InterCriteria Analysis with weight coefficients of objects or criteria

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Abstract: The idea of the InterCriteria Analysis (ICA) was generated by the authors in 2014. For the next (already 9) years, ICA has been an object of intensive research and applications. In the present paper, an extension of the ICA is introduced. In it, the criteria or the objects have their own weight coefficients, or priorities. In the particular case, when these coefficients are equal to 1, the standard ICA appears.
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1 Introduction

In 2014, in [4], the authors introduced the method of InterCriteria Analysis (ICA), using initiallly the name "intercriteria decision making". During the last nine years, the theory and applications of the ICA have been significantly developed and now there are more than a hundred of papers on the theory behind the method or on various applications of the method to real-world datasets and problems.

In the present paper, we extend the idea of ICA, adding weights of the objects or of the criteria. A short description of the program product, realizing the new ICA-algorithm is given and the new algorithm is illustrated by an example. In [2], a survey of the results over ICA is given.

All definitions, related to the Index Matrices (IMs) are used from [1, 3].

2 Algorithms of the InterCriteria Analysis with weights

Here, we will introduce two algorithms of the InterCriteria Analysis with weights: one related to the proximity between the criteria and another related to the proximity between the objects.

2.1 An algorithm for proximity between the criteria in ICA

Let us have criteria $C_1, C_2, \ldots, C_n$ and objects $O_1, O_2, \ldots, O_m$ and let each object O_i have a priority b_i $(1 \leq i \leq m)$. Let the evaluations of the objects with criteria be given by the following IM:

$$
\begin{array}{cc|ccccccc}
 & & C_1 & \ldots & C_k & \ldots & C_l & \ldots & C_n \\
\hline
O_1 & b_1 & a_{C_1,O_1} & \ldots & a_{C_1,O_k} & \ldots & a_{C_1,O_l} & \ldots & a_{C_1,O_n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
O_i & b_i & a_{C_i,O_1} & \ldots & a_{C_i,O_k} & \ldots & a_{C_i,O_l} & \ldots & a_{C_i,O_n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
O_j & b_j & a_{C_m,O_1} & \ldots & a_{C_j,O_k} & \ldots & a_{C_j,O_l} & \ldots & a_{C_j,O_n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
O_m & b_m & a_{C_m,O_1} & \ldots & a_{C_m,O_k} & \ldots & a_{C_m,O_l} & \ldots & a_{C_m,O_n}
\end{array},
$$

where for every i, j, $(1 \leq i < j \leq m, 1 \leq k < l \leq n)$:

- O_i is an object,

- b_i is the weight of object O_i,

- C_k is a criterion,

- a_{C_k,O_i} is a real number or another object, that is comparable with respect to relation R with the other a-objects, so that for each i, j, k: $R(a_{C_k,O_i}, a_{C_k,O_j})$ is defined. Let $\overline{R}$ be the dual relation of R in the sense that if R holds, then $\overline{R}$ does not hold, and vice versa. For example, if "R" is the relation "$<$", then $\overline{R}$ is the relation "$>$", and vice versa; and let the third possible situation about objects a_{C_k,O_i} and a_{C_k,O_j} be they to be equal.

Initially, let the variables $M, N, P = 0$.

For every two pairs (i, j) and (k, l) such that $1 \leq i < j \leq m, 1 \leq k < l \leq n$, we traverse all pairs (i, j), $1 \leq i < j \leq m$, and we check:

(a) whether $a_{C_k, O_i} = a_{C_k, O_j}$. If yes, then $P := P + b_i b_j$; otherwise, we proceed to checking (b).

(b) whether relations $R(a_{C_k, O_i}, a_{C_k, O_j})$ and $R(a_{C_l, O_i}, a_{C_l, O_j})$ are valid simultaneously. If yes, then $M := M + b_i b_j$; otherwise, we proceed to checking (c).

(c) [the last possible case] only one of both relations $R(a_{C_k, O_i}, a_{C_k, O_j})$ and $R(a_{C_l, O_i}, a_{C_l, O_j})$ is simultaneously valid and the other relation is the opposite one. Then $N := N + b_i b_j$.

Let $S = \sum\limits_{1 \leq i < j \leq m} b_i b_j$. Then the intuitionistic fuzzy pair (see [5])

$$\langle \mu, \nu \rangle = \langle \frac{M}{S}, \frac{N}{S} \rangle$$

determines the degrees of nearness and of non-nearness between the criteria C_k and C_l.

Now, we can see immediately, that if $b_i = 1$ for each i $(1 \leq i \leq m)$, then we obtain the standard ICA.

2.2 An algorithm for proximity between the objects in ICA

The algorithm is similar to the above one, but in it there is an important difference. Before application of formulas, similar to above ones, we must normalize the a-elements in interval $[0, 1]$. The reason is the following. Let us have the above IM and let us extract its sub-IM containing only two objects (O and P) and only two criteria (C and D), and let for simplicity the objects do not have weights. Let the objects have the following evaluations with respect to the criteria:

	C	D
O	0.4	300
P	0.1	200

.

If we transpose the matrix above (in order to use the above algorithm), we will obtain the IM

	O	P
C	0.4	0.1
D	300	200

.

Therefore, we see that the two objects are in consonance. But, if we know that the evaluations of criterion C are in the interval $[0, 1]$ and these for criterion D are in interval $[0, 1000]$, then after normalization of the values of the second criterion, we will obtain values

$$a_{D,O} = \frac{300 - 0}{1000 - 0} = 0.3,$$

$$a_{D,P} = \frac{200 - 0}{1000 - 0} = 0.2.$$

Now, the normalized IM will have the form

	O	P
C	0.4	0.1
D	0.3	0.2

and therefore, both objects are in dissonance.

Now, the algorithm obtains the following form.

Let us have criteria $C_1, C_2, \ldots, C_n$ and objects $O_1, O_2, \ldots, O_m$ and let each criterion C_j have a weight coefficient c_j $(1 \leq j \leq n)$. Let the evaluations of the objects with criteria be given by the following IM:

	C_1	$\ldots$	C_k	$\ldots$	C_l	$\ldots$	C_n
	c_1	$\ldots$	c_k	$\ldots$	c_l	$\ldots$	c_n
O_1	a_{C_1,O_1}	$\ldots$	a_{C_1,O_k}	$\ldots$	a_{C_1,O_l}	$\ldots$	a_{C_1,O_n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
O_i	a_{C_i,O_1}	$\ldots$	a_{C_i,O_k}	$\ldots$	a_{C_i,O_l}	$\ldots$	a_{C_i,O_n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
O_j	a_{C_m,O_1}	$\ldots$	a_{C_j,O_k}	$\ldots$	a_{C_j,O_l}	$\ldots$	a_{C_j,O_n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
O_m	a_{C_m,O_1}	$\ldots$	a_{C_m,O_k}	$\ldots$	a_{C_m,O_l}	$\ldots$	a_{C_m,O_n}

where for every i, j, $(1 \leq i < j \leq m, 1 \leq k < l \leq n)$:

- O_i is an object,
- C_k is a criterion,
- c_k is the weight of object C_k,
- a_{C_k,O_i} is a real number or another object, as above.

Let initially, the variables $M, N, P = 0$.

For every two pairs (i, j) and (k, l) such that $1 \leq i < j \leq m, 1 \leq k < l \leq n$, we check:

(a) whether $a_{C_k,O_i} = a_{C_k,O_j}$. If yes, then $P := P + c_k c_l$; otherwise, we proceed to checking (b).

(b) whether relations $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $R(a_{C_l,O_i}, a_{C_l,O_j})$ are valid simultaneously. If yes, then $M := M + c_k c_l$; otherwise, we proceed to checking (c).

(c) [the last possible case] only one of both relations $R(a_{C_k,O_i}, a_{C_k,O_j})$ and $R(a_{C_l,O_i}, a_{C_l,O_j})$ is valid simultaneously. Then $N := M + c_k c_l$.

Let $S = \displaystyle\sum_{1 \leq i < j \leq m} c_k c_l$. Then the intuitionistic fuzzy pair

$$\langle \mu, \nu \rangle = \langle \frac{M}{S}, \frac{N}{S} \rangle$$

determines the degrees of proximity and of remoteness between the objects O_i and O_j.

Now, we can see immediately, that if $c_k = 1$ for each k $(1 \leq k \leq n)$, then we obtain the standard ICA.

3 Input format

The input spreadsheet must contain at least two sheets:

- A data sheet, containing all evaluations of the object by each criterion, ordered as has been previously defined.

- A priority sheet, which must be positioned after the data sheet in the workbook. The criterion an object priorities must be placed in the same positions as the corresponding names occupy in the data sheet.

Figure 1. An example spreadsheet showing the layouts needed by the ICA program.

This example input table was used to generate the following illustrations.

If the input data comes in the form of a tab-separated file, the priority information needs to be appended after the end of the text as follows:

```
<end of table 1>

<priority values>
```

The priorities must match the number of objects or criteria, depending on what is being prioritized.

4 Loading the file into the program

The file is opened normally through the "File → Open workbook..." menu. When the options dialog appears, the user must select the desired type of priorities to be applied to the analysis.

4.1 Using object priorities

If the user checks the "Use object priorities" box, the object priority values are taken into account in the calculation process.
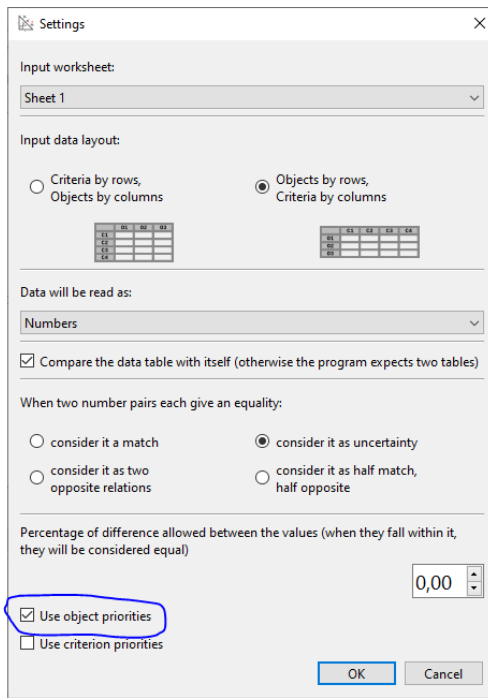
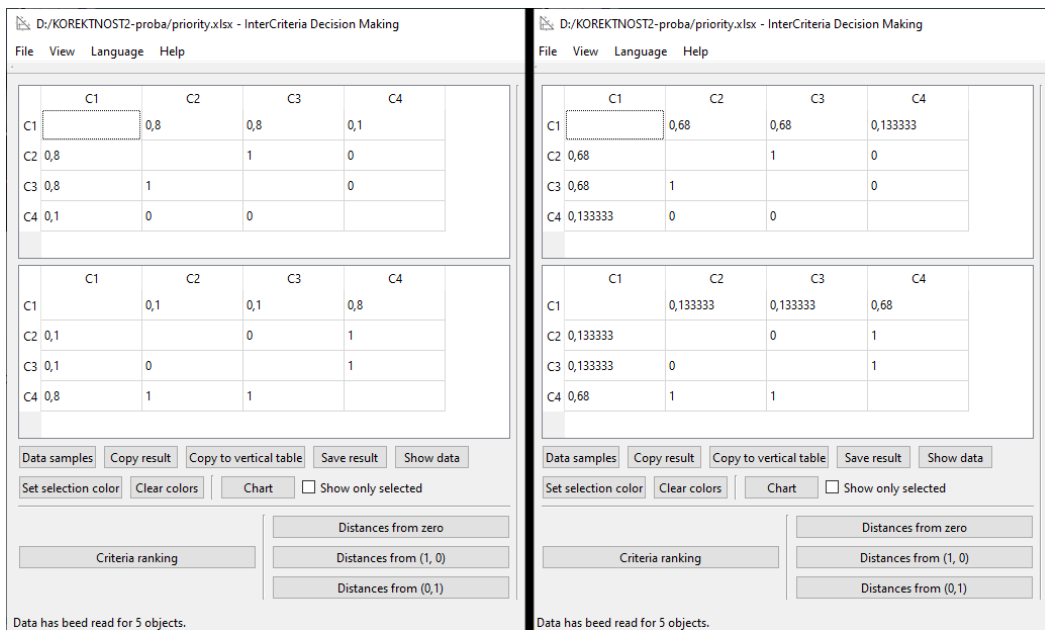Figure 2. Selecting the option to use object priorities.

Figure 3. Results without using priorities (left) compared to results using priorities (right).

If the user needs to check if the input data has been read correctly, they can press the new "View data" button, which opens a window showing the input data as read from the file, along with the priority values in square brackets next to the object name.
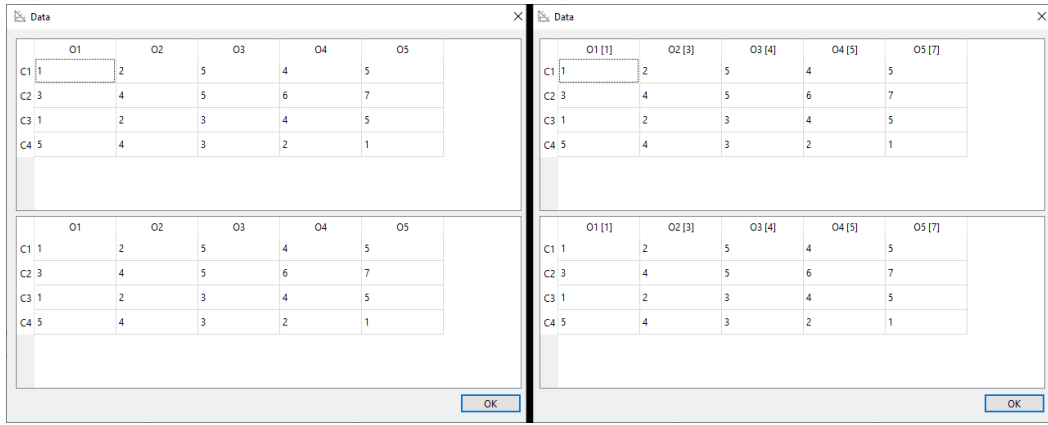
Figure 4. Input without using priorities (left) compared to input data using priorities (right).

4.2   Using criterion priorities

Criterion priorities are applied to the calculation process by swapping the objects for the criteria. Since different criteria have different evaluation ranges, to make the values fit the same range, they must first be normalized for each criterion. The maximum ($\max_i a_{O_i,C_j}$) and minimum ($\min_i a_{O_i,C_j}$) value for each criterion first needs to be found, and then the evaluations are recalculated like this:

$$a_{O_i,C_j} = \frac{a_{O_i,C_j} - \min_i a_{O_i,C_j}}{\max_i a_{O_i,C_j} - \min_i a_{O_i,C_j}}$$

Then, objects and criteria are swapped, and the recular calculations are performed, as if the objects are the criteria and vice versa, with the criterion priorities taking the place of object priorities. To show the normalized values, the user can press the "Show data" button.
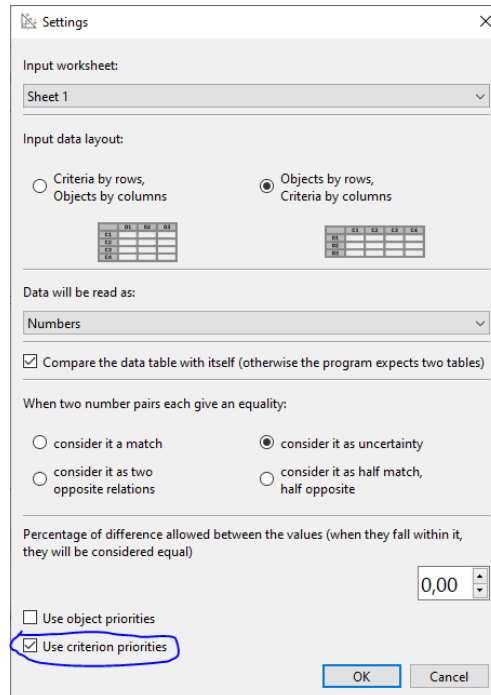
Figure 5. Selecting the option to use criteria priorities.

Figure 6. Normalized data by criterion.

Figure 7. Results from ICA using criteria priorities.

5 Conclusion

Nine years after the introduction of the InterCriteria Analysis by the authors, an extension of the method is described. In next research, the modification of ICA will be applied to distinct datasets and real-life problems, and the differences with the standard case will be discussed.

Acknowledgements

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