

Research on intuitionistic fuzzy implications. Part 2

Nora Angelova¹, Krassimir Atanassov²
and Vassia Atanassova³

¹ Faculty of Mathematics and Informatics, Sofia University
5 James Bourchier Blvd., 1164 Sofia, Bulgaria
e-mail: noraa@fmi.uni-sofia.bg

² Dept. of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
e-mail: krat@bas.bg

³ Dept. of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
e-mail: vassia.atanassova@gmail.com

Received: 21 January 2022

Revised: 18 June 2022

Accepted: 23 June 2022

Abstract: Continuing the research from [2], here we give the list of the axioms that are satisfied by the intuitionistic fuzzy implications, introduced in Part 1 of the present research. Given the large number of implications that have already been defined and the naturally arising question about their comparability and usability, we discuss the criteria of usability of implications and outline 44 ones that satisfy at least 28 from the discussed 38 axioms, that is about $1/4$ of all implications that satisfy at least about $3/4$ of all axioms. Finally, we show the relationships between each pair of implications in the form of a graph.

Keywords: Intuitionistic fuzzy implication, Intuitionistic fuzzy pair, Intuitionistic fuzzy set, Logic axiom.

2020 Mathematics Subject Classification: 03E72.

1 Introduction

In Part 1 of the present research [2], giving short remarks on the results related to the intuitionistic fuzzy implications, for the first time in fuzzy sets theory, we introduced the relations between separate implications.

All necessary definitions and notations are given in Part 1, but below, we will repeat a part of them, wherever this is appropriate.

2 Main results

First, we give the list of logic axioms for which all presently defined intuitionistic fuzzy implications will be tested. These axioms (see [4]) are intuitionistic fuzzy adaptations of:

- the axioms of Klir and Yuan [5], except for Axiom A9: \rightarrow is a continuous function,
- modifications of Klir and Yuan's axioms A3, A4, A5, A7 (two forms) and A8 (see [4]),
- the axioms of intuitionistic logic [6],
- the axioms of Kolmogorov (see, e.g. [7]),
- relations between logical constants (from A31 to A38).

Let

$$O^* = \langle 0, 1 \rangle, \quad U^* = \langle 0, 0 \rangle, \quad E^* = \langle 1, 0 \rangle.$$

We use the following axioms:

$$A1 \quad (\forall x, y)((x \leq y) \rightarrow (\forall z)(x \rightarrow z \geq y \rightarrow z)),$$

$$A2 \quad (\forall x, y)((x \leq y) \rightarrow (\forall z)(z \rightarrow x \leq z \rightarrow y)),$$

$$A3 \quad (\forall y)(O^* \rightarrow y = E^*),$$

$$A4 \quad (\forall y)(O^* \rightarrow y \text{ is an IFT}),$$

$$A5 \quad (\forall y)(E^* \rightarrow y = y),$$

$$A6 \quad (\forall y)(E^* \rightarrow y \leq y),$$

$$A7 \quad (\forall x)(x \rightarrow x = E^*),$$

$$A8 \quad (\forall x)(x \rightarrow x \text{ is an IFT}),$$

$$A9 \quad (\forall x, y, z)(x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)),$$

$$A10 \quad (\forall x, y)(x \rightarrow y = 1 \text{ if and only if } x \leq y),$$

$$A11 \quad (\forall x, y)((x \leq y) \rightarrow (x \rightarrow y = E^*)),$$

$$A12 \quad (\forall x, y)(x \rightarrow y = \neg y \rightarrow \neg x),$$

$$A13 \quad (\forall x, y)(x \rightarrow y = \neg\neg(\neg y \rightarrow \neg x)),$$

$$A14 \quad A \rightarrow (B \rightarrow A),$$

$$A15 \quad A \rightarrow (B \rightarrow (A \wedge B)),$$

- A16 $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$,
- A17 $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$,
- A18 $A \rightarrow \neg\neg A$,
- A19 $\neg(A \wedge \neg A)$,
- A20 $(\neg A \vee B) \rightarrow (A \rightarrow B)$,
- A21 $\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$,
- A22 $(\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$,
- A23 $(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)$,
- A24 $(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$,
- A25 $(A \rightarrow \neg B) \rightarrow (B \rightarrow \neg A)$,
- A26 $\neg\neg\neg A \rightarrow \neg A$,
- A27 $\neg A \rightarrow \neg\neg\neg A$,
- A28 $\neg\neg(A \rightarrow B) \rightarrow (A \rightarrow \neg\neg B)$,
- A29 $(C \rightarrow A) \rightarrow ((C \rightarrow (A \rightarrow B)) \rightarrow (C \rightarrow B))$.
- A30 $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$,
- A31 $(B \rightarrow C) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$,
- A32 $(A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$,
- A33 $O^* \rightarrow O^*$,
- A34 $O^* \rightarrow U^*$,
- A35 $O^* \rightarrow E^*$,
- A36 $E^* \rightarrow O^*$,
- A37 $E^* \rightarrow U^*$,
- A38 $E^* \rightarrow E^*$.

The full list of 182 implications that will be a subject of investigation is given in Part 1 of the present research, [2]. To analyse their satisfiability of the 38 axioms listed above, we use the software, described in [1], and the results are given in Table 1, where “+” and “-” denote whether the respective axiom has been satisfied by the respective implication, or not.

Remark. *In the course of the present research, the authors discovered the following pairs (and one triple) of coinciding implications:*

- \rightarrow_{16} and \rightarrow_{181} ,
- \rightarrow_{40} and \rightarrow_{173} ,
- \rightarrow_{41} and \rightarrow_{183} ,
- \rightarrow_{178} and \rightarrow_{188} ,
- \rightarrow_{179} and \rightarrow_{189} ,
- \rightarrow_{180} , \rightarrow_{187} and \rightarrow_{190} .

The difference of some of these implications was determined to be a result of their different lexicographic records, or as a result of misprinted numeration of implications at the moment of publication. The authors propose that in future only the respective first implication from the pair (triple) be used and identified, i.e., \rightarrow_{16} instead of \rightarrow_{181} , \rightarrow_{40} instead of \rightarrow_{173} , and so forth.

Table 1. Relationships between logic axioms and IF implications

	\rightarrow_1	\rightarrow_2	\rightarrow_3	\rightarrow_4	\rightarrow_5	\rightarrow_6	\rightarrow_7	\rightarrow_8	\rightarrow_9	\rightarrow_{10}	\rightarrow_{11}	\rightarrow_{12}	\rightarrow_{13}
1	-	+	+	+	+	-	-	+	-	-	+	+	+
2	+	+	+	+	+	+	-	+	+	+	+	+	+
3	+	+	+	+	+	+	-	+	+	+	+	+	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	+	-	+	+	+	+	+	-	+	+	+	-	+
6	+	+	+	+	+	+	+	+	+	+	+	+	+
7	-	+	+	-	-	-	-	+	-	-	+	-	-
8	+	+	+	+	+	+	+	+	+	-	+	-	+
9	-	-	+	+	+	-	-	-	-	-	+	+	+
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	+	+	-	-	-	-	+	-	-	+	-	-
12	-	-	-	+	+	-	+	-	-	-	-	-	+
13	-	-	-	+	+	-	+	-	-	-	-	-	+
14	+	-	+	+	+	+	+	-	+	-	+	-	+
15	+	-	+	+	+	+	-	-	-	-	+	-	+
16	+	-	+	+	+	+	-	-	+	-	+	+	+
17	+	+	+	+	+	-	-	-	+	-	+	+	+
18	+	+	+	+	+	+	+	+	+	-	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	-	+	+	+	+	-	-	+	-	+	-	+
21	+	+	+	+	+	+	+	+	+	-	+	+	+
22	+	+	+	+	+	+	+	+	+	-	+	+	+
23	+	+	+	+	+	+	+	+	+	-	+	+	+
24	+	+	+	+	+	+	+	+	+	-	+	+	+
25	+	+	+	+	+	+	+	+	+	-	+	+	+
26	+	+	+	+	+	+	+	+	+	-	+	+	+
27	+	+	+	+	+	+	+	+	+	-	+	+	+
28	+	+	+	+	+	+	+	+	+	-	+	+	+
29	+	+	+	+	+	-	-	-	+	-	+	+	+
30	+	+	+	+	+	+	+	+	+	-	+	+	+
31	+	+	+	+	+	-	-	-	+	-	+	+	+
32	+	+	+	+	+	-	+	+	+	-	+	+	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	+	+	+	+	+	+	+	+	-	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{14}	\rightarrow_{15}	\rightarrow_{16}	\rightarrow_{17}	\rightarrow_{18}	\rightarrow_{19}	\rightarrow_{20}	\rightarrow_{21}	\rightarrow_{22}	\rightarrow_{23}	\rightarrow_{24}	\rightarrow_{25}	\rightarrow_{26}
1	+	+	+	-	+	+	+	-	+	+	+	+	+
2	+	+	+	+	+	+	+	-	+	+	+	+	+
3	+	+	+	+	+	+	+	+	+	+	+	+	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	+	-	+	+	+	+	-	-	-	-	-	-	+
6	+	+	+	+	+	+	-	-	-	-	+	+	+
7	+	+	-	-	-	-	+	-	-	+	+	-	-
8	+	+	-	+	+	-	+	+	+	+	+	-	-
9	+	-	+	+	+	+	+	-	+	+	-	+	+
10	+	+	-	-	-	-	-	-	-	-	+	-	-
11	+	+	-	-	-	-	+	-	-	+	+	-	-
12	-	-	-	-	-	-	+	-	+	+	-	-	-
13	-	-	-	-	-	-	+	-	+	+	-	-	-
14	+	-	-	+	+	-	+	+	+	+	-	-	-
15	+	-	-	+	+	-	+	+	+	+	-	-	-
16	+	-	-	+	+	-	+	-	+	+	-	+	-
17	+	-	-	+	+	-	+	+	+	+	+	+	-
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	-	-	+	+	-	+	+	+	+	-	-	-
21	+	+	+	+	+	+	+	+	+	+	+	+	+
22	+	+	+	+	+	+	+	+	+	+	+	+	+
23	+	+	+	+	+	+	+	+	+	+	+	+	+
24	+	+	+	+	+	+	+	+	+	+	+	+	+
25	+	+	+	+	+	+	+	-	+	+	+	+	+
26	+	+	+	+	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	+	+	+	+	+	+	+	+	+	+	+	+	+
29	+	-	-	+	+	-	+	+	+	+	+	+	-
30	+	+	-	+	+	-	+	+	+	+	+	+	-
31	+	-	-	+	+	-	+	+	+	+	+	+	-
32	+	+	+	+	+	+	+	+	+	+	+	+	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	-	+	+	+	+	-	+	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{27}	\rightarrow_{28}	\rightarrow_{29}	\rightarrow_{30}	\rightarrow_{31}	\rightarrow_{32}	\rightarrow_{33}	\rightarrow_{34}	\rightarrow_{35}	\rightarrow_{36}	\rightarrow_{37}	\rightarrow_{38}	\rightarrow_{39}
1	+	+	+	-	+	+	+	+	+	-	+	-	-
2	+	+	+	+	+	+	+	+	+	-	+	+	+
3	+	+	+	+	+	+	+	+	+	-	+	+	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	+	-	-	-	-	-	-	-	-	-	-	-
6	-	+	-	-	-	-	-	-	+	-	-	-	-
7	-	-	-	-	+	+	-	+	-	-	+	-	-
8	+	+	+	+	+	+	+	+	+	+	+	+	-
9	+	+	-	-	+	+	+	+	-	+	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	+	+	-	+	-	-	+	-	-
12	+	-	-	-	-	-	-	-	-	-	-	-	-
13	+	-	-	-	-	-	-	-	-	-	-	-	-
14	+	+	+	+	+	+	+	+	+	+	-	+	-
15	+	+	+	+	+	+	+	+	-	+	-	+	-
16	+	+	+	+	+	+	+	+	+	+	-	+	-
17	+	+	+	-	-	-	-	-	+	-	-	-	-
18	+	+	+	+	+	+	+	+	+	+	+	+	-
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	+	+	+	+	+	+	+	+	+	-	+	-
21	+	+	+	+	+	+	+	+	+	+	+	+	-
22	+	+	+	+	+	+	+	+	+	+	+	+	-
23	+	+	+	+	+	+	+	+	+	+	+	+	-
24	+	+	+	-	-	-	-	-	+	-	-	-	-
25	+	+	+	+	+	+	+	+	+	+	+	+	-
26	+	+	+	+	+	+	+	+	+	+	+	+	-
27	+	+	+	+	+	+	+	+	+	+	+	+	-
28	+	+	+	-	-	-	-	-	+	-	-	-	-
29	+	+	+	-	-	-	-	-	+	-	-	-	-
30	+	+	+	+	+	+	+	+	+	+	+	+	-
31	+	+	+	-	-	-	-	-	-	-	-	-	-
32	+	+	+	-	-	-	-	-	+	-	-	-	-
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	+	+	+	+	+	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{40}	\rightarrow_{41}	\rightarrow_{42}	\rightarrow_{43}	\rightarrow_{44}	\rightarrow_{45}	\rightarrow_{46}	\rightarrow_{47}	\rightarrow_{48}	\rightarrow_{49}	\rightarrow_{50}	\rightarrow_{51}	\rightarrow_{52}
1	+	+	+	+	+	+	-	+	+	+	+	-	+
2	+	+	+	+	+	+	+	+	+	+	+	-	+
3	+	+	+	+	+	+	+	+	+	+	+	-	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	+	+	+	+	+	+	+
7	+	-	+	-	-	-	-	-	-	-	-	-	-
8	+	-	+	+	+	+	-	-	-	-	-	-	-
9	-	+	+	-	-	-	-	-	+	+	+	+	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	+	-	+	-	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	+	+	+	+	-	-	-	-	-	-	-
15	-	-	+	+	+	+	-	-	-	-	-	-	-
16	-	-	+	+	+	+	+	-	+	+	+	+	-
17	-	-	-	-	-	-	+	-	+	+	+	+	-
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	-	-	+	+	+	+	-	-	-	-	-	-	-
21	+	+	+	+	+	+	+	+	+	+	+	+	+
22	+	+	+	+	+	+	+	+	+	+	+	+	+
23	+	+	+	+	+	+	+	+	+	+	+	+	+
24	-	-	-	-	-	-	+	+	+	+	+	+	+
25	+	+	+	+	+	+	+	+	+	+	+	+	+
26	+	+	+	+	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	-	-	-	-	-	-	+	+	+	+	+	+	+
29	-	-	-	-	-	-	+	-	+	+	+	+	-
30	+	-	+	+	+	+	+	+	+	+	+	+	+
31	-	-	-	-	-	-	+	-	+	+	+	+	-
32	-	-	-	-	-	-	+	+	+	+	+	+	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	+	+	+	-	-	-	-	-	-	-
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{53}	\rightarrow_{54}	\rightarrow_{55}	\rightarrow_{56}	\rightarrow_{57}	\rightarrow_{58}	\rightarrow_{59}	\rightarrow_{60}	\rightarrow_{61}	\rightarrow_{62}	\rightarrow_{63}	\rightarrow_{64}	\rightarrow_{65}
1	-	-	+	+	+	+	+	+	+	+	+	+	+
2	+	+	+	+	+	+	+	+	-	+	+	-	+
3	+	+	+	+	+	+	+	+	-	+	+	-	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	-	-	+	-	-	+	-
6	+	+	+	+	-	+	+	-	+	-	-	+	-
7	-	-	-	-	-	-	-	-	-	+	+	-	+
8	-	-	-	-	-	-	-	-	+	+	+	+	+
9	-	-	-	+	+	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	+	+	-	+
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-	+	-	-	+	-
15	-	-	-	-	-	-	-	-	+	-	-	+	-
16	+	-	-	-	+	-	-	-	+	-	-	+	-
17	+	-	+	-	+	-	-	-	+	-	-	-	-
18	+	-	+	+	+	-	-	-	+	+	+	+	+
19	+	+	+	+	+	-	-	-	+	+	+	+	+
20	-	-	-	-	-	-	-	-	+	-	-	+	-
21	+	-	+	+	+	-	-	-	+	+	+	+	+
22	+	-	+	+	+	-	-	-	+	+	+	+	+
23	+	-	+	+	+	-	-	-	+	+	+	+	+
24	+	-	+	+	+	-	-	-	+	+	-	+	+
25	+	-	+	+	+	-	-	-	+	-	-	+	+
26	+	-	+	+	+	-	-	-	+	+	+	+	+
27	+	-	+	+	+	-	-	-	+	+	+	+	+
28	+	-	+	+	+	-	-	-	+	+	+	+	+
29	+	-	+	-	+	-	-	-	+	-	-	-	-
30	+	-	+	-	+	-	-	-	+	-	-	+	+
31	+	-	+	-	+	-	-	-	+	+	-	-	-
32	+	-	+	+	+	-	-	-	+	-	-	-	-
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	-	-	-	-	-	-	-	-	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{66}	\rightarrow_{67}	\rightarrow_{68}	\rightarrow_{69}	\rightarrow_{70}	\rightarrow_{71}	\rightarrow_{72}	\rightarrow_{73}	\rightarrow_{74}	\rightarrow_{75}	\rightarrow_{76}	\rightarrow_{77}	\rightarrow_{78}
1	+	+	+	+	+	+	+	+	+	-	+	+	+
2	-	-	+	+	+	-	+	+	+	-	+	+	+
3	-	+	+	+	+	+	+	+	+	+	+	+	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	+	-	-	-	-	-	-	-	-	-	-	-
6	-	+	-	+	-	-	+	+	-	-	+	+	+
7	-	-	+	+	-	-	-	-	+	-	-	+	-
8	+	-	+	+	-	+	+	-	+	+	+	+	-
9	-	-	-	-	-	-	-	-	+	-	+	+	-
10	-	-	-	+	-	-	-	-	-	-	-	-	-
11	-	-	+	+	-	-	-	-	+	-	-	+	-
12	-	-	-	-	-	-	-	-	+	-	+	+	-
13	-	-	-	-	-	-	-	-	+	-	+	+	-
14	+	-	-	-	-	+	+	-	+	+	+	+	-
15	+	-	-	-	-	+	-	-	+	+	+	+	-
16	+	-	-	-	-	+	+	-	+	+	+	+	-
17	+	-	-	-	-	+	-	-	+	+	+	+	-
18	+	-	+	+	-	+	+	-	+	+	+	+	-
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	-	-	-	-	+	+	-	+	+	+	+	-
21	+	-	+	+	-	+	+	-	+	+	+	+	-
22	+	-	+	+	-	+	+	-	+	+	+	+	-
23	+	-	+	+	-	+	+	-	+	+	+	+	-
24	+	-	-	-	-	+	+	-	+	+	+	+	-
25	+	-	-	-	-	+	+	-	+	+	+	+	-
26	+	-	+	+	-	+	+	-	+	+	+	+	-
27	+	-	+	+	-	+	+	-	+	+	+	+	-
28	+	-	+	+	-	+	+	-	+	+	+	+	-
29	+	-	-	-	-	+	-	-	+	+	+	+	-
30	+	-	-	-	-	+	+	-	+	+	+	+	-
31	+	-	-	-	-	+	-	-	+	+	+	+	-
32	+	-	-	-	-	+	-	-	+	+	+	+	-
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	-	+	+	-	-	+	+	-	-	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{79}	\rightarrow_{80}	\rightarrow_{81}	\rightarrow_{82}	\rightarrow_{83}	\rightarrow_{84}	\rightarrow_{85}	\rightarrow_{86}	\rightarrow_{87}	\rightarrow_{88}	\rightarrow_{89}	\rightarrow_{90}	\rightarrow_{91}
1	+	+	+	+	+	+	+	+	+	+	+	+	+
2	+	+	+	-	+	+	-	-	+	+	+	+	-
3	+	+	+	-	+	+	-	+	+	+	+	+	-
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	-	-	-	-	-	-	-
6	+	+	+	-	-	-	-	-	-	-	-	-	+
7	-	-	-	-	+	+	-	-	-	+	-	-	-
8	+	+	+	+	+	+	+	-	-	+	+	+	-
9	+	-	+	-	-	-	-	-	-	+	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	+	+	-	-	-	+	-	-	-
12	+	-	-	-	-	-	-	-	-	-	-	-	-
13	+	-	-	-	-	-	-	-	-	-	-	-	-
14	+	+	+	+	-	-	+	-	-	+	+	+	-
15	+	-	+	+	-	-	+	-	-	+	-	+	-
16	+	+	+	+	-	-	+	-	-	+	+	+	+
17	+	-	+	-	-	-	-	-	-	-	-	-	+
18	+	+	+	+	+	+	+	-	-	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	+	+	+	-	-	+	-	-	+	+	+	-
21	+	+	+	+	+	+	+	-	-	+	+	+	+
22	+	+	+	+	+	+	+	-	-	+	+	+	+
23	+	+	+	+	+	+	+	-	-	+	+	+	+
24	+	+	+	-	-	-	-	-	-	-	-	-	+
25	+	+	+	+	+	-	+	-	-	+	+	+	+
26	+	+	+	+	+	+	+	-	-	+	+	+	+
27	+	+	+	+	+	+	+	-	-	+	+	+	+
28	+	+	+	-	-	-	-	-	-	-	-	-	+
29	+	-	+	-	-	-	-	-	-	-	-	-	+
30	+	+	+	+	-	-	+	-	-	+	+	+	+
31	+	-	+	-	-	-	-	-	-	-	-	-	+
32	+	-	+	-	-	-	-	-	-	-	-	-	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	+	+	+	+	+	+	+	+	+	-
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{92}	\rightarrow_{93}	\rightarrow_{94}	\rightarrow_{95}	\rightarrow_{96}	\rightarrow_{97}	\rightarrow_{98}	\rightarrow_{99}	\rightarrow_{100}	\rightarrow_{101}	\rightarrow_{102}	\rightarrow_{103}	\rightarrow_{104}
1	+	+	+	+	+	+	+	+	+	+	+	+	+
2	+	+	-	-	+	+	+	+	+	+	+	+	+
3	+	+	-	+	+	+	+	+	-	-	+	-	-
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	-	-	-	-	-	-	-
6	+	+	+	+	+	+	+	+	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	+	+	+	+	+
9	-	-	-	-	-	+	-	-	+	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	+	+	+	+	+
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-	+	+	+	+	+
15	-	-	-	-	-	-	-	-	+	+	+	+	+
16	-	-	+	-	-	+	-	-	+	+	+	+	+
17	-	-	+	-	-	+	-	-	+	+	+	-	-
18	-	+	+	-	-	+	+	+	+	+	+	+	+
19	-	+	+	+	+	+	+	+	+	+	+	+	+
20	-	-	-	-	-	-	-	-	+	+	+	+	+
21	-	+	+	-	-	+	+	-	+	+	+	+	+
22	-	+	+	-	-	+	+	-	+	+	+	+	+
23	-	+	+	-	-	+	+	-	+	+	+	+	+
24	-	-	+	-	-	+	-	-	+	+	+	-	+
25	-	-	+	-	-	+	-	-	+	+	+	+	+
26	-	+	+	-	-	+	+	-	+	+	+	+	+
27	-	+	+	-	-	+	+	+	+	+	+	+	+
28	-	+	+	-	-	+	-	-	+	+	+	-	+
29	-	-	+	-	-	+	-	-	+	+	+	-	-
30	-	-	+	-	-	+	+	-	+	+	+	+	+
31	-	-	+	-	-	+	-	-	+	+	+	-	-
32	-	-	+	-	-	+	-	-	+	+	+	-	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	+	+	+	+	+
37	-	-	-	-	-	-	-	-	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{105}	\rightarrow_{106}	\rightarrow_{107}	\rightarrow_{108}	\rightarrow_{109}	\rightarrow_{110}	\rightarrow_{111}	\rightarrow_{112}	\rightarrow_{113}	\rightarrow_{114}	\rightarrow_{115}	\rightarrow_{116}	\rightarrow_{117}
1	+	+	+	+	-	-	-	-	-	-	-	-	-
2	+	+	+	+	+	+	-	+	-	+	+	-	+
3	+	-	-	+	+	+	+	+	-	+	+	+	-
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	+	+	-	+	-	-	-	-	-
6	-	-	-	-	+	+	-	+	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-
8	+	-	-	-	+	+	+	+	+	+	+	+	+
9	-	-	-	-	-	+	-	+	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	+	+	+	+	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	+	-	-	-	+	+	+	+	+	+	+	+	+
15	+	-	-	-	+	+	+	+	+	+	+	+	+
16	+	+	+	-	+	+	+	+	+	+	+	+	+
17	-	+	-	-	+	+	+	+	+	-	-	-	-
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	-	+	-	+	+	+	+	+	+	+	+	+
21	+	-	+	-	+	+	+	+	+	+	+	+	+
22	+	-	+	-	+	+	+	+	+	+	+	+	+
23	+	+	+	-	+	+	+	+	+	+	+	+	+
24	+	-	-	-	+	+	+	+	+	-	-	-	+
25	+	+	+	-	+	+	+	+	+	+	+	+	+
26	+	+	+	-	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	+	-	+	-	+	+	+	+	+	-	-	-	+
29	-	+	-	-	+	+	+	+	+	-	-	-	-
30	+	+	+	+	+	+	+	+	+	+	+	+	+
31	-	-	-	-	+	+	+	+	+	-	-	-	+
32	+	+	-	-	+	+	+	+	+	-	-	-	-
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	+	+	+	+	-	-	-	-	-	-	-	-	-
37	+	+	+	+	+	+	+	+	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{118}	\rightarrow_{119}	\rightarrow_{120}	\rightarrow_{121}	\rightarrow_{122}	\rightarrow_{123}	\rightarrow_{124}	\rightarrow_{125}	\rightarrow_{126}	\rightarrow_{127}	\rightarrow_{128}	\rightarrow_{129}	\rightarrow_{130}
1	-	-	-	-	-	-	+	+	-	+	-	+	+
2	-	+	+	-	+	-	-	-	-	-	-	-	-
3	-	+	+	+	+	-	-	+	+	+	-	-	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	+	+	-	+	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-
8	+	-	-	-	-	-	+	+	+	+	+	+	+
9	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	+	-	-	-	-	-	+	+	+	+	+	+	+
15	+	-	-	-	-	-	+	+	+	+	+	+	+
16	+	-	+	+	+	-	+	+	+	+	+	+	+
17	+	-	-	+	-	-	+	+	+	+	+	-	-
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	-	-	-	-	-	+	+	+	+	+	+	+
21	+	+	+	+	+	+	+	+	+	+	+	+	+
22	+	+	+	+	+	+	+	+	+	+	+	+	+
23	+	+	+	+	+	+	+	+	+	+	+	+	+
24	+	-	+	+	+	-	+	+	+	+	+	-	-
25	+	-	+	+	+	-	+	+	+	+	+	+	+
26	+	+	+	+	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	+	+	+	+	+	-	+	+	+	+	+	-	-
29	+	-	-	+	-	-	+	+	+	+	+	-	-
30	+	+	+	+	+	-	+	+	+	+	+	+	+
31	+	-	-	+	-	-	+	+	+	+	+	-	-
32	+	+	-	+	-	-	+	+	+	+	+	-	-
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	-	-	-	-	-	+	+	+	+	+	+	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{131}	\rightarrow_{132}	\rightarrow_{133}	\rightarrow_{134}	\rightarrow_{135}	\rightarrow_{136}	\rightarrow_{137}	\rightarrow_{138}	\rightarrow_{139}	\rightarrow_{140}	\rightarrow_{141}	\rightarrow_{142}	\rightarrow_{143}
1	-	+	-	+	+	-	+	-	+	-	+	-	-
2	-	-	-	-	-	-	-	-	+	-	+	-	-
3	+	+	-	-	+	+	+	-	-	-	-	-	-
4	+	+	+	+	+	+	+	+	+	-	+	-	-
5	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	+	+	+	-	+
7	-	-	-	-	-	-	-	-	-	-	-	-	-
8	+	+	+	-	-	-	-	-	+	-	+	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	+	+	+	-	-	-	-	-	-	-	-	-	-
15	+	-	+	-	-	-	-	-	-	-	-	-	-
16	+	+	+	+	+	+	+	-	-	-	-	-	-
17	-	+	-	+	+	+	+	-	-	-	-	-	-
18	+	+	+	+	+	+	+	-	-	-	-	-	-
19	+	+	+	+	+	+	+	+	-	-	-	-	-
20	+	+	+	-	-	-	-	-	-	-	-	-	-
21	+	+	+	+	+	+	+	-	+	-	+	-	-
22	+	+	+	+	+	+	+	-	+	-	+	-	-
23	+	+	+	+	+	+	+	-	+	-	+	-	-
24	-	+	+	+	+	+	+	-	-	-	-	-	-
25	+	+	+	+	+	+	+	-	-	-	-	-	-
26	+	+	+	+	+	+	+	-	-	-	+	-	-
27	+	+	+	+	+	+	+	-	-	-	-	-	-
28	-	+	+	+	+	+	+	-	-	-	-	-	-
29	-	+	-	+	+	+	+	-	-	-	-	-	-
30	+	+	+	+	+	+	+	-	-	-	-	-	-
31	-	-	+	+	+	+	+	-	-	-	-	-	-
32	-	+	-	+	+	+	+	-	-	-	-	-	-
33	+	+	+	+	+	+	+	+	+	-	+	-	-
34	+	+	+	+	+	+	+	+	+	+	+	+	-
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	-	-	-	-	-	-	-	-	-	-
38	+	+	+	+	+	+	+	+	+	-	+	-	-

Table 1 (Continued from previous page)

	\rightarrow_{144}	\rightarrow_{145}	\rightarrow_{146}	\rightarrow_{147}	\rightarrow_{148}	\rightarrow_{149}	\rightarrow_{166}	\rightarrow_{167}	\rightarrow_{168}	\rightarrow_{169}	\rightarrow_{170}	\rightarrow_{171}	\rightarrow_{172}
1	-	-	+	+	+	+	-	-	-	-	-	+	+
2	-	-	+	+	+	+	+	+	+	+	+	+	+
3	-	-	-	-	-	-	+	+	+	+	+	+	+
4	-	-	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	-	-	-	+	-	-	-	-	-	-
6	-	+	-	+	-	+	+	+	-	+	-	+	+
7	-	-	-	-	-	-	-	-	-	-	-	+	+
8	-	-	+	+	+	-	+	+	+	-	+	+	+
9	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	+	+
12	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	+	+	+	-	+	-	-
15	-	-	-	-	-	-	+	+	+	-	+	-	-
16	-	-	-	-	-	-	+	+	+	+	+	-	-
17	-	-	-	-	-	-	+	+	-	+	+	-	+
18	-	-	-	-	-	-	+	+	+	+	+	+	+
19	-	-	-	-	-	-	+	+	+	+	+	+	+
20	-	-	-	-	-	-	+	+	+	-	+	-	-
21	-	-	+	+	+	+	+	+	+	+	+	+	+
22	-	-	+	+	+	+	+	+	+	+	+	+	+
23	-	-	+	+	+	+	+	+	+	+	+	+	+
24	-	-	-	-	-	-	+	+	-	+	+	+	+
25	-	-	-	-	-	-	+	+	+	+	+	+	+
26	-	-	+	+	+	+	+	+	+	+	+	+	+
27	-	-	-	-	-	-	+	+	+	+	+	+	+
28	-	-	-	-	-	-	+	+	-	+	+	+	+
29	-	-	-	-	-	-	+	+	-	+	+	-	+
30	-	-	-	-	-	-	+	+	+	+	+	+	+
31	-	-	-	-	-	-	+	+	-	+	+	-	+
32	-	-	-	-	-	-	+	+	-	+	+	+	+
33	-	-	+	+	+	+	+	+	+	+	+	+	+
34	+	-	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	-	-	+	-	+	-	+	-	+	-	+	-	-
38	-	-	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{173}	\rightarrow_{174}	\rightarrow_{175}	\rightarrow_{176}	\rightarrow_{177}	\rightarrow_{178}	\rightarrow_{179}	\rightarrow_{180}	\rightarrow_{181}	\rightarrow_{182}	\rightarrow_{183}	\rightarrow_{184}	\rightarrow_{185}
1	+	+	+	+	+	+	+	+	+	+	+	+	+
2	+	+	+	+	+	+	+	+	+	+	+	+	+
3	+	+	+	+	+	+	+	+	+	+	+	+	+
4	+	+	+	+	+	+	+	+	+	+	+	+	+
5	-	-	-	+	-	-	-	-	+	-	-	-	-
6	-	+	-	+	+	-	+	-	+	+	-	-	+
7	+	-	+	+	+	+	-	+	-	-	-	-	-
8	+	-	+	+	+	+	-	+	-	-	-	-	-
9	-	-	-	-	+	+	+	+	+	+	+	+	+
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	+	-	+	+	+	+	-	+	-	-	-	-	-
12	-	-	-	-	+	-	-	+	-	-	-	-	-
13	-	-	-	-	+	-	-	+	-	-	-	-	-
14	-	-	-	+	+	+	-	+	-	-	-	-	-
15	-	-	-	+	+	+	-	+	-	-	-	-	-
16	-	-	-	+	+	+	+	+	-	-	-	-	-
17	-	+	+	-	+	-	+	+	-	-	-	-	-
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	-	-	-	+	+	+	-	+	-	-	-	-	-
21	+	+	+	+	+	+	+	+	+	+	+	+	+
22	+	+	+	+	+	+	+	+	+	+	+	+	+
23	+	+	+	+	+	+	+	+	+	+	+	+	+
24	-	+	+	+	+	-	+	+	+	+	-	+	+
25	+	+	+	+	+	+	+	+	+	+	+	+	+
26	+	+	+	+	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	-	+	+	-	+	-	+	+	+	+	-	+	+
29	-	+	+	-	+	-	+	+	-	-	-	-	-
30	+	+	+	+	+	+	+	+	-	-	-	-	-
31	-	+	+	+	+	-	+	+	-	-	-	-	-
32	-	+	+	-	+	-	+	+	+	+	-	+	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	-	+	+	-	+	-	+	+	-	+	+	-
38	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 (Continued from previous page)

	\rightarrow_{186}	\rightarrow_{187}	\rightarrow_{188}	\rightarrow_{189}	\rightarrow_{190}	\rightarrow_{191}	\rightarrow_{192}	\rightarrow_{193}	\rightarrow_{194}	\rightarrow_{195}	\rightarrow_{196}	\rightarrow_{197}	\rightarrow_{198}
1	+	+	+	+	+	-	+	+	+	+	+	+	+
2	+	+	+	+	+	+	-	-	-	-	-	-	-
3	+	+	+	+	+	+	-	-	-	-	-	-	-
4	+	+	+	+	+	+	+	-	+	+	+	+	+
5	-	-	-	-	-	-	+	-	-	-	-	-	-
6	-	-	-	+	-	+	+	+	-	+	-	+	-
7	+	+	+	-	+	+	-	-	-	-	-	-	-
8	+	+	+	-	+	+	+	-	+	+	+	-	+
9	-	+	+	+	+	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-
11	+	+	+	-	+	+	-	-	-	-	-	-	-
12	-	+	-	-	+	-	-	-	-	-	-	-	-
13	-	+	-	-	+	-	-	-	-	-	-	-	-
14	+	+	+	-	+	+	+	-	+	+	+	-	+
15	+	+	+	-	+	+	+	-	+	+	+	-	+
16	+	+	+	+	+	-	+	-	+	+	+	+	+
17	-	+	-	+	+	-	+	-	-	+	-	+	+
18	+	+	+	+	+	+	+	+	+	+	+	+	+
19	+	+	+	+	+	+	+	+	+	+	+	+	+
20	+	+	+	-	+	+	+	-	+	+	+	-	+
21	+	+	+	+	+	+	+	+	+	+	+	+	+
22	+	+	+	+	+	+	+	+	+	+	+	+	+
23	+	+	+	+	+	+	+	+	+	+	+	+	+
24	+	+	-	+	+	+	+	+	-	+	-	+	+
25	+	+	+	+	+	+	+	+	+	+	+	+	+
26	+	+	+	+	+	+	+	+	+	+	+	+	+
27	+	+	+	+	+	+	+	+	+	+	+	+	+
28	+	+	-	+	+	+	+	+	-	+	-	+	+
29	-	+	-	+	+	-	+	-	-	+	-	+	+
30	+	+	+	+	+	-	+	-	+	+	+	+	+
31	+	+	-	+	+	-	+	+	-	+	-	+	+
32	-	+	-	+	+	+	+	+	-	+	-	+	+
33	+	+	+	+	+	+	+	+	+	+	+	+	+
34	+	+	+	+	+	+	+	-	+	+	+	+	+
35	+	+	+	+	+	+	+	+	+	+	+	+	+
36	-	-	-	-	-	-	-	-	-	-	-	-	-
37	+	+	+	-	+	-	+	-	+	-	+	-	+
38	+	+	+	+	+	+	+	+	+	+	+	+	+

3 Shortlisting the most suitable intuitionistic fuzzy implications

In this research, we are specifically interested to outline which of all the 182 implications in the table (from \rightarrow_1 to \rightarrow_{149} and from \rightarrow_{160} to \rightarrow_{198}) are most appropriate to work with in practice. We define this implication’s usability as the property to satisfy the largest number of axioms (objective, quantifiable criterion), and—in addition—with the most concise record that is easy to work with (secondary, more subjective but not less important criterion).

To get a “bird-eye” impression of the implications’ satisfying of the axioms, we give the Figure 1 which is another way of presenting the information from Table 1, where all satisfied axioms (“+”) are marked in red and all unsatisfied axioms (“−”) are marked in blue. The axioms sorted by the number of implications that satisfy them is given in the form of a histogram in the next Figure 2.

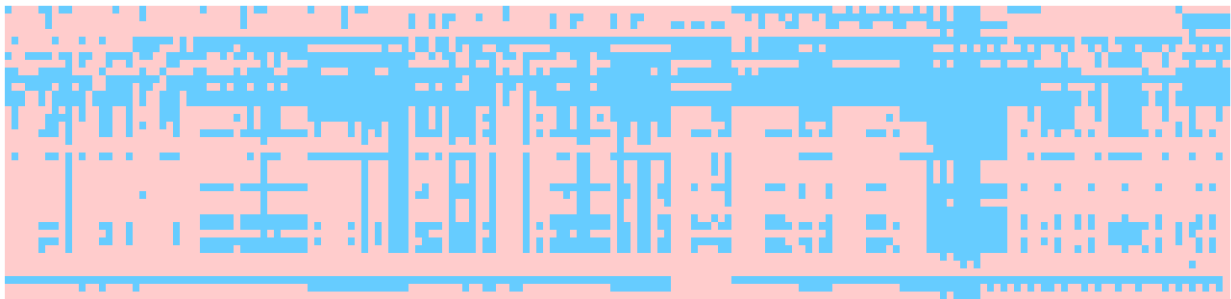


Figure 1. The total list of 182 implications (columns) by the 38 implications (rows), with the satisfied axioms (“+”) marked in red and the unsatisfied axioms (“−”) marked in blue.

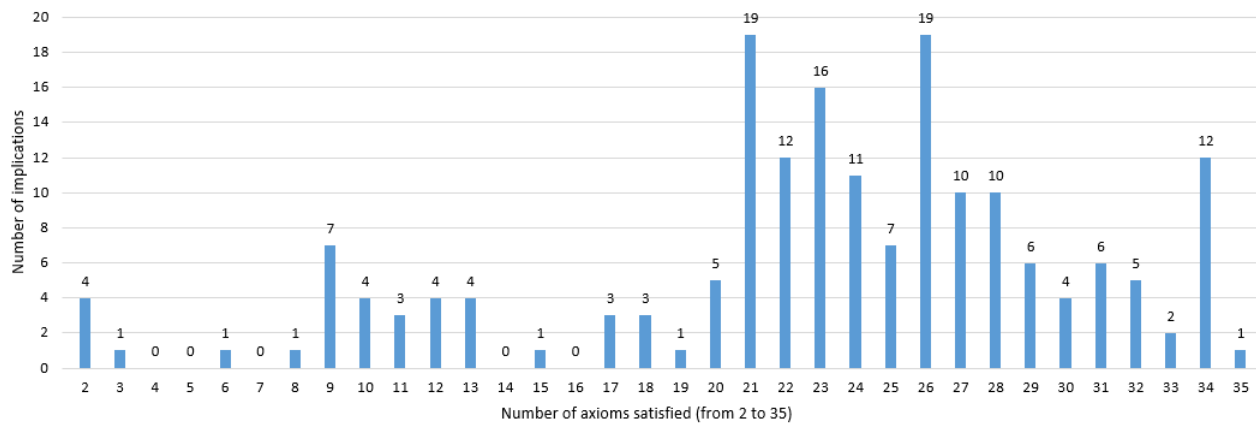


Figure 2. The number of implications per number of satisfied axioms (up to 35 out of 38)

Starting from the highest number of satisfied axioms per implication (that is, starting from the rightmost part of histogram of Figure 2), we selected the upper quartile, top 25% or the 44 implications (*per the Remark in Section 2, implications $\rightarrow_{187}, \rightarrow_{190}$ are to be excluded, so the total number of 46 is reduced to 44*), which satisfy at least 28 out of 38 axioms. Alternatively, selecting the top decile, that is approximately the top 20 implications that satisfy at least 32 out of 38 axioms, is considered rather restrictive, but in some cases it may be a valid option.

These 44 implications we call “suitable” implications, and we are listing them below with the number of satisfied axioms each, sorted in decreasing order:

- 1 implication satisfying 35 axioms: \rightarrow_{14}
10 implications satisfying 34 axioms: $\rightarrow_3, \rightarrow_4, \rightarrow_5, \rightarrow_{11}, \rightarrow_{13}, \rightarrow_{23}, \rightarrow_{74}, \rightarrow_{77}, \rightarrow_{177}, \rightarrow_{180}$,
2 implications satisfying 33 axioms: $\rightarrow_{20}, \rightarrow_{79}$,
5 implications satisfying 32 axioms: $\rightarrow_{18}, \rightarrow_{22}, \rightarrow_{27}, \rightarrow_{28}, \rightarrow_{76}$,
6 implications satisfying 31 axioms: $\rightarrow_{17}, \rightarrow_{81}, \rightarrow_{100}, \rightarrow_{102}, \rightarrow_{110}, \rightarrow_{112}$,
3 implications satisfying 30 axioms: $\rightarrow_{101}, \rightarrow_{109}, \rightarrow_{166}$,
7 implications satisfying 29 axioms: $\rightarrow_1, \rightarrow_9, \rightarrow_{24}, \rightarrow_{29}, \rightarrow_{61}, \rightarrow_{176}, \rightarrow_{192}$,
10 implications satisfying 28 axioms: $\rightarrow_2, \rightarrow_{35}, \rightarrow_{71}, \rightarrow_{105}, \rightarrow_{125}, \rightarrow_{127}, \rightarrow_{167}, \rightarrow_{170}, \rightarrow_{186}, \rightarrow_{198}$.

These implications are given in Table 2.

Table 2. List of the suitable intuitionistic fuzzy implications

\rightarrow_1	$\langle \max(b, \min(a, c)), \min(a, d) \rangle$
\rightarrow_2	$\langle \overline{\text{sg}}(a - c), d \text{sg}(a - c) \rangle$
\rightarrow_3	$\langle 1 - (1 - c) \text{sg}(a - c), d \text{sg}(a - c) \rangle$
\rightarrow_4	$\langle \max(b, c), \min(a, d) \rangle$
\rightarrow_5	$\langle \min(1, b + c), \max(0, a + d - 1) \rangle$
\rightarrow_9	$\langle b + a^2c, ab + a^2d \rangle$
\rightarrow_{11}	$\langle 1 - (1 - c) \text{sg}(a - c), d \text{sg}(a - c) \text{sg}(d - b) \rangle$
\rightarrow_{13}	$\langle b + c - bc, ad \rangle$
\rightarrow_{14}	$\langle 1 - (1 - c) \text{sg}(a - c) - d \overline{\text{sg}}(a - c) \text{sg}(d - b), d \text{sg}(d - b) \rangle$
\rightarrow_{17}	$\langle \max(b, c), \min(ab + a^2, d) \rangle$
\rightarrow_{18}	$\langle \max(b, c), \min(1 - b, d) \rangle$
\rightarrow_{20}	$\langle \max(\overline{\text{sg}}(a), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(c)) \rangle$
\rightarrow_{22}	$\langle \max(b, 1 - d), 1 - \max(b, 1 - d) \rangle$
\rightarrow_{23}	$\langle 1 - \min(\text{sg}(1 - b), \overline{\text{sg}}(1 - d)), \min(\text{sg}(1 - b), \overline{\text{sg}}(1 - d)) \rangle$
\rightarrow_{24}	$\langle \overline{\text{sg}}(a - c) \overline{\text{sg}}(d - b), \text{sg}(a - c) \text{sg}(d - b) \rangle$
\rightarrow_{27}	$\langle \max(\overline{\text{sg}}(1 - b), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(1 - d)) \rangle$
\rightarrow_{28}	$\langle \max(\overline{\text{sg}}(1 - b), c), \min(a, d) \rangle$
\rightarrow_{29}	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), \min(a, \overline{\text{sg}}(1 - d)) \rangle$
\rightarrow_{35}	$\langle 1 - ad, ad \rangle$
\rightarrow_{61}	$\langle \max(c, \min(b, d)), \min(a, d) \rangle$
\rightarrow_{71}	$\langle \max(b, c), \min(cd + d^2, a) \rangle$
\rightarrow_{74}	$\langle \max(\text{sg}(b), \overline{\text{sg}}(d)), \min(\overline{\text{sg}}(b), \text{sg}(d)) \rangle$
\rightarrow_{76}	$\langle \max(c, 1 - a), \min(1 - c, a) \rangle$
\rightarrow_{77}	$\langle (1 - \min(\overline{\text{sg}}(1 - a), \text{sg}(1 - c))), \min(\overline{\text{sg}}(1 - a), \text{sg}(1 - c)) \rangle$

(Continued on next page)

Table 2 (Continued from previous page)

\rightarrow_{79}	$\langle \max(\overline{\text{sg}}(1 - c), \text{sg}(b)), \min(\text{sg}(d), \overline{\text{sg}}(1 - a)) \rangle$
\rightarrow_{81}	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), \min(d, \overline{\text{sg}}(1 - a)) \rangle$
\rightarrow_{100}	$\langle \max(b \text{sg}(a), c), \min(a \text{sg}(b), d) \rangle$
\rightarrow_{101}	$\langle \max(b \text{sg}(a), c \text{sg}(d)), \min(a \text{sg}(b), \text{sg}(c)d) \rangle$
\rightarrow_{102}	$\langle \max(b, c \text{sg}(d)), \min(a, \text{sg}(c)d) \rangle$
\rightarrow_{105}	$\langle \max(1 - a, \min(1 - d, \text{sg}(d))), \min(a, d, \text{sg}(1 - d)) \rangle$
\rightarrow_{109}	$\langle b + \min(\overline{\text{sg}}(1 - a), c), ab + \min(\overline{\text{sg}}(1 - a), d) \rangle$
\rightarrow_{110}	$\langle \max(b, c), \min(ab + \overline{\text{sg}}(1 - a), d) \rangle$
\rightarrow_{112}	$\langle b + c - bc, ab + \overline{\text{sg}}(1 - a)d \rangle$
\rightarrow_{125}	$\langle \max(b, c), \min(cd + \overline{\text{sg}}(1 - d), a) \rangle$
\rightarrow_{127}	$\langle b + c - bc, (cd + \overline{\text{sg}}(1 - d))a \rangle$
\rightarrow_{166}	$\langle \max(b, \min(a, c)), \min(a, \max(b, d)) \rangle$
\rightarrow_{167}	$\langle \max(1 - a, \min(a, c)), \min(a, 1 - \min(a, c)) \rangle$
\rightarrow_{170}	$\langle \max(b, \min(1 - b, 1 - d)), 1 - \max(b, \min(1 - b, 1 - d)) \rangle$
\rightarrow_{176}	$\langle \overline{\text{sg}}(a - c) + \text{sg}(a - c) \max(b, c), \text{sg}(a - c) \min(a, d) \rangle$
\rightarrow_{177}	$\langle \overline{\text{sg}}(a - c) + \text{sg}(a - c) \max(1 - a, c), \text{sg}(a - c) \min(a, 1 - c) \rangle$
\rightarrow_{180}	$\langle \overline{\text{sg}}(d - b) + \text{sg}(d - b) \max(b, 1 - d), \text{sg}(d - b) \min(1 - b, d) \rangle$
\rightarrow_{186}	$\langle \overline{\text{sg}}(d - b) + \text{sg}(d - b) \max(b, c), \text{sg}(d - b) \min(a, d) \rangle$
\rightarrow_{192}	$\langle \max(c, \min(b, d)), \min(d, \max(a, c)) \rangle$
\rightarrow_{198}	$\langle \max(1 - d, \min(b, d)), 1 - \max(1 - d, \min(b, d)) \rangle$

Finally, in Figure 3, an oriented graph is shown, with nodes corresponding to the different implications and oriented arcs where the top-down location of the nodes reflects the order from stronger to weaker implications. The nodes in the figure that correspond to implications \rightarrow_{20} , \rightarrow_{27} , \rightarrow_{29} , \rightarrow_{35} , \rightarrow_{74} , \rightarrow_{79} , \rightarrow_{81} , \rightarrow_{100} , \rightarrow_{101} , \rightarrow_{102} , \rightarrow_{105} are omitted, because they are not connected with any other nodes.

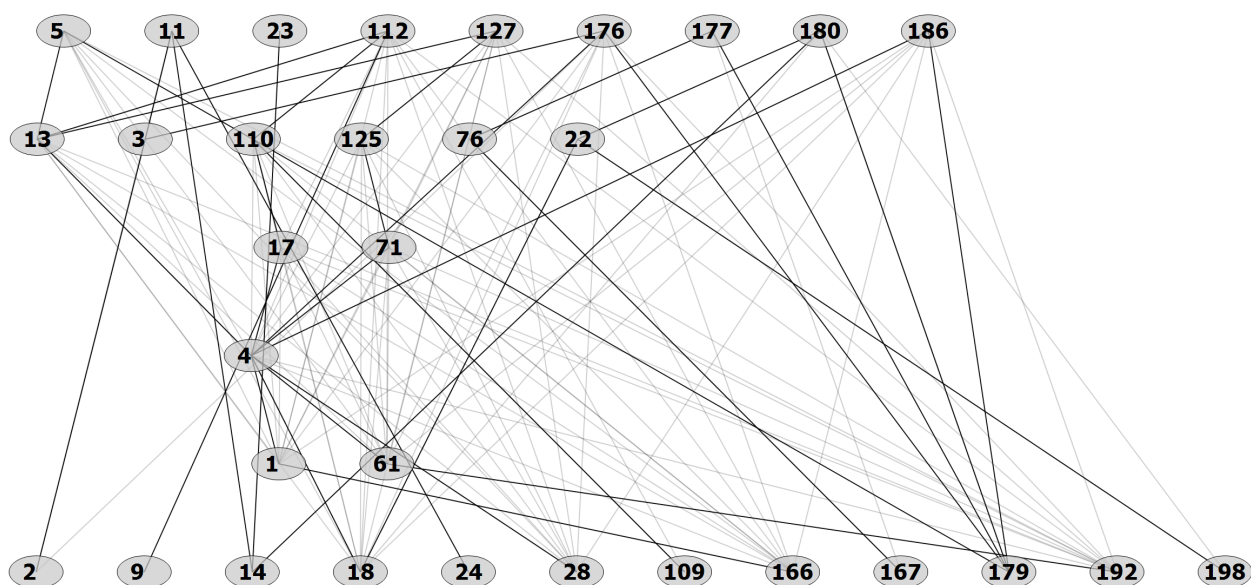


Figure 3. Oriented graph depicting the connections between the “suitable” implications.

4 Conclusion: ideas for the future

In future, we will prepare a similar graph for all intuitionistic fuzzy implications that can potentially find real applications, e.g. in the intuitionistic fuzzy sets-based approach of InterCriteria Analysis, in decision-support systems, Data Mining tools, PROLOG-type languages, and others.

In [3] we started study the properties of the intuitionistic fuzzy negations. In the next research, we will continue the research initiated here, but for the case of negations. Finally, one more idea for future research is to look at the presented data from the axioms perspective and assess which of them are least and most satisfied and what conclusions regarding the intuitionistic fuzzy implications can be made in this regard.

Acknowledgement

The authors are grateful for the support provided by the Bulgarian National Science Fund under Grant number KP-06-N22-1/2018 “Theoretical research and applications of InterCriteria Analysis”.

References

- [1] Angelova, N. (2021). IFSTOOL - Software for Intuitionistic Fuzzy Sets Necessity, Possibility and Circle Operators. *Advances in Intelligent Systems and Computing*, Vol. 1081, 76–81, doi:10.1007/978-3-030-47024-1_9
- [2] Angelova, N., & Atanassov, K. (2021). Research on intuitionistic fuzzy implications. *Notes on Intuitionistic Fuzzy Sets*, 27(2), 20–93.

- [3] Angelova, N., & Atanassov, K. (2021). Research on intuitionistic fuzzy negations. *Notes on Intuitionistic Fuzzy Sets*, 27(3), 18–31.
- [4] Atanassov, K. (2017). *Intuitionistic Fuzzy Logics*. Springer, Cham.
- [5] Klir, G., & Yuan, B. (1995). *Fuzzy Sets and Fuzzy Logic*. Prentice Hall, New Jersey.
- [6] Rasiowa, H., & Sikorski, R. (1963). *The Mathematics of Metamathematics*. Polish Academy of Sciences, Warsaw.
- [7] Tabakov, M. (1986). *Logics and Axiomatics*. Nauka i Izkustvo, Sofia.