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Centroid of an intuitionistic fuzzy number

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Abstract: In this paper a formula for finding the centroid of an intuitionistic fuzzy number is introduced and its properties are studied.
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1 Introduction

Yager [13] proposed a method which computes for each fuzzy number, a crisp measure, as its centroid. Other than Yager, a number of researchers like Murakami et al. [9], Cheng [5], Wang et al. [12] have also studied the concept of centroid of fuzzy numbers. Centroid of a fuzzy number f is its geometric center and is given by the formula $\int_{-\infty}^{\infty} xf(x)dx \Big/ \int_{-\infty}^{\infty} f(x)dx$ [4, 12]. Corresponding to every intuitionistic fuzzy number we introduce its crisp equivalent using its membership function and non membership function. The new scoring method has wide applications in various fields such as economics, intuitionistic fuzzy decision making etc. First we give a brief review of preliminaries.

2 Preliminaries

Definition 2.1. [7] Let M be a fuzzy subset of set of real numbers $\mathbb{R}$. Then M is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \phi$ such that

$$\mu_M(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty) \end{cases}$$

where l is a function from $(-\infty, a)$ to $[0,1]$ that is monotonic increasing, continuous from the right, and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; r is a function from (b, ∞) to $[0,1]$ that is monotonic decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (\omega_2, \infty)$.

Definition 2.2. [10] [Triangular fuzzy number] A fuzzy number M is defined to be a triangular fuzzy number (TFN) if its membership function $\mu_M : \mathbb{R} \to [0,1]$ is equal to

$$\mu_M(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } x \in [a,b] \\ \frac{c-x}{c-b} & \text{if } x \in [b,c] \\ 0 & \text{otherwise} \end{cases}$$

where $a \leq b \leq c$. This fuzzy number is denoted by (a,b,c).

Definition 2.3. [1] [Intuitionistic fuzzy sets] Let $X \neq \phi$ be a given set. An Intuitionistic fuzzy set in X is an object given by

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ satisfy the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for every $x \in X$.

Different research works [3, 6, 10, 11] were done over intuitionistic fuzzy numbers (IFNs). IFN is the generalization of fuzzy number and so it can be represented in the following manner.

Definition 2.4. [6] [Intuitionistic fuzzy numbers] An intuitionistic fuzzy subset

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in \mathbb{R}\}$$

of the real line $\mathbb{R}$ is called an IFN if the following holds:

(i) There exist $b \in \mathbb{R}$ such that $\mu_A(b) = 1$ and $\nu_A(b) = 0$

(ii) μ_A is a continuous mapping from $\mathbb{R} \to [0,1]$ and for every $x \in \mathbb{R}$, the relation $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds

(iii) The membership and non-membership functions of A is of the following form:

$$\mu_A(x) = \begin{cases} 0, & -\infty < x \leq a \\ f(x), & a \leq x \leq b \\ 1, & x = b \\ g(x), & b \leq x \leq c \\ 0, & c \leq x < \infty \end{cases}$$

$$\nu_A(x) = \begin{cases} 1, & -\infty < x \leq e \\ h(x), & e \leq x \leq b,\ 0 \leq f(x) + h(x) \leq 1 \\ 0, & x = b \\ k(x), & b \leq x \leq g, 0 \leq g(x) + k(x) \leq 1 \\ 1, & g \leq x < \infty \end{cases}$$

where f, g, h, k are functions from $\mathbb{R} \to [0, 1]$, f and k are strictly increasing functions and g and h are strictly decreasing functions.

It is worth noting that each IFN $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in \mathbb{R}\}$ is a conjunction of two fuzzy numbers: A^+ with a membership function $\mu_{A^+}(x) = \mu_A(x)$ and A^- with a membership function $\mu_{A^-}(x) = 1 - \nu_A(x)$.

Note. Here $e \leq a$ and $c \leq g$. For $x \leq b$, if $e > a$, then there exists real number m such that $a < m < e$,

$$\mu_A(x) + \nu_A(x) \leq 1 \Rightarrow f(m) \leq 0,$$

a contradiction. Hence $e \leq a$. Similarly $c \leq g$.

Definition 2.5 (Triangular intuitionistic fuzzy number)**.** An IFN may be defined as a triangular intuitionistic fuzzy number (TIFN) if and only if its membership and non-membership function takes the following form:

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases}$$

$$\nu_A(x) = \begin{cases} 1, & x < e \\ \frac{b-x}{b-e}, & e \leq x \leq b \\ \frac{x-b}{g-b}, & b \leq x \leq g \\ 1, & x > g. \end{cases}$$

where $e \leq a$ and $c \leq g$. Symbolically TIFN A is represented as $\{(a, b, c); (e, b, g)\}$.

3 Centroid of an intuitionistic fuzzy number

Here, we introduce a new fuzzy number p and its score is taken as the centroid of the intuitionistic fuzzy number.

Definition 3.1. Let $p : \mathbb{R} \to [0, 1]$ be defined by

$$p(x) = \frac{(\mu - \nu)(x) + 1}{2}$$

where μ and ν are the membership and non-membership functions of IFN A.

Proposition 3.1. *p is a fuzzy number.*

Proof. Range of $\mu - \nu$ lies in $[-1, 1]$.

$$p(x) = \begin{cases} 0, & x < e \\ \frac{1-h(x)}{2}, & e \le x < a \\ \frac{f(x)-h(x)+1}{2}, & a \le x < b \\ \frac{g(x)-k(x)+1}{2}, & b \le x < c \\ \frac{1-k(x)}{2}, & c \le x < g \\ 0, & x \ge g \end{cases}$$

$h(b) = 1$. Therefore

$$p(x) = \begin{cases} 1 & \text{for } x = b \\ l(x) & \text{for } x \in (-\infty, b) \\ r(x) & \text{for } x \in (b, \infty) \end{cases}$$

where

$$l(x) = \begin{cases} 0, & x < e \\ \frac{1-h(x)}{2}, & e \le x < a \\ \frac{f(x)-h(x)+1}{2} & a \le x < b \end{cases}$$

l is a function from $(-\infty, b)$ to $[0, 1]$ continuous and monotonically increasing and

$$r(x) = \begin{cases} \frac{g(x)-k(x)+1}{2}, & b < x \le c \\ \frac{1-k(x)}{2}, & c < x \le g \\ 0, & x > g \end{cases}$$

r is a function from (b, ∞) to $[0, 1]$, continuous and monotonically decreasing.

Hence p is a fuzzy number.

Proposition 3.2. *Centroid of p is*

$$\frac{\int_e^a \frac{1-h(x)}{2} x dx + \int_a^b \frac{f(x)-h(x)+1}{2} x dx + \int_b^c \frac{g(x)-k(x)+1}{2} x dx + \int_c^g \frac{1-k(x)}{2} x dx}{\int_e^a \frac{1-h(x)}{2} dx + \int_a^b \frac{f(x)-h(x)+1}{2} dx + \int_b^c \frac{g(x)-k(x)+1}{2} dx + \int_c^g \frac{1-k(x)}{2} dx} \tag{3.1}$$

Remark 3.1. (3.1) *is taken as the centroid of the IFN A.*

Remark 3.2. *If $\nu_A(x) = 1 - \mu_A(x)$, then IFN A is a fuzzy number. Its centroid is*

$$\frac{\int_a^b x f(x) dx + \int_b^c x g(x) dx}{\int_a^b f(x) dx + \int_b^c g(x) dx} \quad \textit{by } (3.1)$$

Hence formula for centroid of IFN introduced here is generalization of that of fuzzy number.

Remark 3.3. *The centroid of TIFN $\{(a, b, c); (e, b, g)\}$ is*

$$\frac{1}{3}\left[\frac{(g-e)(b-2g-2e) + (c-a)(a+b+c) + 3(g^2-e^2)}{g-e+c-a}\right]$$

Proof. Centroid of the TIFN is

$$\frac{\int_e^a x(\frac{\frac{x-b}{b-e}+1}{2})dx + \int_a^b (\frac{\frac{x-a}{b-a}-\frac{b-x}{b-e}+1}{2})xdx + \int_b^c (\frac{\frac{x-c}{b-c}-\frac{b-x}{b-g}+1}{2})xdx + \int_c^g x(\frac{\frac{x-b}{b-g}+1}{2})dx}{\int_e^a (\frac{\frac{x-b}{b-e}+1}{2})dx + \int_a^b (\frac{\frac{x-a}{b-a}-\frac{b-x}{b-e}+1}{2})dx + \int_b^c (\frac{\frac{x-c}{b-c}-\frac{b-x}{b-g}+1}{2})dx + \int_c^g (\frac{\frac{x-b}{b-g}+1}{2})dx}$$

by (3.1),

$$= \frac{1}{3}\Big[\frac{(g-e)(b-2g-2e)+(c-a)(a+b+c)+3(g^2-e^2)}{g-e+c-a}\Big]. \tag{3.2}$$

Remark 3.4. If $\nu_A(x) = 1-\mu_A(x)$, then TIFN $\{(a,b,c);(e,b,g)\}$ will become the TFN (a,b,c). Then $e = a$ and $g = c$.
By (3.2), centroid $= \frac{a+b+c}{3}$.

Proposition 3.3. *The centroid of TIFN* $\{(a,b,c);(e,b,g)\}$ *is* b *if* $b-a = c-b$ *and* $b-e = g-b$.

Proof. Let $b-a = c-b = k$ and $b-e = g-b = l$,

$$\text{Centroid } = \frac{1}{3}\Big[\frac{-6bl+6bk+12bl}{6(k+l)}\Big] = b.$$

4 Significance of the proposed method

Here we are using both membership grade and non-membership grade to obtain a single score. If we obtain a membership score and a non membership score for IFNs, (as described in [10], IFNs are treated in a different manner there) ranking of IFNs fails if membership score of $M_1 \leq$ membership score of M_2 and non membership score of $M_1 \leq$ non-membership score of M_2 where M_1 and M_2 are two IFNs. Getting a single score involving both membership and non-membership grades overcome that situation.

This new scoring method has wide applications in various fields. In economics, for extending equilibrium in the fuzzy sense to intuitionistic fuzzy sense, these notions can be used.

5 Conclusion

A formula for finding centroid of an IFN is introduced and it is proved that it is a generalization of formula in the literature.

References

[1] Atanassov, K. T. *Intuitionistic fuzzy sets*, Physica-Verlag, Heidelberg, New York, 1999.

[2] Atanassov, K. T. Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 20, 1986, No. 1, 87–96.

[3] Ban, A. Trapezoidal approximations of intuitionistic fuzzy numbers expressed by value, ambiguity, width and weighted expected value, *Notes on Intuitionistic Fuzzy Sets*, Vol. 14, 2008, No. 1, 38–47.

[4] Chen, S. J., C. L. Hwang, *Fuzzy multiple attribute decision making*, Springer-Verlag, Heidelberg, 1992.

[5] Cheng, C. H. A new approach for ranking fuzzy numbers by distance method, *Fuzzy Sets and Systems*, Vol. 95, 1998, 307–317.

[6] Guha, D., D. Chakraborthy, A theoretical development of distance measure for intuitionistic fuzzy numbers, *International Journal of Mathematics and Mathematical Sciences*, Vol. 2010, Article ID 949143, 25 pages.

[7] Klir, G. J., B. Yuan, *Fuzzy sets and fuzzy logic Theory and applications*, Prentice Hall, New York, 2003.

[8] Koutsoyiannis, A. *Modern Microeconomics*, Macmillan Press Ltd, 1979.

[9] Murakami, S., H. Maeda, S. Imamura, Fuzzy decision analysis on the development of centralized regional energy control system, *Proceedings of IFAC Symposium on Fuzzy Information, Knowledge Representation and Decision Analysis*, 1983, 363–368.

[10] Nayagam, V. L. G., G. Venkateshwari and G. Sivaraman, Ranking of intuitionistic fuzzy numbers, *Proceedings of IEEE International Conference on Fuzzy Systems*, Hong Kong, 2008, 1971–1974.

[11] Nayagam, V. L. G., G. Venkateshwari, G. Sivaraman, Modified ranking of intuitionistic fuzzy numbers, *Notes on Intuitionistic Fuzzy Sets*, Vol. 17, 2011, No. 1, 5–22.

[12] Wang, Y. M., J. B. Yang, D. L. Xu, K. S. Chin, On the centroids of fuzzy numbers, *Fuzzy Sets and Systems*, Vol. 157, 2006, 919–926.

[13] Yager, R. R. On a general class of fuzzy connectives, *Fuzzy Sets and Systems*, Vol. 4, 1980, 235–242.

[14] Zadeh, L. A. Fuzzy Sets, *Information and Control*, Vol. 8, 1965, 338–353.