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Intuitionistic Fuzzy Interpretations of Conway's Game of Life. Part 2: Modal and Extended Modal Transformations of the Game Field

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Abstract: Intuitionistic fuzzy sets (IFSs) are an extension of Zadeh's fuzzy sets, which introduce a degree of membership and a degree of non-membership whose sum is equal to or less than 1 and the complement to 1 is called a degree of uncertainty.

Conway's Game of Life is a popular heuristic zero-player game, devised by John Horton Conway in 1970. Its "universe" is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive or dead. Every cell interacts with its eight neighbours, which are the cells that are directly horizontally, vertically, or diagonally adjacent. In a stepwise manner, the state of each cell in the grid preserves or alternates with respect to a given list of rules.

In a previous authors' research, for each cell its IF estimation was defined as a pair consisting of the degrees l_p and l_a, namely degrees of presence and absence of life, where $l_p + l_a \leq 1$.

The article proposes intuitionistic fuzzy approaches to modifying of the Game of Life cells' states. These approaches are based on the IF modal and extended modal operators.
Keywords: Conway's Game of Life, Intuitionistic fuzzy sets

1 Introduction

Conway's Game of Life is devised by John Horton Conway in 1970, and already 40 years it is an object of research, software implementions and modifications. In [1] there is a list of many papers devoted to Conway's Game of Life. In [6], the authors introduced a modification of this game, based on the idea of the intuitionistic fuzziness (see [2]).

Here, in continuation of the ideas from [6], we will discuss the possibility to use some of the modal and topological operators, defined over IFSs.

The standard Conway's Game of Life has a "universe" which is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive or dead, or as we learned the game in the middle of 1970s, in the square there is an asterisk, or not.

Every cell interacts with its eight neighbours, namely the cells that are directly adjacent either in horizontal, vertical, or diagonal direction. In a stepwise manner, the state of each cell in the grid preserves or alternates with respect to a given list of rules.

Here, we will discuss some versions of the game in which we will keep the condition for the necessary number of existing neighbours asterisks for birth or dying of an asterisk in some square. For our aims, we will use elements of intuitionistic fuzzy set theory (see, [2]).

2 Previous Results: Intuitionistic Fuzzy Criteria of Existence, Birth and Death of an Asterisk, and Roles for Changing of the Game-field

Following [6], we will introduce some definitions.

Let us have a plane tesselated with squares. Let in some of these squares there be symbols "*", meaning that the squares are "alive". Now we will extend this construction of the Game of Life to some new forms.

Let us assume that the square $\langle i, j \rangle$ is assigned a pair of real numbers $\langle \mu_{i,j}, \nu_{i,j} \rangle$, so that $\mu_{i,j} + \nu_{i,j} \leq 1$. We can call the numbers $\mu_{i,j}$ and $\nu_{i,j}$ degree of existence and degree of non-existence of symbol "*" in square $\langle i, j \rangle$. Therefore, $\pi(i, j) = 1 - \mu_{i,j} - \nu_{i,j} \leq 1$ will correspond to the degree of uncertainty, e.g., lack of information about existence of an asterisk in the respective square.

In [6], first, we formulated a series of different criteria for correctness of the intuitionistic fuzzy interpretations that will include as a particular case the standard game.

We will suppose that there exists an asterisk in square $\langle i, j \rangle$ if:

- **(1.1)** $\mu_{i,j} > 0.5$. Therefore $\nu_{i,j} < 0.5$. In the particular case, when $\mu_{i,j} = 1 > 0.5$ we obtain $\nu_{i,j} = 0 < 0.5$, i.e., the standard existence of the asterisk.
- **(1.2)** $\mu_{i,j} \geq 0.5$. Therefore $\nu_{i,j} \leq 0.5$. Obviously, if case (1.1) is valid, then case (1.2) also will be valid.
- **(1.3)** $\mu_{i,j} > \nu_{i,j}$. Obviously, case (1.1) is particular case of the present one, but case (1.2) is not included in the currently discussed case for $\mu_{i,j} = 0.5 = \nu_{i,j}$.
- **(1.4)** $\mu_{i,j} \geq \nu_{i,j}$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of the present one.
- **(1.5)** $\mu_{i,j} > 0$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of the present one, but case (1.4) is not included in the currently discussed case for $\mu_{i,j} = 0.0 = \nu_{i,j}$.
- **(1.6)** $\nu_{i,j} < 1$. Obviously, cases (1.1), (1.2) and (1.3) are particular cases of the present one, but case (1.5) is not included in the currently discussed case for $\mu_{i,j} = 0.0$.

From these criteria it follows that if one is valid – let it be the s-th criterion ($1 \leq s \leq 6$) then we can assert that the asterisk exists with respect to the s-th criterion and therefore, it will exist with respect to all other criteria, the validity of which follows from the validity of the s-th criterion.

On the other hand, if the s-th criterion is not valid, then we will say that the asterisk does not exist with respect to the s-th criterion. It is very important that in this case the square may not be empty. It is suitable to tell that the square $\langle i, j\rangle$ is totally empty, if its degrees of existence and non-existence are $\langle 0, 1\rangle$.

The square is s-full if it contains an asterisk with respect to the s-th criterion and that the square is s-empty if it does not satisfy the s-th criterion.

For the aims of the game-method for modelling, it will be suitable to use (with respect to the type of the concrete model) one of the first four criteria for existence of an asterisk. Let us say for each fixed square $\langle i, j\rangle$ that therein is an asterisk due to the s-th criterion for $1 \leq s \leq 4$), if this criterion confirms the existence of an asterisk.

In [6], four criteria for the birth of an asterisk and four criteria for the death of an asterisk are given. Also, seven intuitionistic fuzzy rules for changing of the game-field are introduced there. Two of these rules are extensions of the standard game-rules, while the rest five rules do not have analogues in the standard game.

3 Transformations of the Game-field by Modal and Extended Modal Operators

Let a set E be fixed. An Intuitionistic Fuzzy Set (IFS) A in E is an object of the following form (see [2]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let A be an IFS and let $\alpha, \beta \in [0, 1]$.

We shall define the following modal operators (see, e.g., [2]):

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\};$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\};$$

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x)\rangle | x \in E\};$$

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1;$$

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$H^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$

$$J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle | x \in E\}.$$

The first two operators are analogous of the operators in the modal logic (see, e.g., [7]). The next seven operators are extensions of the first two. It is important to denote that they do not have analogues in the modal logic.

3.1 Global transformations of the game field by modal and extended modal operators

Now, we can apply each one of the nine modal operators over the game field F. Having in mind the above discussed IF-interpretations of the asterisk, we see that we can construct the set

$$A = \{\langle\langle i,j\rangle, \mu_{i,j}, \nu_{i,j}\rangle | \langle i,j\rangle \in F\}.$$

On one hand, it corresponds to the game-field, while on the other, it is an IFS. Now, if we apply some of the modal operators over IFS A, we will change the game-field in a this way. For this aim we must determine in a suitable way the values of arguments α and β of the extended modal operators. So, we will transform globally the game field. The first two (standard) modal operators can be used only for this type of transformations.

Let for a fixed cell $\langle i,j\rangle$ the subset of A that contains its neighboard cells be

$$A_{(i,j)} = \{\langle\langle p,q\rangle, \mu_{p,q}, \nu_{p,q}\rangle | \langle p,q\rangle \in F \;\&\; p \in \{i-1, i, i+1\} \;\&\; q \in \{j-1, j, j+1\} \& \; \langle p,q\rangle \neq \langle i,j\rangle\}.$$

3.2 Global transformations of the game field by modal and extended modal operators

These transformations are related to the way of determining of the argumenrs α and β of the extended modal operators. In the first case (previous subsection) they were given outside the game. Here, they will be generated on the bases of the IF-values of the asterisks.

The transformations can be two types. In the first case the values of the asterisks that stay in the eight neighbours of a fixed asterisk will determine the values of the α and β-parameters. There are three cases for the calculation of these parameters.

6.1[1] (pesimistic parameter values):

$$\alpha = \min_{\langle p,q\rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \max_{\langle p,q\rangle \in A_{(i,j)}} \nu_{p,q}.$$

6.2 (average parameter values):

$$\alpha = \frac{1}{8} \sum_{\langle p,q\rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \frac{1}{8} \sum_{\langle p,q\rangle \in A_{(i,j)}} \nu_{p,q}.$$

6.3 (optimistic parameter values):

$$\alpha = \max_{\langle p,q\rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\beta = \min_{\langle p,q\rangle \in A_{(i,j)}} \nu_{p,q}.$$

[1]In this paper we continue the numeration of [6]

The α and β-values determined by (6.1) - (6.3) are used as parameters of a previously fixed extended modal operator that is applied over the IF-values of the fixed asterisk.

In the second case, the values of the asterisks that stay in the eight neighbours of a fixed asterisk will determine the values of the μ^* and ν^*-parameters for the asterisk in cell $\langle i, j \rangle$, while its original values $\mu_{i,j}$ and $\nu_{i,j}$ will be used for α and β-parameters, i.e, the extended modal operators that will be used for the transformation of the game field for cell $\langle i, j \rangle$ will have values $\alpha = \mu_{i,j}$ and $\beta = \nu_{i,j}$.

The μ^* and ν^* parameters will be calculated in the following three ways.

7.1 (pessimistic parameter values):

$$\mu^*_{i,j} = \min_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu^*_{i,j} = \max_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

6.2 (average parameter values):

$$\mu^*_{i,j} = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu^*_{i,j} = \frac{1}{8} \sum_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

6.3 (optimistic parameter values):

$$\mu^*_{i,j} = \max_{\langle p,q \rangle \in A_{(i,j)}} \mu_{p,q},$$

$$\nu^*_{i,j} = \min_{\langle p,q \rangle \in A_{(i,j)}} \nu_{p,q}.$$

4 Conclusion

In the next authors' research new modifications of this game will be described. We will modify the rules of the game, as we did this in our previous research, e.g. [3, 4, 5].

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