Notes on Intuitionistic Fuzzy Sets
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Properties of fuzzy chromatic numbers in intuitionistic fuzzy graphs

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Abstract: The theory of fuzzy coloring is analyzed with its properties in consideration of the Intuitionistic Fuzzy Graphs (IFGs) nature. The fuzzy chromatic number obtained by applying fuzzy coloring technique to the vertices and edges of IFGs are examined.
Keywords: Fuzzy coloring, Strong fuzzy colors, Fuzzy chromatic number, Intuitionistic fuzzy graphs.
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1 Introduction

A graph is a combination of vertices and edges. Coloring of the vertices and edges of a graph is known as graph coloring. Coloring applies in many fields like telecommunications, networking, etc. In 1852, Francis Guthrie [7], came up with the idea of coloring conjecture. Then, in the mid 1960's, Behzad [4] and Vizing [22] independently introduced the concept of total coloring. Fuzzy graphs was introduced by A. Rosenfeld in the year 1975 [19] and coloring to these fuzzy graphs was invented by Changiz Eslachi and B. N. Onagh in 2006 [8]. In the year 2009, S. Lavanya and R. Sattanathan invented fuzzy total coloring of fuzzy graphs [13]. The introduction of Intuitionistic Fuzzy Sets (IFSs) was by K. T. Atanassov in 1983 [1] (see also [2], [3]) and then Intuitionistic Fuzzy Graphs (IFGs) were invented by A. Shannon and K. T. Atanassov in the year 1994 [21]. Their properties were examined by R. Parvathi and M. G. Karunambigai in 2006 [16] and coloring to these IFGs was introduced by Prasanna *et al.* in 2017 [17]. Earlier, in 2015, a new coloring procedure named "fuzzy coloring" was introduced by Samanta *et al.* [20] and it was applied to fuzzy graphs. This motivated R. Buvaneswari and P. Revathy [5] in the year 2023, to apply the concept of fuzzy coloring to IFGs. The fuzzy coloring properties when applied to IFGs are discussed in this paper.

2 Preliminaries

Definition 2.1. [21] $\hat{G} = (V, E)$ is an IFG, where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_i, \nu_i : V \to [0, 1]$ notate levels of membership and non-membership of an element $\mu_i, \nu_i \in V$ and $0 \leq \mu_i + \nu_i \leq 1$, for all $v_i \in V$ $(i = 1, 2, \ldots, n)$.

(ii) $E \subseteq V \times V$, where $\mu_{ij} : V \times V \to [0, 1]$ and $\nu_{ij} : V \times V \to [0, 1]$ such that $\mu_{ij} \leq \min(\mu_i, \mu_j)$, $v_{ij} \leq \max(v_i, v_j)$ and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, for all $(v_i, v_j) \in E$.

Here, the triplets $\langle v_i, \mu_i, \nu_i \rangle$ and $\langle e_{ij}, \mu_{ij}, \nu_{ij} \rangle$ notate the degrees of membership and non-membership of the vertices v_i and the edge relations $e_{ij} = (v_i, v_j) \in V \times V$ of an IFG.

Note 2.1. [10]

(1) When $\mu_{ij} = v_{ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise, there exists an edge between v_i and v_j.

(2) If one of the inequalities in (i) or (ii) is not satisfied, then $\hat{G}$ is not an IFG.

Definition 2.2. [5] An IFG with no edges is called as a null IFG.

Definition 2.3. [5] An IFG with only a single vertex is called as a trivial IFG.

Definition 2.4. [10] If $\mu_{ij} = \min(\mu_i, \mu_j), \nu_{ij} = \max(\nu_i, \nu_j)$ for every $\nu_i, \nu_j \in E$, then an IFG is known as a strong IFG.

Definition 2.5. [5] Existence of a path amidst all pairs of vertices in an IFG is known as connected IFG.

Definition 2.6. [10] An IFG $\hat{G} = (V, E)$ is a complete IFG, if $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\nu_{ij} = \max(\nu_i, \nu_j)$ for every $\nu_i, \nu_j \in V$.

Definition 2.7. [5] An IFG which has vertices $v_1, \ldots, v_n$ and also $n \geq 3$, i.e., there should be minimum three vertices and three edges to form a cycle. This shows that two edges will eventually create a cycle starting at a vertex which implies a cycle IFG.

Definition 2.8. [11] For all feasible paths in an IFG between v_i and v_j, if $v_i, v_j \in V \subseteq G$, the $\mu-$ *and* $\nu-$*strengths of connectedness* is given by $CONN_{\mu(G)}(v_i,\ v_j) = \max\{S_\mu\}$ and $CONN_{\nu(G)}(v_i, v_j) = \min\{S_\nu\}$.

Definition 2.9. [11] If $\mu_{ij} \geq CONN_{\mu(G)}(v_i, v_j)$ and $\nu_{ij} \leq CONN_{\nu(G)}(v_i, v_j)$ for all $v_i, v_j \in V$, then an arc (v_i, v_j) is known as a *strong arc* of an IFG.

Note 2.2. [12] *Let* $\hat{G} = (V, E)$ *be a connected and strong IFG. Then every edge* $e_{ij} \in E$ *in* G *is strong.*

Definition 2.10. [11] If $\mu_{ij} < CONN_{\mu(G)}(v_i, v_j)$ and $\nu_{ij} > CONN_{\nu(G)}(v_i, v_j)$ for all $v_i, v_j \in V$, then an arc (v_i, v_j) is known as a *weak arc* of an IFG.

Definition 2.11. [11] If $\mu_{ij} > CONN_{\mu(G)}(v_i,\ v_j)$ and $\nu_{ij} < CONN_{\nu(G)}(v_i, v_j)$, then an arc (v_i, v_j) is called as α-*strong arc* in an IFG.

Definition 2.12. [11] If $\mu_{ij} = CONN_{\mu(G)}(v_i,\ v_j)$ and $\nu_{ij} = CONN_{\nu(G)}(v_i,\ v_j)$, then an arc (v_i, v_j) is called as β-*strong arc* in an IFG.

Definition 2.13. [11] If $\mu_{ij} < CONN_{\mu(G)}(v_i,\ v_j)$ and $\nu_{ij} > CONN_{\nu(G)}(v_i,\ v_j)$, then an arc (v_i, v_j) is called as δ-*weak arc* in an IFG.

Definition 2.14. [20] The distinct colors stated by $\hat{C} = \{c_1, c_2, \ldots, c_n\}, n \geq 1$. A fuzzy set $(\hat{C}, f)$, a fuzzy color set $f : \hat{C} \to [0, 1]$, with $f(c_i)$, the membership value of color c_i, quantity of color used per unit of quality blend. Tuning of a fuzzy color $\hat{c}_i = (c_i, f(c_i))$ gives the distinct fuzzy color c_i with various solidaties. A fuzzy color has a membership value 1.

Note 2.3. $(C_k, j), j = 0, 0.1, \ldots, 1$ *denotes the distinct fuzzy colors used to color the vertices and edges with the solidity (depth)* j*, whose membership values of the color always lies between* 0 *and* 1*, depending on the amount of the white mixture done with the distinct fuzzy color.*

Definition 2.15. [5] Let $\hat{G} = (V, E)$ be an IFG. The ordered pair $I[\chi_F(\hat{G})] = \{x, (C_k, 1)\}$ is said to be fuzzy vertex chromatic index, where x is the fuzzy vertex chromatic number denoting the least number of distinct fuzzy colors needed in coloring the vertices of an IFG $\hat{G}$.

Definition 2.16. [5] Let $\hat{G} = (V, E)$ be an IFG. The ordered pair $I[\chi'_F] = \{x', (C_k, 1)\}$ is said to be fuzzy edge chromatic index, where x' is the fuzzy edge chromatic number denoting the least number of distinct fuzzy colors needed in coloring the edges of an IFG $\hat{G}$.

Definition 2.17. [5] Let $\hat{G} = (V, E)$ be an IFG. The ordered pair $I[\chi^T_F] = \{x^T, (C_k, 1)\}$ is said to be total fuzzy chromatic index, where x^T is the total fuzzy chromatic number denoting the minimal number of distinct fuzzy colors needed in coloring the vertices and edges of an IFG $\hat{G}$.

3 Fuzzy coloring in IFGs

Different fuzzy colors are taken into consideration for an IFG to be fuzzy colored. Any two colors can usually be combined to create a new color. However, white color when combined with any distinct color, the color remains unchanged, but the reliability (solidity) of the color reduces. This color obtained is known as "fuzzy color". No two adjacent vertices [15] or edges of a graph, fuzzy graph, or an IFG can typically have the same color when applied to them. Since it is forbidden to color two adjacent vertices with the same color, by decreasing the solidity of the color, the adjacent vertices can be colored with the fuzzy colors formed from a single distinct color, in the event that there are no edges or weak edges connecting them. This coloring technique is termed as "Fuzzy Coloring". Considering a distinct color, C_k and $\omega(\leq 1)$ color units C_k is mixed with $1-\omega$ units of the color white, that blend is said as a quality blend of the color C_k and that consequent color obtained is the fuzzy color of the color C_k with a membership value ω.

Fuzzy color can be continued more in number with a single distinct color and different levels of white mix level giving outputs as different mild shades of the considered color. In case, if red is the color chosen, a "fuzzy red"color is formed by blending the color red of 0.9 units with color white of 0.1 unit. The "fuzzy red"color is denoted as "R"and so, it also can be denoted as $(R, 0.9)$, where R denotes the color red. In addition, another fuzzy red color $(R, 0.8)$ is created by blending a color red of 0.8 units and a color white of 0.2 units, and, $(R, 1)$ represents the fuzzy red color with its maximum solidity. This coloring can be notated as $(R, 0.a)$, $(R, 0.b)$, $(R, 0.c)$, and so on.

The fuzzy coloring to an IFG, is done in consideration of the edges [14] with the cases of all strong edges, with some of the strong edges and no edges or with weak edges.

Case 1: Fuzzy coloring of IFGs with all strong edges

Fuzzy coloring of an IFG with all strong edges is the same as coloring of both the crisp graph and fuzzy graph. It's two vertices are to be colored by two different fuzzy colors with its maximum solidity. For example, $(G, 1)$ and $(R, 1)$.

Case 2: Fuzzy coloring of IFGs with some strong edges

In an IFG with some strong edges, choose a vertex V and color with a strong distinct fuzzy color with membership value 1, say, $(C, 1)$, and its adjacent vertices with no strong edge are colored with its fuzzy colors C_k by diminishing its reliability (solidity). And once again, if a strong edge occurs in any of the connections between the two vertices, choose another fuzzy color with membership value 1 and repeat the procedure.

Case 3: Fuzzy coloring of IFGs with no edges or weak edges

In an IFG, where no edge connecting any two vertices exists, choose any vertex V and shade with a strong distinct fuzzy color, say, $(C, 1)$, and its adjacent vertices are to be colored with fuzzy colors C_K, say, $(C, 0.a)$ and so on.

3.1 Fuzzy chromatic number by fuzzy coloring in IFGs

Let $\hat{G}$ be an IFG. The vertex chromatic number of $\hat{G}$ [18] is the least number of colors required to color the vertex notated as $\chi(\hat{G})$ and the least number of fuzzy colors used in vertex coloring gives fuzzy vertex chromatic number notated by $\chi_F(\hat{G})$. The edge chromatic number of $\hat{G}$ is the least number of colors used in edge coloring and is notated by $\chi'(\hat{G})$, and the least number of fuzzy colors used in edge coloring gives the fuzzy edge chromatic number denoted by $\chi'_F(\hat{G})$, and total chromatic number of $\hat{G}$ is the least number of colors opted in both the vertex and edge coloring notated by $\chi^T(\hat{G})$ and the least fuzzy colors used in vertex and edge coloring gives the total fuzzy chromatic number by $\chi^T_F(\hat{G})$. Based on number of vertices and edges of $\hat{G}$, $\chi_F(\hat{G})$, $\chi'_F(\hat{G})$ and $\chi^T_F(\hat{G})$ are calculated.

Theorem 3.1. *If $\hat{G}$ is a strong IFG such that its underlying crisp graph is a path P_n of length n, then $\chi_F(\hat{G}) = \chi^S_F(\hat{G})$.*

Proof. Let $\hat{G}$ be a strong IFG with a path of length n. The adjacent vertices of an IFG are colored by two distinct fuzzy colors, say $(R, 1)$ and $(G, 1)$ and here the colors would be definitely strong fuzzy colors with a maximum membership value 1 due to the strong edge. Here, $\chi_F(\hat{G})$ and $\chi^S_F(\hat{G})$ denote the fuzzy chromatic number and the strong fuzzy chromatic number. Therefore, $\chi_F(\hat{G}) = \chi^S_F(\hat{G})$. □

Corollary 3.1. *If $\hat{G}$ is an IFG with that its underlying crisp graph is a path P_n of length n, then $\chi_F(\hat{G}) = 2$.*

Theorem 3.2. *The fuzzy edge chromatic number of the complement of a complete IFG is* 0*.*

Proof. Let $\hat{G}$ be a complete IFG. An IFG $\hat{G} = \big((\mu_i, \nu_i), (\mu_{ij}, \nu_{ij})\big)$ is called complete IFG, when, $\mu_{ij} = \mu_i \wedge \mu_j$ and $\nu_{ij} = \nu_i \vee \nu_j$, for any $i, j \in V, i \neq j$. The complement of an IFG, $\hat{G} = (V, E)$ is an IFG, $\hat{G} = \big((\overline{\mu_i, \nu_i}), (\overline{\mu_{ij}, \nu_{ij}})\big)$, where $\overline{\mu_{ij}} = \mu_i \wedge \mu_j - \mu_{ij}$ and $\overline{\nu_{ij}} = \nu_i \vee \nu_j - \nu_{ij}$, for any $i, j \in E$ [23].

$$\overline{\mu_{ij}} = \mu_i \wedge \mu_j - \mu_{ij}, \quad \overline{\mu_{ij}} = \mu_i \wedge \mu_j - \mu_i \wedge \mu_j, \quad \overline{\mu_{ij}} = 0;$$
$$\overline{\nu_{ij}} = \nu_i \vee \nu_j - \nu_{ij}, \quad \overline{\nu_{ij}} = \nu_i \vee \nu_j - \nu_i \vee \nu_j, \quad \overline{\nu_{ij}} = 0.$$

Thus, the complement of the complete IFG $\hat{G}$ does not have any edge.
Hence, ${\chi_F}'(\hat{G}) = 0$. □

Theorem 3.3. *In a null or trivial IFG $\hat{G}$, $\chi_F(\hat{G}) > \chi^S_F(\hat{G})$.*

Proof. The vertices of a null IFG can be colored with fuzzy colors obtained from a single distinct color. Therefore, $\chi_F(\hat{G}) > \chi^s_F(\hat{G})$. □

Example 3.1. Consider a null IFG, $\hat{G} = (V, E)$ with the vertices $V = \{v_1, v_2\}$.
From Figure 3.1, it is clear that $\chi_F(\hat{G}) = 2$ and $\chi^s_F(\hat{G}) = 1$. Therefore, $\chi_F(\hat{G}) > \chi^s_F(\hat{G})$.

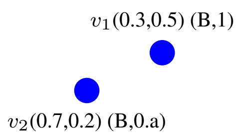

Figure 3.1

Theorem 3.4. *If $\hat{G}$ is a trivial IFG, then $\chi_F(\hat{G}) = \chi_F^S(\hat{G})$.*

Proof. The single vertex of a trivial IFG can be fuzzy colored with the maximum solidity 1. Therefore, $\chi_F(\hat{G}) = \chi_F^s(\hat{G})$. □

Example 3.2. Consider a trivial IFG, $\hat{G} = (V, E)$ with a vertex v_1.
From Figure 3.2, it is clear that $\chi_F(\hat{G}) = \chi_F^S(\hat{G}) = 1$. Therefore, $\chi_F(\hat{G}) = \chi_F^s(\hat{G})$.

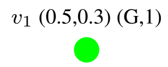

Figure 3.2

Note 3.1. [9] *$\chi'_F(\hat{G}) = 0$ in both null or trivial IFGs, as there are no edges in it.*

Theorem 3.5. *If $\hat{G}$ is an IFG, then, $\chi_F(\hat{G}) = (N, (C_k, W)) \leq n$, such that $0 < W \leq 1$.*

Proof. Let $\hat{G} = (V, E)$ be an IFG. Let n denote the number of vertices or edges. Let N denote the number of fuzzy colors. Let W denote the weights (solidities) of the fuzzy colors. If the vertices and edges of $\hat{G}$ are fuzzy colored, then the distinct fuzzy colors used to color the vertices or edges of an IFG are less than the number of vertices or edges. This implies, the fuzzy chromatic number is less than or equal to its vertices and edges of $\hat{G}$.
Therefore, $\chi_F(\hat{G}) = (N, (C_k, W)) \leq n$ such that $0 < W \leq 1$. □

Lemma 3.1. *$\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$ is the strong fuzzy chromatic number of an IFG which is complete such that W=1.*

Proof. Let $\hat{G} = (V, E)$ be a complete IFG, then $\mu_{ij} \leq \mu_i \wedge \mu_j$ and $\nu_{ij} \leq \nu_i \vee \nu_j$ for any $i, j \in V, i \neq j$. Let N_s denote the number of strong fuzzy colors. Let C_k denote the fuzzy color used. Let W denote the weights (solidities) of the fuzzy colors used. We know that $\frac{\mu_{ij}}{\mu_i \wedge \mu_j} = 1$ and $\frac{\nu_{ij}}{\nu_i \wedge \nu_j} = 1$.

Since the IFG is complete, N distinct fuzzy colors are required if there are n vertices. The membership value of each fuzzy color is 1. The weights (solidities) of the distinct fuzzy colors used at each vertices and edges will be $\underbrace{1 + 1 + \cdots}_{N \text{ times}} = N$. Thus, $\chi_F = (N_s, (C_k, W)_s)$ is the fuzzy chromatic number of a complete IFG such that $W = 1$. □

Note 3.2. *A complete IFG with α-strong and β-strong arcs can be both fuzzy colored and strongly fuzzy colored, i.e., $W \leq 1$.*

Note 3.3. [11] *A complete IFG contains no δ-weak arcs.*

Theorem 3.6. *Let $\hat{G}$ be a strong IFG with the fuzzy chromatic number $\chi_F(\hat{G}) = (N, (C_k, W))$ and the strong fuzzy chromatic number is $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$, then $N \leq N_s$ and $\sum(C_k, W) \geq \sum(C_k, W)_s$.*

Proof. Let $\hat{G} = (V, E)$ be an IFG. Let N and N_s denote the number of fuzzy colors and strong fuzzy colors. $\sum(C_k, W)$ is the summation of all the membership values of fuzzy colors obtained from a single distinct color and $\sum(C_k, W)_s$ is the summation of all the maximum membership values of all strong fuzzy colors.

Case 1: If all the edges are strong, then the fuzzy chromatic number and strong fuzzy chromatic number of $\hat{G}$ will be the same, i.e., $N_s = N$ and $\sum(C_k, W)_s = \sum(C_k, W)$.

Case 2: When some of the edges are strong in an IFG, then the IFG can be both fuzzy colored and strongly fuzzy colored. There exists some of the strong fuzzy colors and some with reduced solidity. The strong edge can be colored by the strong fuzzy color with the membership value 1 and the remaining edges by solidity reduced fuzzy color with membership value of fuzzy color less than 1, i.e., $N < N_s$ and $\sum(C_k, W) > \sum(C_k, W)_s$.

Case 3: When there are weak edges or no edges, then any edge is to be strongly fuzzy colored with membership value 1 and the remaining edges by solidity reduced fuzzy colors with membership value of the fuzzy color less than 1. Therefore, $N_s = 1$ and $N < N_s$ is obvious and also $\sum(C_k, W) > \sum(C_k, W)_s$.

From the three cases, we get $N \leq N_s$ and $\sum(C_k, W) \geq \sum(C_k, W)_s$. □

Lemma 3.2. *Let $\hat{G}$ be an IFG with a strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$. Then $\sum(C_k, W)_s \geq 0.5 * N_s$.*

Proof. Let $\hat{G} = (V, E)$ be an IFG with the strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$. The IFG is strongly fuzzy colored by N_s number of strong distinct fuzzy colors. $\sum(C_k, W)_s$ is the summation of all maximum membership values of weights (solidities) of all the strong fuzzy colors and the maximum membership value 1 will be the same for all the distinct strong fuzzy colors. □

Example 3.3. Consider an IFG, $\hat{G} = (V, E)$ with the vertices $V = \{v_1, v_2, v_3, v_4\}$.
From Figure 3.3, we get $\sum(C_k, W)_s = 4$ and $N_s = 4$. Figure 3.3 implies $\sum(C_k, W)_s \geq 0.5 * N_s$.

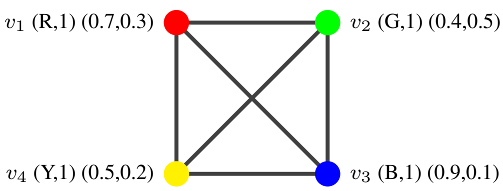

Figure 3.3

Lemma 3.3. *Let $\hat{G}$ be an IFG with a strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$. Then $\frac{N_s}{2} \leq \sum(C_k, W)_s$.*

Example 3.4. Consider an IFG, $\hat{G} = (V, E)$ with the vertices $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. From Figure 3.4, we get $\sum(C_k, W)_s = 3$ and $N_s = 3$. Figure 3.4 illustrates $\frac{N_s}{2} \leq \sum(C_k, W)_s$.

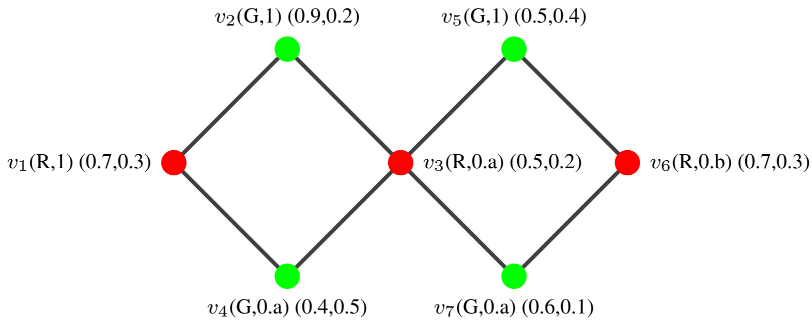

Figure 3.4

Lemma 3.4. *In any IFG $\hat{G}$ other than a strong and complete with a fuzzy chromatic number $\chi_F(\hat{G}) = (N, (C_k, W))$ and a strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$,*

$$\frac{\sum(C_k, W)_s - \sum(C_k, W)}{N_s - N} \geq 0.5$$

Example 3.5. Consider an IFG, $\hat{G} = (V, E)$ other than a strong and complete with the vertices $V = \{v_1, v_2, v_3, v_4\}$.

Let $\hat{G}$ be an IFG with the fuzzy chromatic number $\chi_F(\hat{G}) = (N, (C_k, W))$ and a strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$. We get $N_s \neq N$, i.e., $(N_s - N) \geq 0$.

Also, $(N_s - N)$ is the difference between the number of fuzzy colors and the number of strong fuzzy colors. And $\sum(C_k, W)_s - \sum(C_k, W)$ is the difference between the weights (solidities) of the strong fuzzy colors and fuzzy colors at its vertices and edges.
From Figure 3.5, we get, $\sum(C_k, W)_s = 4$, $\sum(C_k, W) = 0$, $N_s = 4$ and $N = 0$.
Figure 3.5 implies

$$\frac{\sum(C_k, W)_s - \sum(C_k, W)}{N_s - N} \geq 0.5 \, .$$

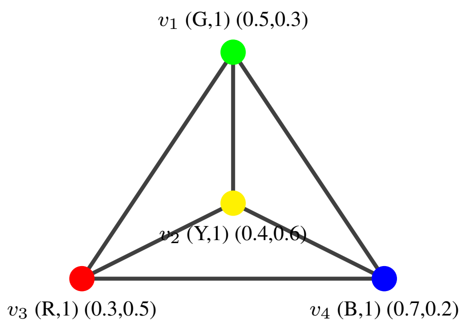

Figure 3.5

Lemma 3.5. *Let $\hat{G}$ be a cycle IFG with a fuzzy chromatic number $\chi_F(\hat{G}) = (N, (C_k, W))$ and a strong fuzzy chromatic number $\chi_F^s(\hat{G}) = (N_s, (C_k, W)_s)$. When the number of edges are even in an IF cycle, it gives $N = 4$ and $N_s = 2$ and when the number of edges are odd, then $N = 2$ and $N_s = 3$.*

Example 3.6. Consider two cycle IFGs, $\hat{G} = (V, E)$ with even and odd number of vertices $V = \{v_1, v_2, \ldots, v_n\}$.

Figure 3.6 gives $N = 4$ and $N_s = 2$.

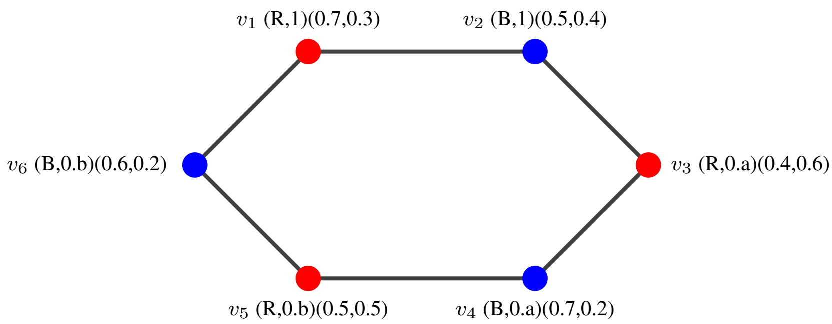

Figure 3.6

Figure 3.7 gives $N = 2$ and $N_s = 3$.

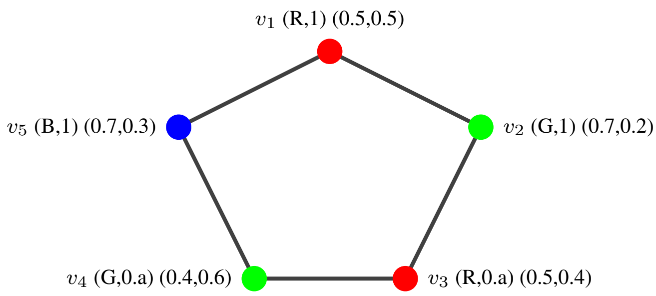

Figure 3.7

Theorem 3.7. *An IFG $\hat{G}$ having α-strong, β-strong and δ- weak arcs can be both fuzzy colored and strongly fuzzy colored.*

Example 3.7. Consider the Figure 3.8 which has α-strong, β-strong and δ-weak arcs. The arcs (v_1, v_4), (v_2, v_3) and (v_4, v_5) are α-strong arcs, (v_1, v_3) and (v_3, v_4) are β-strong arcs, (v_1, v_2) and (v_3, v_5) are δ-weak arcs.

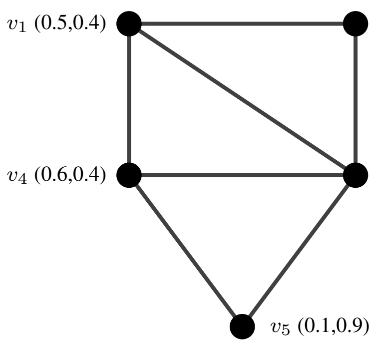

Figure 3.8

Figure 3.9 shows that the α-strong arcs are colored by strong fuzzy colors, β-strong and δ-weak arcs are colored starting with a strong fuzzy color and then with fuzzy colors of reduced solidity. This enables to identify the path of the arcs.

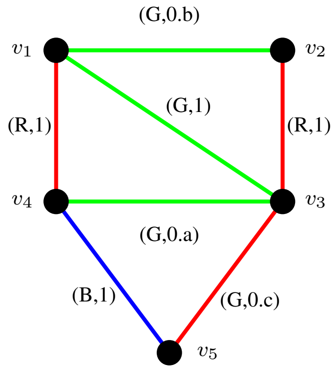

Figure 3.9

Thus, an IFG $\hat{G}$ having α-strong, β-strong and δ-weak arcs can be both fuzzy colored and strongly fuzzy colored.

Remark 3.1. ([6])

(i) An IFG with only α-strong arcs can only be strong fuzzy colored.

(ii) An IFG with either β-strong or δ-weak arcs can be both fuzzy colored and strongly fuzzy colored.

4 Conclusion

The properties of fuzzy coloring applied to IFGs are analysed along with its fuzzy chromatic number and strong fuzzy chromatic number theoretically. Thus, the concept of fuzzy coloring highly depends on the reliability (solidity) of the fuzzy colors applied at its vertices and edges of an IFG. This work can be further extended to IF environment.

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