of the ICA is introduced. In it, the criteria or the objects have their own weight coefficients, or priorities. In the particular case, when these coefficients are equal to 1, the standard ICA appears.

**Keywords:** InterCriteria Analysis, Intuitionistic fuzzy sets, Weight coefficients, Criteria, Objects. **2020 Mathematics Subject Classification:** 03E72.



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### **1** Introduction

In 2014, in [4], the authors introduced the method of InterCriteria Analysis (ICA), using initially the name "intercriteria decision making". During the last nine years, the theory and applications of the ICA have been significantly developed and now there are more than a hundred of papers on the theory behind the method or on various applications of the method to real-world datasets and problems.

In the present paper, we extend the idea of ICA, adding weights of the objects or of the criteria. A short description of the program product, realizing the new ICA-algorithm is given and the new algorithm is illustrated by an example. In [2], a survey of the results over ICA is given.

All definitions, related to the Index Matrices (IMs) are used from [1, 3].

#### 2 Algorithms of the InterCriteria Analysis with weights

Here, we will introduce two algorithms of the InterCriteria Analysis with weights: one related to the proximity between the criteria and another related to the proximity between the objects.

#### 2.1 An algorithm for proximity between the criteria in ICA

Let us have criteria  $C_1, C_2, \ldots, C_n$  and objects  $O_1, O_2, \ldots, O_m$  and let each object  $O_i$  have a priority  $b_i$   $(1 \le i \le m)$ . Let the evaluations of the objects with criteria be given by the following IM:

		$C_1$	•••	$C_k$	•••	$C_l$		$C_n$
$O_1$	$b_1$	$a_{C_1,O_1}$		$a_{C_1,O_k}$	•••	$a_{C_1,O_l}$		$a_{C_1,O_n}$
÷	÷	÷	÷	÷	÷	÷	÷	÷
$O_i$	$b_i$	$a_{C_i,O_1}$		$a_{C_i,O_k}$		$a_{C_i,O_l}$		$a_{C_i,O_n}$
÷	÷	÷	:	÷	÷	÷	÷	÷
$O_j$	$b_j$	$a_{C_m,O_1}$		$a_{C_j,O_k}$		$a_{C_j,O_l}$		$a_{C_j,O_n}$
÷	÷	÷	:	÷	÷	÷	÷	÷
$O_m$	$b_m$	$a_{C_m,O_1}$		$a_{C_m,O_k}$		$a_{C_m,O_l}$	•••	$a_{C_m,O_n}$

where for every  $i, j, (1 \le i < j \le m, 1 \le k < l \le n)$ :

- $O_i$  is an object,
- $b_i$  is the weight of object  $O_i$ ,
- $C_k$  is a criterion,
- a<sub>Ck,Oi</sub> is a real number or another object, that is comparable with respect to relation R with the other a-objects, so that for each i, j, k: R(a<sub>Ck,Oi</sub>, a<sub>Ck,Oj</sub>) is defined. Let R be the dual relation of R in the sense that if R holds, then R does not hold, and vice versa. For example, if "R" is the relation "<", then R is the relation ">", and vice versa; and let the third possible situation about objects a<sub>Ck,Oi</sub> and a<sub>Ck,Oi</sub> be they to be equal.

Initially, let the variables M, N, P = 0.

For every two pairs (i, j) and (k, l) such that  $1 \le i < j \le m, 1 \le k < l \le n$ , we traverse all pairs  $(i, j), 1 \le i < j \le m$ , and we check:

- (a) whether  $a_{C_k,O_i} = a_{C_k,O_j}$ . If yes, then  $P := P + b_i b_j$ ; otherwise, we proceed to checking (b).
- (b) whether relations  $R(a_{C_k,O_i}, a_{C_k,O_j})$  and  $R(a_{C_l,O_i}, a_{C_l,O_j})$  are valid simultaneously. If yes, then  $M := M + b_i b_j$ ; otherwise, we proceed to checking (c).
- (c) [the last possible case] only one of both relations  $R(a_{C_k,O_i}, a_{C_k,O_j})$  and  $R(a_{C_l,O_i}, a_{C_l,O_j})$  is simultaneously valid and the other relation is the opposite one. Then  $N := N + b_i b_j$ .

Let 
$$S = \sum_{1 \le i < j \le m} b_i b_j$$
. Then the intuitionistic fuzzy pair (see [5])

$$\langle \mu, \nu \rangle = \langle \frac{M}{S}, \frac{N}{S} \rangle$$

determines the degrees of nearness and of non-nearness between the criteria  $C_k$  and  $C_l$ .

Now, we can see immediately, that if  $b_i = 1$  for each  $i \ (1 \le i \le m)$ , then we obtain the standard ICA.

#### 2.2 An algorithm for proximity between the objects in ICA

The algorithm is similar to the above one, but in it there is an important difference. Before application of formulas, similar to above ones, we must normalize the *a*-elements in interval [0, 1]. The reason is the following. Let us have the above IM and let us extract its sub-IM containing only two objects (*O* and *P*) and only two criteria (*C* and *D*), and let for simplicity the objects do not have weights. Let the objects have the following evaluations with respect to the criteria:

$$\begin{array}{c|cc}
C & D \\
\hline
O & 0.4 & 300 \\
P & 0.1 & 200 \\
\end{array}$$

If we transpose the matrix above (in order to use the above algorithm), we will obtain the IM

$$\begin{array}{c|cc} O & P \\ \hline C & 0.4 & 0.1 \\ D & 300 & 200 \\ \end{array}$$

Therefore, we see that the two objects are in consonance. But, if we know that the evaluations of criterion C are in the interval [0, 1] and these for criterion D are in interval [0, 1000], then after normalization of the values of the second criterion, we will obtain values

$$a_{D,O} = \frac{300 - 0}{1000 - 0} = 0.3,$$
  
 $a_{D,P} = \frac{200 - 0}{1000 - 0} = 0.2.$ 

Now, the normalized IM will have the form

	O	P
C	0.4	0.1
D	0.3	0.2

and therefore, both objects are in dissonance.

Now, the algorithm obtains the following form.

Let us have criteria  $C_1, C_2, \ldots, C_n$  and objects  $O_1, O_2, \ldots, O_m$  and let each criterion  $C_j$  have a weight coefficient  $c_j$   $(1 \le j \le n)$ . Let the evaluations of the objects with criteria be given by the following IM:

	$C_1$		$C_k$		$C_l$		$C_n$
	$c_1$		$c_k$		$c_l$		$c_n$
$O_1$	$a_{C_1,O_1}$		$a_{C_1,O_k}$		$a_{C_1,O_l}$		$a_{C_1,O_n}$
÷	•	÷	÷	÷	:	÷	÷
$O_i$	$a_{C_i,O_1}$		$a_{C_i,O_k}$		$a_{C_i,O_l}$		$a_{C_i,O_n}$ ,
÷	÷	÷	÷	÷	÷	÷	÷
$O_j$	$a_{C_m,O_1}$		$a_{C_j,O_k}$		$a_{C_j,O_l}$		$a_{C_j,O_n}$
÷	:	:	÷	÷	:	÷	÷
$O_m$	$a_{C_m,O_1}$		$a_{C_m,O_k}$		$a_{C_m,O_l}$		$a_{C_m,O_n}$

where for every  $i, j, (1 \le i < j \le m, 1 \le k < l \le n)$ :

- $O_i$  is an object,
- $C_k$  is a criterion,
- $c_k$  is the weight of object  $C_k$ ,
- $a_{C_k,O_i}$  is a real number or another object, as above.

Let initially, the variables M, N, P = 0.

For every two pairs (i, j) and (k, l) such that  $1 \le i < j \le m, 1 \le k < l \le n$ , we check:

- (a) whether  $a_{C_k,O_i} = a_{C_k,O_j}$ . If yes, then  $P := P + c_k c_l$ ; otherwise, we proceed to checking (b).
- (b) whether relations  $R(a_{C_k,O_i}, a_{C_k,O_j})$  and  $R(a_{C_l,O_i}, a_{C_l,O_j})$  are valid simultaneously. If yes, then  $M := M + c_k c_l$ ; otherwise, we proceed to checking (c).
- (c) [the last possible case] only one of both relations  $R(a_{C_k,O_i}, a_{C_k,O_j})$  and  $R(a_{C_l,O_i}, a_{C_l,O_j})$  is valid simultaneously. Then  $N := M + c_k c_l$ .

Let  $S = \sum_{1 \le i < j \le m} c_k c_l$ . Then the intuitionistic fuzzy pair

$$\langle \mu, \nu \rangle = \langle \frac{M}{S}, \frac{N}{S} \rangle$$

determines the degrees of proximity and of remoteness between the objects  $O_i$  and  $O_j$ .

Now, we can see immediately, that if  $c_k = 1$  for each  $k \ (1 \le k \le n)$ , then we obtain the standard ICA.

## 3 Input format

The input spreadsheet must contain at least two sheets:

- A data sheet, containing all evaluations of the object by each criterion, ordered as has been previously defined.
- A priority sheet, which must be positioned after the data sheet in the workbook. The criterion an object priorities must be placed in the same positions as the corresponding names occupy in the data sheet.

	Α	В	с	D	E	F		А	B		С		D	E	F
1		C1	C2	C3	C4		1			7		2	3	1	
2	01	1	3	1	5		2	1					Criterio	n prioritie	s
3	02	2	4	2	4		3	3							
4	O3	5	5	3	3		4	4							
5	04	4	6	4	2		5	5							
6	O5	5	7	5	1		6	7							
7							7	Obio	t nri	oritio					
8							8	Objec	t pro	onue	5				
O.							0								
	< >	Shee	t 1 Shee	et 2	+		<	>	1	Sheet	1	She	et 2	+	

Figure 1. An example spreadsheet showing the layouts needed by the ICA program.

This example input table was used to generate the following illustrations.

If the input data comes in the form of a tab-separated file, the priority information needs to be appended after the end of the text as follows:

```
<end of table 1>
<priority values>
```

The priorities must match the number of objects or criteria, depending on what is being prioritized.

### 4 Loading the file into the program

The file is opened normally through the "File  $\rightarrow$  Open workbook..." menu. When the options dialog appears, the user must select the desired type of priorities to be applied to the analysis.

#### 4.1 Using object priorities

If the user checks the "Use object priorities" box, the object priority values are taken into account in the calculation process.

Settings	×
Input worksheet:	
Sheet 1	~
Input data layout:	
Criteria by rows, Objects by columns	<ul> <li>Objects by rows, Criteria by columns</li> </ul>
05         02         03           61	61         62         63         64           61         62         63         64           62         64         64         64           63         64         64         64
Data will be read as:	
Numbers	~
Compare the data table with itself (	otherwise the program expects two tables)
When two number pairs each give an e	equality:
○ consider it a match	Oconsider it as uncertainty
<ul> <li>consider it as two opposite relations</li> </ul>	<ul> <li>consider it as half match, half opposite</li> </ul>
Percentage of difference allowed betw they will be considered equal)	een the values (when they fall within it,
	0,00 -
Use object priorities	
Use criterion priorities	OK Cancel

Figure 2. Selecting the option to use object priorities.

ile	View Langu	iage Help			File	View Langu	Jage Help		
	C1	C2	C3	C4		C1	C2	C3	C4
C1		0,8	0,8	0,1	C1		0,68	0,68	0,133333
C2	0,8		1	0	C2	0,68		1	0
C3	0,8	1		0	C3	0,68	1		0
C4	0,1	0	0		C4	0,133333	0	0	
	C1	C2	C3	C4		C1	C2	C3	C4
C1		0,1	0,1	0,8	C1		0,133333	0,133333	0,68
C2	0,1		0	1	C2	0,133333		0	1
C3	0,1	0		1	C3	0,133333	0		1
C4	0,8	1	1		C4	0,68	1	1	
C2 C3 C4 Dat	0,1 0,1 0,8	0 1	0 1	1 1	C2 C3 C4	0,133333 0,133333 0,68	0 1	0 1	1 1 Save result
Set	selection color	Clear colors	Chart 9	how only selected	Set	selection color	Clear colors	Chart 🗌	Show only selected
				Distances from zero					Distances from ze
	Crite	ria ranking		Distances from (1, 0)		Crite	ria ranking		Distances from (1
				Distances from (0,1)					Distances from (0

Figure 3. Results without using priorities (left) compared to results using priorities (right).

If the user needs to check if the input data has been read correctly, they can press the new "View data" button, which opens a window showing the input data as read from the file, along with the priority values in square brackets next to the object name.

🖄 Dat	a					X 🖹 Da	ta					
	01	02	03	04	05		01 [1]	O2 [3]	O3 [4]	O4 [5]	O5 [7]	
C1 1		2	5	4	5	C1 1		2	5	4	5	
C2 3		4	5	6	7	C2 3		4	5	6	7	
C3 1		2	3	4	5	C3 1		2	3	4	5	
C4 5		4	3	2	1	C4 5		4	3	2	1	
C1 1		2	5	4	5	C1 1		2	5	4	5	
		-	-	*				-	-	*	-	
C2 3		4	3			C2 5		4	2	0	/ •	
C3 1		2	3	4	5	C3 1		2	3	4	5	
		4	3	2	1	C4 5		4	3	2	1	

Figure 4. Input without using priorities (left) compared to input data using priorities (right).

#### 4.2 Using criterion priorities

Criterion priorities are applied to the calculation process by swapping the objects for the criteria. Since different criteria have different evaluation ranges, to make the values fit the same range, they must first be normalized for each criterion. The maximum  $(\max_i a_{O_i,C_j})$  and minimum  $(\min_i a_{O_i,C_j})$  value for each criterion first needs to be found, and then the evaluations are recalculated like this:

$$a_{O_i,C_j} = \frac{a_{O_i,C_j} - \min_i a_{O_i,C_j}}{\max_i a_{O_i,C_i} - \min_i a_{O_i,C_i}}$$

Then, objects and criteria are swapped, and the recular calculations are performed, as if the objects are the criteria and vice versa, with the criterion priorities taking the place of object priorities. To show the normalized values, the user can press the "Show data" button.

Settings	×
Input worksheet:	
Sheet 1	~
Input data layout:	
Criteria by rows, Objects by columns	<ul> <li>Objects by rows, Criteria by columns</li> </ul>
01         02         01           c1	61 62 63 64 61
Data will be read as:	
Numbers	~
Compare the data table with it	self (otherwise the program expects two tables)
O consider it a match	<ul> <li>consider it as uncertainty</li> </ul>
<ul> <li>consider it as two opposite relations</li> </ul>	O consider it as half match, half opposite
Percentage of difference allowed they will be considered equal)	between the values (when they fall within it,
	0,00
Use object priorities	0,00 🕄
Use object priorities	0,00 -

Figure 5. Selecting the option to use criteria priorities.

	C1 [7]	C2 [2]	C3 [3]	C4 [1]
01	0	0	0	1
02	0,25	0,25	0,25	0,75
03	1	0,5	0,5	0,5
04	0,75	0,75	0,75	0,25
~				
05	1	1	1	0
05	C1 [7]	1 C2 [2] 0	1 C3 [3] 0	0 C4[1] 1
05	C1 [7] 0 0,25	1 C2 [2] 0 0,25	1 C3 [3] 0 0,25	0 C4 [1] 1 0,75
05 01 02 03	C1 [7] 0 0,25 1	C2 [2] 0 0,25 0,5	1 C3 [3] 0 0,25 0,5	0 C4 [1] 1 0,75 0,5
01 02 03 04	C1 (7) 0 0,25 1 0,75	C2 [2] 0 0,25 0,5 0,75	1 C3 [3] 0 0,25 0,5 0,75	0 C4[1] 1 0,75 0,5 0,25



ا ﷺ ile	D:/KOREKTNOST2- View Language	proba/priority.xlsx ge Help	: - InterCriteria De	cision Making			- 0	
01 02 03 04	01 0,226415 0 0	02 0,226415 0 0 0	03 0 0 0,132075	04 0 0 0,132075	O5 0 0,132075 0,226415	01 02 03 04 05	0 01 0 02 0 03 0 04 0 05	
05	0	0 02 0	0,132075 O3 0,132075	0,226415 O4 0,226415	O5 0,226415			
02 03	0	0,132075	0,132075	0,226415 0	0,226415			
04	0,226415	0,226415	0	0	0			
Dat Set	a samples Cop	y result Copy to Clear colors	vertical table S Chart S	ave result Show	/ data	_		
	Crite	ria ranking		Distances f	rom zero rom (1, 0)		Prediction	

Figure 7. Results from ICA using criteria priorities.

# 5 Conclusion

Nine years after the introduction of the InterCriteria Analysis by the authors, an extension of the method is described. In next research, the modification of ICA will be applied to distinct datasets and real-life problems, and the differences with the standard case will be discussed.

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