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Generalized Net Model of a Special Type of Expert Systems

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Abstract: Generalized net models describing functioning and the results of the work of two types – standard and a special type of expert systems, are given. The process of the generalized net model functioning is illustrated.
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1 Introduction

In a series of papers, collected in monographs [2, 5, 6], different types of Expert Systems (ESs; see, e.g. [4, 7]) are described by in terms of Generalized Nets (GNs; see [1, 3]). Here, we will construct a new GN-model of an ES, which is an extension of the standard ESs.

The hypotheses of each of the standard ESs are some variables. Here, we will discuss the case, when the hypotheses have a complex form. Now, it is a Boolean expressions of different variables, each of which can be interpreted as a separate hypothesis. For example, the complex hypothesis can have the form $A\&((B \rightarrow (C \vee D)) \rightarrow \neg E)$.

In the beginning, we will describe the GN that is universal for all ESs from production type having Data Base (DB) of the facts and Knowledge Base (KB) of the rules.

2 GN that is universal for the ESs from production type

In [2], it is constructed the first GN which is fully independent on the forms of the ESs whose functioning and results of work are represented by this net. It is shown on Fig. 1.

For clarity, the places are marked by three different symbols: a, b and c, such that:

- the token α together with its descendants from all generations after splitting will go to the a-places;
- the token β will move along transfer to b-places;
- the token γ will move along transfer to c-places.

In the construction below, the tokens' characteristics will be represented by ordered tuples whose first component is in turn a vector of natural numbers. When tokens will split, they will be marked with the number of the current split, keeping the previous numeration, i.e., if the first component of a token characteristic is $\langle s_1, s_2, ..., s_{k-1}\rangle$ ($k \geq 0; s_1, s_2, ..., s_{k-1}$ - natural numbers), then its next characteristic will be $\langle s_1, s_2, ..., s_{k-1}, s_k\rangle$, where the natural number s_k will correspond to the number of the tokens's current splitting.

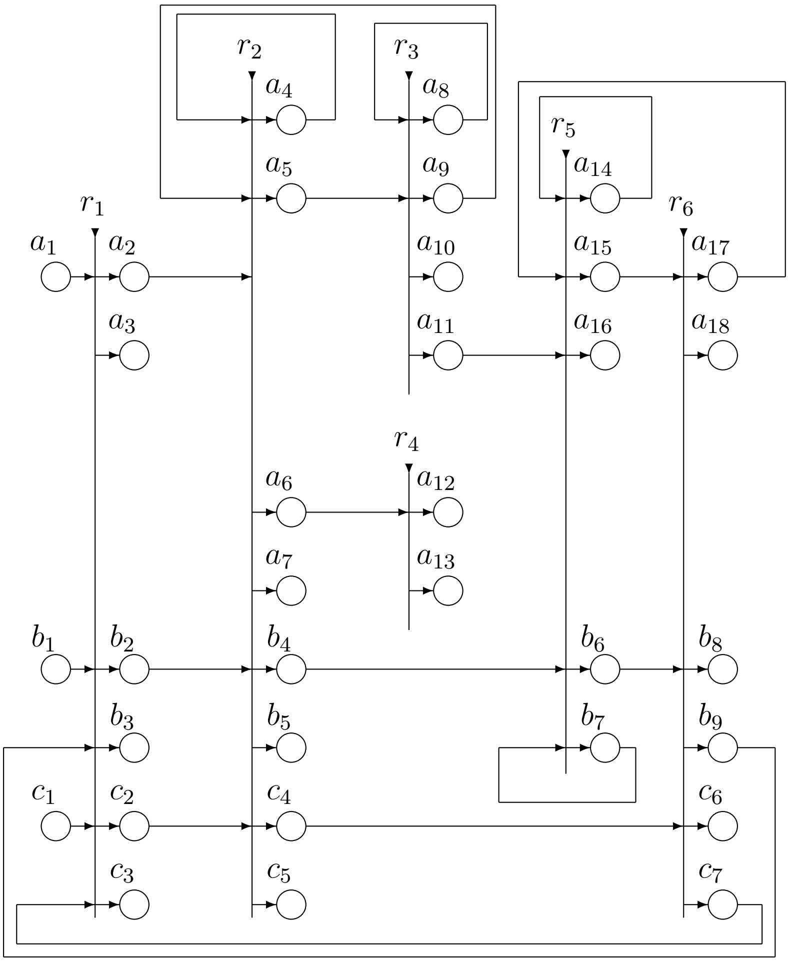

Fig. 1.

Let Δ be the DB of a given ES.

Let token α enter place a_1 of the GN with an initial characteristic

$$x_0^\alpha = \text{“}\langle p, H\rangle\text{”},$$

where p is the current number of the α-token which enters place a_1 and H is a hypothesis.

Let token β enter place b_1 with an initial characteristic

$$x_0^\beta = \text{“}\Delta\text{”}.$$

Let token γ enter the place c_1 with an initial characteristic

$$x_0^\gamma = \text{“}R\text{”},$$

where $R = \{R_1, ..., R_n\}$ is the list of the rules in the KB. Each rule R_i has the form ($1 \leq i \leq n$):

$$R_i = \langle C_i; A_{i,1}, ..., A_{i,s_i}\rangle,$$

where C_i is the consequent and $A_{i,1}, ..., A_{i,s_i}$ are the elements of the conjunction which forms the antecedent, x_i denotes the i-th characteristic of the $\alpha-$token with the highest priority in a given place.

The transitions of the GN are the following (see Fig. 1).

$$Z_1 = \langle\{a_1, b_1, b_9, c_1, c_7\}, \{a_2, a_3, b_2, b_3, c_2, c_3\}, r_1\rangle,$$

where

$$r_1 = \begin{array}{c|cccccc} & a_2 & a_3 & b_2 & b_3 & c_2 & c_3 \\ \hline a_1 & r_{a_1,a_2} & r_{a_1,a_3} & false & false & false & false \\ b_1 & false & false & r_{b_1,b_2} & r_{b_1,b_3} & false & false \\ b_9 & false & false & r_{b_9,b_2} & r_{b_9,b_3} & false & false \\ c_1 & false & false & false & false & r_{c_1,c_2} & r_{c_1,c_3} \\ c_7 & false & false & false & false & r_{c_7,c_2} & r_{c_7,c_3} \end{array}$$

and
$r_{a_1,a_2} = r_{b_1,b_2} = r_{b_9,b_2} = r_{c_1,c_2} = r_{c_7,c_2} =$ “$pr_2 x_0^\alpha \not\in x_0^\beta$”,
$r_{a_1,a_3} = \neg r_{a_1,a_2}$,
$r_{b_1,b_3} = r_{b_9,b_3} = r_{c_1,c_3} = r_{c_7,c_3} = \neg r_{a_1,a_2}$ & “there are no new α-tokens that must enter place a_1”,
where $\neg P$ is a negation of predicate P.

The tokens obtain the characteristic

$$\text{“}\langle pr_1 x_0^\alpha, !pr_2 x_0^\alpha\rangle\text{”}$$

in place a_3, and they do not obtain any characteristic in the other output places, where $!F$ denotes that fact F is valid, while $\neg!F$ denotes that it is not valid.

$$Z_2 = \langle\{a_2, a_4, a_9, b_2, c_2\}, \{a_4, a_5, a_6, a_7, b_4, b_5, c_4, c_5\}, r_2\rangle,$$

where

$$r_2 = \begin{array}{c|cccccccc} & a_4 & a_5 & a_6 & a_7 & b_4 & b_5 & c_4 & c_5 \\ \hline a_2 & r_{a_2,a_4} & r_{a_2,a_5} & r_{a_2,a_6} & false & false & false & false & false \\ a_4 & r_{a_4,a_4} & r_{a_4,a_5} & false & r_{a_4,a_7} & false & false & false & false \\ a_9 & r_{a_9,a_4} & r_{a_9,a_5} & r_{a_9,a_6} & r_{a_9,a_7} & false & false & false & false \\ b_2 & false & false & false & false & r_{b_2,b_4} & r_{b_2,b_5} & false & false \\ c_2 & false & false & false & false & false & false & r_{c_2,c_4} & r_{c_2,c_5} \end{array}$$

and
$r_{a_2,a_4} = r_{a_4,a_4} = r_{a_9,a_4} =$ "$\underline{numb}(pr_1 x_0^\gamma, pr_2 x_{last}^\alpha) > \underline{nc}(a_4, \alpha) + 1$"& $\neg r_{a_4,a_7}$,
$r_{a_2,a_5} = r_{a_4,a_5} = r_{a_9,a_5} = r_{b_2,b_4} = r_{c_2,c_4} =$ "$\underline{numb}(pr_1 x_0^\gamma, pr_2 x_{last}^\alpha) > \underline{nc}(a_4, \alpha)$" & $\neg r_{a_4,a_7}$,
$r_{a_2,a_6} = r_{a_9,a_6} = r_{b_2,b_5} = r_{c_2,c_5} =$ "$\underline{numb}(pr_1 x_0^\gamma, pr_2 x_{last}^\alpha) = 0$" & $\neg r_{a_4,a_7}$,
$r_{a_4,a_7} = r_{a_9,a_7} =$ "the current α-token has $\underline{pv}(\underline{pv}(pr_1 x_{last}^\alpha))$-kin in at least one of the places a_{11}, a_{14} or a_{15}",
where the functions $\underline{numb}, \underline{nc}$ and $\underline{pv}$ mean the following:

- $\underline{numb}(Y, y)$ is the number of the occurrences of the element y in the ordered set Y,
- $\underline{nc}(l, \alpha)$ is the number of cycles of the token α in place l,
- $\underline{pv}(\langle s_1, s_2, ..., s_{k-1}, s_k \rangle) = \langle s_1, s_2, ..., s_{k-1} \rangle$.

The tokens obtain the characteristics

$$\text{“}\langle\langle pr_1 x_{last}^\alpha; \underline{nc}(l_4, \alpha) + 1)\rangle, \{A_1, ..., A_i | \langle pr_2 x_{last}^\alpha, \{A_1, ..., A_i\}\rangle \in x_0^\gamma \text{ is}$$

$$\text{appearences for a } \underline{nc}(l_4, \alpha) + 1) - \text{step}\} - x_{last}^\beta\rangle\text{”}$$

in place a_5 and

$$\text{“}\langle pr_1 x_{last}^\alpha, \neg ! pr_2 x_{last}^\alpha\rangle\text{”}$$

in place a_6, and they do not obtain any characteristic in the other output places.

We must note that the output place priorities must satisfy the following inequality:

$$\pi_L(a_7) > \pi_L(a_6) > \pi_L(a_5) > \pi_L(a_4).$$

$$Z_3 = \langle \{a_5, a_8\}, \{a_8, a_9, a_{10}, a_{11}\}, r_3 \rangle\rangle,$$

where

$$r_3 = \begin{array}{c|cccc} & a_8 & a_9 & a_{10} & a_{11} \\ \hline a_5 & r_{a_5,a_8} & r_{a_5,a_9} & r_{a_5,a_{10}} & r_{a_5,a_{11}} \\ a_8 & r_{a_8,a_8} & r_{a_8,a_9} & r_{a_8,a_{10}} & false \end{array}$$

where
$r_{a_5,a_8} = r_{a_8,a_8} =$ "$\underline{card}(pr_2 x_{last}^\alpha) > \underline{nc}(a_8, \alpha) + 1$"&$\neg r(a_5, a_{10})$,
$r_{a_5,a_9} = r_{a_8,a_9} =$ "$\underline{card}(pr_2 x_{last}^\alpha) > \underline{nc}(a_8, \alpha)$"&$\neg r_{a_5,a_{10}}$,
$r_{a_5,a_{10}} = r_{a_8,a_{10}} =$ "in the places a_6, a_{12} or a_{13} resides a token which is a last-kin of the token with the highest priority" ,
$r_{a_5,a_{11}} =$ "$\underline{card}(x_{last}^\alpha) = 0$"&$\neg r_{a_5,a_{10}}$.

The tokens do not obtain any characteristic in places a_8 and a_{10} and they obtain the characteristics

$$\text{“}\langle\langle pr_1 x_{last}^\alpha; \underline{nc}(a_8, \alpha) + 1)\rangle, (\underline{nc}(a_8, \alpha) + 1)\text{-th element of the set } pr_2 x_{last}^\alpha\rangle\text{”}$$

in place a_9 and

$$\text{“}\langle pr_1 x_{last}^\alpha, ! pr_2 x_{last}^\alpha\rangle\text{”}$$

in place a_{11}.

We must note that the output place priorities must satisfy the following inequality:

$$\pi_L(a_{11}) > \pi_L(a_{10}) > \pi_L(a_9) > \pi_L(a_8).$$

$$Z_4 = \langle\{a_6\}, \{a_{12}, a_{13}\}, r_4\rangle,$$

where

$$r_4 = \begin{array}{c|cc} & a_{12} & a_{13} \\ \hline a_6 & r_{a_6,a_{12}} & r_{a_6,a_{13}} \end{array}$$

where
$r_{a_6,a_{12}} =$ "there are tokens outside places a_{14}, a_{15},..., a_{18}",
$r_{a_6,a_{13}} = \neg r_{a_6,a_{12}}$.

The tokens do not obtain any characteristic in place a_{12}. They obtain the characteristic

$$\text{“}\langle pr_a x^{\alpha}_{last}, \neg!pr_2 x^{\alpha}_0\rangle\text{”}$$

in place a_{13}.

$$Z_5 = \langle\{a_{11}, a_{14}, a_{17}, b_4, b_7\}, \{a_{14}, a_{15}, a_{16}, b_6, b_7\}, r_5\rangle,$$

where

$$r_5 = \begin{array}{c|ccccc} & a_{14} & a_{15} & a_{16} & b_6 & b_7 \\ \hline a_{11} & r_{a_{11},a_{14}} & r_{a_{11},a_{15}} & r_{a_{11},a_{16}} & false & false \\ a_{14} & r_{a_{14},a_{14}} & r_{a_{14},a_{15}} & r_{a_{14},a_{16}} & false & false \\ a_{17} & r_{a_{17},a_{14}} & r_{a_{17},a_{15}} & r_{a_{17},a_{16}} & false & false \\ b_4 & false & false & false & r_{b_4,b_6} & r_{b_4,b_7} \\ b_7 & false & false & false & r_{b_7,b_6} & r_{b_7,b_7} \end{array}$$

where
$r_{a_{11},a_{14}} = r_{a_{14},a_{14}} = r_{a_{17},a_{14}} =$ "all last homogeneous kins of the token are in places $a_4, a_5, a_8, a_9, a_{11}$ or a_{14} and there are no last homogeneous kins in places a_6, a_{12} or a_{13}",
$r_{a_{11},a_{15}} = r_{a_{14},a_{15}} = r_{a_{17},a_{15}} =$"the token does not have last homogeneous kins",
$r_{a_{11},a_{16}} = r_{a_{14},a_{16}} = r_{a_{17},a_{16}} =$"the token has last homogeneous kins in places a_6, a_{12} or a_{13}",
$r_{b_4,b_6} = r_{b_7,b_6} =$ "all interior a-places, with the possible exception of places a_{15} and a_{17} are empty",
$r_{b_4,b_7} = r_{b_7,b_7} = \neg r_{b_4,b_6}$.

All last kins merge in place a_{14} and the resulting token obtains no characteristic; the tokens obtain the characteristics

$$\text{“}\langle \underline{pv}(\underline{pv}(pr_1 x^{\alpha}_{last})), !pr_2 x^{\alpha}_{last-2})\rangle\text{”}$$

in place a_{15} and

$$\text{“}x^{\beta}_{last} \cup \{pr_2 x^{\alpha}_{last-2}\}\text{”}$$

in place b_7 and they do not obtain any characteristic in places a_{16} and b_6.

We must note that the β-token obtains the above mentioned characteristic in place b_7 which symbolises that the new (local) fact is added to the DB, only if this extension of the DB is possible. Otherwise, the β-token will not obtain any characteristic in place b_7.

$$Z_6 = \langle\{a_{15}, b_6, c_4\}, \{a_{17}, a_{18}, b_8, b_9, c_6, c_7\}, r_6\rangle,$$

where

$$r_6 = \begin{array}{c|cccccc} & a_{17} & a_{18} & b_8 & b_9 & c_6 & c_7 \\ \hline a_{15} & r_{a_{15},a_{17}} & r_{a_{15},a_{18}} & false & false & false & false \\ b_6 & false & false & r_{b_6,b_8} & r_{b_6,b_9} & false & false \\ c_4 & false & false & false & false & r_{c_4,c_6} & r_{c_4,c_7} \end{array}$$

where
$r_{a_{15},a_{17}} = r_{b_6,b_9} = r_{c_4,c_7} = \neg r_{a_{15},a_{18}}$,
$r_{a_{15},a_{18}} = r_{b_6,b_8} = r_{c_4,c_6} =$ "$pr_1 x^{\alpha}_{last} = pr_1 x^{\alpha}_0$".

The tokens do not obtain any characteristic in places $a_{17}, b_8, b_9, c_6, c_7$ and they obtain the characteristic

$$\text{“}\langle x^{\alpha}_{last}, !pr_2 x^{\alpha}_0\rangle\text{”}$$

in place a_{18}.

The GN described here has the following universal property: it does not depend on the particular modelled production system. The only constraint posed on it is the above-mentioned conditon concerning the type of the rule – namely, that the members of the antecedents of the ES-rules must be conjunctions of positive variables.

3 A new GN model

The new GN, that we will construct, describes a (hypothetic) ES that can check the validity of hypotheses, having the form of Boolean expressions. The new GN includes as a subnet the above described GN, that we will denote by E_3.

Let, as above, a token α enter place p_1 of the new GN with an initial characteristic

$$x^{\alpha}_0 = \text{“}\langle p, H\rangle\text{”},$$

where p is the current number of the α-token which enters place p_1 and H is a hypothesis, that has the form of Boolean expressions.

The new GN (see Fig. 2) has two new transitions, that we will describe below.

$$T_1 = \langle\{p_1, p_3\}, \{a_1, p_2, p_3\}, s_1)\rangle,$$

where

$$s_1 = \begin{array}{c|ccc} & a_1 & p_2 & p_3 \\ \hline p_1 & false & true & true \\ p_4 & true & false & W_{4,3} \end{array},$$

where
$W_{4,3} =$ "the current part of the expression contains at least one variable".

Token α from place p_1 splits to two tokens: α, that enters place p_3 with a characteristic

"list of the variables of the expression"

and ω, that enters place p_2 without a new characteristic.

Token α from place p_3 also splits to two tokens: α and ε. Token α enters place a_1 with a characteristic

"q-th member of the variables list of the expression", where q is the current number of the α-token which enters place a_1 and as we mentioned in the beginning, the q-th member of the list is a hypothesis, that the GN E_3 will check.

Token ε enters place p_3 with the initial α-characteristic, i.e, the original expression.

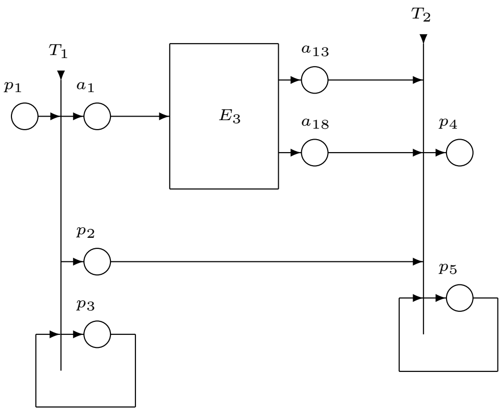

Fig. 2.

$$T_2 = \langle \{a_{13}, a_{18}, p_2, p_5\}, \{p_4, p_5\}, s_2\rangle,$$

where

$$s_2 = \begin{array}{c|cc} & p_4 & p_5 \\ \hline a_{13} & false & true \\ a_{18} & false & true \\ p_2 & false & true \\ p_5 & W_{5,4} & W_{5,5} \end{array}$$

where
$W_{5,4}$ = "all places of GN E_3 are empty",
$W_{5,5} = \neg W_{5,4}$.

Token ε from place p_2 enters place p_5 without any characteristic. This token will unite with each α token trom place a_{13} or a_{18} and on the separate steps it will obtain as a characteristic the final characteristic of the α-token. So, when the truth-values of all hypotheses, represented as α-tokens, are checked, token ε will collect all their values and after this predicate $W_{5,4}$ will obtain truth-value *true*. Then, token ε will enter place p_4 with final characteristic

"truth-value of the initial (complex) hypothesis".

We will illustrate the work of the present GN-model with the following very simple example.

Let the hypothesis have the above mentioned form $A\&((B \to (C \vee D)) \to \neg E)$.

Let the ES's DB be $\{B, F, G\}$ and the ES's KB contain rules (written in logical form):

...

$$R_u : B\&G \to C$$

...

$$R_v : C\&F \to D$$

...

$$R_w : P\&B \to E$$

...

$$R_x : C\&D \to \neg A$$

...

Let no consequent be equal to $\neg E$.

The whole GN will start work when a token α enters with the above characteristic. It will enter place p_3 and after this it will generate five tokens $\alpha_1, \alpha_2, ..., \alpha_5$ that will have, respectively, characteristics $A, B, C, D, \neg E$. The GN E_3 will calculate the truth-values of these hypotheses and will determine for them that:

- hypothesis A is not valid, i.e., it has truth-value $false$;
- hypothesis B is valid, because it is a fact from the DB, i.e., its truth-value is $true$;
- hypotheses C and D are valid, because their validity follows from the validity of DB facts and their truth-value is $true$;
- hypothesis $\neg E$ is not valid, i.e., it has truth-value $false$, because it does not follow from the KB-rules.

Token ε will obtain these values in place p_5 and after this in place p_4 it will obtain as a characteristic the expression

$$false \ \& \ ((true \to (true \vee true)) \to false)$$

that has a final truth-value $false$.

On the other hand, if the initial (complex) hypothesis was $\neg A \vee ((B \to (C\&D)) \to \neg E)$ then token ε had to finish with the characteristic $true$.

4 Conclusion

The constructed here GN-model illustrates the possibilities of the generalized nets apparatus to describe the functioning and the results of the work of expert systems. In a next research we will do the next step, complicating the form of the initial hypotheses. So, the new generalized nets will illustrate more adequately the possibility for describing data mining processes by generalized nets tools.

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