**Notes on Intuitionistic Fuzzy Sets**
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# Degrees and regularity of intuitionistic fuzzy semihypergraphs

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**Abstract:** This research work takes a new paradigm on the hypergraph concept which is a combination of a hypergraph and a semigraph. A semihypergraph is a connected hypergraph in which each hyperedge must have at least three vertices and any two hyperedges have at least one vertex in common. In a semihypergraph, vertices are classified as end, middle or middle-end vertices. This distinction, combined with membership and non-membership values, enables a more granular examination of vertices and their degrees in Intuitionistic Fuzzy Semihypergraphs (IFSHGs).

This paper proposes four types of degrees: degree, end vertex degree, adjacent degree and consecutive adjacent degree on an IFSHG. Each degree reflects specific patterns within the intuitioistic fuzzy semihypergraphs. Additionally, three types of sizes are also defined: size, crisp size and pseudo size of IFSHGs. Concepts such as regular and totally regular IFSHGs with their properties are also defined.

**Keywords:** Intuitionistic fuzzy semihypergraphs (IFSHGs), Degree, End vertex degree, Adjacent degree, Consecutive adjacent degree, Size, Regular, Totally regular.
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# 1 Introduction

Graph theory is a rich and dynamic field of mathematics that studies the relationships between discrete objects through graphs, which are composed of vertices and edges. This area has numerous applications across various domains, including computer science, biology, and social sciences. In particular, the history of graph theory is found in 1735, when the Königsberg bridge problem was resolved by Swiss Mathematician Leonhard Euler.

In numerous real-world applications, relationships between items are more problematic; in these cases, graph theory is unable to handle the interactions when we examine more than two objects. In this case, hypergraph concepts are applied to represent complex relationships among objects.

The concepts of graph and hypergraph theory were introduced by Berge [3] in 1976. The notions of fuzzy graph and fuzzy hypergraph were developed by Moderson and Nair in [5]. In 1983, Atanassov [2] first proposed the idea of an intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1994, he and Shannon proposed the concept of an Intuitionistic Fuzzy Graph (IFG) in [15]. In 2006, intuitionistic fuzzy graphs were further researched by Parvathi and Karunambigai [9], and in 2009, intuitionistic fuzzy hypergraph were introduced [10]. The required and sufficient conditions for regular fuzzy graphs and entirely regular fuzzy graphs are given in [8]. Some of the characteristics of regular and irregular intuitionistic fuzzy hypergraphs were covered by Pradeepa and Vimala in [12] and [11].

Sampathkumar in [14] has introduced semigraph and defined various degrees of a vertex $v$ in a semigraph. In 2020, fuzzy semigraph has been introduced by Radha and Renganathan [13].

As an extension of semigraphs, the authors in [4] has proposed a new concept on semihypergraph. In [7] introduced Intuitionistic fuzzy semihypergraphs. Papers [1, 6, 16] motivated to analyse different categories of degree and regular IFSHGs and their properties. This paper investigates the key characteristics of degree and regular IFSHGs. Section 2 provides preliminary definitions and notations. In Section 3, we delve into four types of degrees and three types of sizes of IFSHGs exploring their properties with relevant examples. Additionally, Section 4, determines the properties of regular and totally regular IFSHGs. Section 5 concludes the paper.

# 2 Preliminaries

**Definition 2.1.** [4] A *semihypergraph* is a connected hypergraph $H_s = (V, E_{h_j}, <)$, where $V = \{v_i/i=1, 2, ..., n\}$ is a non-empty, vertex order preserving finite set and $E_{h_j} = \{E_{h_1}, E_{h_2}, ..., E_{h_p}\}$ such that $E_{h_j} \in V$, where, $j = \{1, 2, \ldots, p\}$ with minimum of three vertices satisfying the conditions:

$(i)$ $E_{h_j} \neq \emptyset$ and $\cup E_{h_j} = V, 1 \leq j \leq p$.

$(ii)$ Each pair of hyperedges have minimum of one vertex in common.

$(iii)$ The order of vertices $(v_1, v_2, \ldots, v_n)$ in hyperedge is equal to the reverse order of vertices $(u_1, u_2, \ldots, u_m)$ if and only if

- $n = m$, and
- either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

**Definition 2.2.** In a semihypergraph, for a vertex $v_i \in H_s$ various types of degrees of $v_i$ are defined as follows:

$(i)$ *Degree* $\deg(v_i)$: The number of hyperedges incident to the vertex $v_i$;

$(ii)$ *Endvertex degree* $\deg_e(v_i)$: The number of hyperedges incident to the vertex $v_i$ as an end vertex;

$(iii)$ *Adjacent degree* $\deg_a(v_i)$: The number of vertices adjacent to the vertex $v_i$;

$(iv)$ *Consecutive Adjacent degree* $\deg_{ca}$: The number of vertices which are consecutively adjacent to $v_i$.

**Definition 2.3.** [7] Consider a semihypergraph $H_s = (V, E_{h_j}, <)$. An *IFSHG* is defined and denoted as $\mathcal{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ is a connected semihypergraph, where:

$(i)$ $V = \{v_i/i = 1, 2, \ldots, n\}$ such that $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ for each element $v_i \in V$, where $V$ be a finite, non-empty and vertex order preserving finite set;

$(ii)$ For every consecutive vertices $(C_v)$ of hyperedges, $\mu^c_{ij} : V \times V \to [0,1]$ and $\nu^c_{ij} : V \times V \to [0,1]$ are such that

$$\mu^c_{ij}(v_i, v_j) \leq \min(\mu_i(v_i, v_j)) \quad \& \quad \nu^c_{ij}(v_i, v_j) \leq \max(\nu_i(v_i, v_j))$$

where $0 \leq \mu^c_{ij}(v_i, v_j) + \nu^c_{ij}(v_i, v_j) \leq 1$;

$(iii)$ For every $E_{h_j} \subseteq V \times V$, $\mu_{ij} : V \times V \to [0,1]$ and $\nu_{ij} : V \times V \to [0,1]$ such that

$$\mu_{ij}(E_{h_j}) = \min(\mu^c_{ij}(v_i, v_j)) \leq \min(\mu_i(v_{ei}, v_{ej}))$$

$$\& \; \nu_{ij}(E_{h_j}) = \max(\nu^c_{ij}(v_i, v_j)) \leq \max(\nu_i(v_{ei}, v_{ej})),$$

for all $E_{h_j}, j = 1, 2, 3, \ldots, n$ where $0 \leq \mu_{ij}(E_{h_j}) + \nu_{ij}(E_{h_j}) \leq 1$ and $(v_{ei}, v_{ej})$ represents end vertices of hyperedge in $\mathcal{I}_{sh}$. Here, $\langle \mu_i, \nu_i \rangle$ denotes the membership and non-membership values of vertices, $\langle \mu^c_{ij}, \nu^c_{ij} \rangle$ denotes the membership and non-membership values of consecutive vertices, while $\langle \mu_{ij}, \nu_{ij} \rangle$ denotes the membership and non-membership values of hyperedges, respectively.

**Definition 2.4.** [7] The hyperedge $E_{h_j}$ in IFSHG satisfying the following conditions:

$i)$ $\mu^c_{ij}(v_i, v_j) = \min(\mu_i(v_i, v_j))$ and $\nu^c_{ij}(v_i, v_j) = \max(\nu_i(v_i, v_j)), \forall i \neq j$,

$ii)$ $\mu(E_{h_j}) = \min(\mu_i(v_1, v_n))$, and $\nu(E_{h_j}) = \max(\nu_i(v_1, v_n)), \forall (v_i, v_j) \in C_v \subseteq E_{h_j}$,

is said to be an *Effective hyperedge* in $\mathcal{I}_{sh}$.

**Definition 2.5.** [7] If all the hyperedges of $\mathcal{I}_{sh}$ are effective hyperedges, then an IFSHG is known as an *effective IFSHG*.

**Note.** Throughout this paper, it is assumed that the fourth Cartesian product,

$$V_1 \times_4 V_2 = \{\langle (v_1, v_2), \min(\mu_1, \mu_2), \max(\nu_1, \nu_2) \rangle | v_1 \in V_1, v_2 \in V_2\},$$

is used to find the membership and non-membership values of consecutive vertices $\langle \mu^c_{ij}, \nu^c_{ij} \rangle$ and hyperedges $\langle \mu_{ij}, \nu_{ij} \rangle$.

# 3 Classification of degrees and sizes of IFSHGs

**Definition 3.1.** Consider an IFSHG $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$. Let $v_i$ be any vertex on $V$. The different categories of degrees of vertex $v_i$ are defined as follows:

- *Degree* of a vertex $v_i$ in $\mathfrak{I}_{sh}$ is obtained by summing the $\langle \mu_{ij}, \nu_{ij} \rangle$ values of all hyperedges $E_{h_j}$ which contain the vertex $v_i$ either as an end vertex, a middle vertex or a middle-end vertex, and is denoted by $d(v_i) = \sum\limits_{v_i \in E_{h_j} \subseteq E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$.
- *End Vertex Degree* of any vertex $v_i$ in $V$ is calculated by adding the $\langle \mu_{ij}, \nu_{ij} \rangle$ values of all hyperedges $E_{h_j}$, where $v_i$ is an end vertex and is denoted by $d_e(v_i) = \sum\limits_{E_{h_j} \subseteq E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$, such that $v_i$ is an end vertex of corresponding hyperedges.
- Let $p_n = (v_i, v_{i+1}, \ldots, v_{j-1}, v_j)$ be a partial hyperedge of $E_{h_j}$ and $P = \{p_1, p_2, \ldots, p_n\}$ be a set of all partial hyperedges in $\mathfrak{I}_{sh}$. The membership and non-membership values of each partial edge are defined by the function $k : P \to [0, 1]$, such that

  $$k(p_n) = k(v_i, v_{i+1}, \ldots, v_{j-1}, v_j) = \langle \min(\mu^c_{ij}(v_i, v_j)), \max(\nu^c_{ij}(v_i, v_j)) \rangle$$

  *Adjacent Degree* of a vertex $v_i$ is the total values of all partial hyperedges $p_n$ with $v_i$ as an end vertex. It is denoted as $d_a(v_i) = \sum k(p_n)$.
- *Consecutive Adjacent Degree* of a vertex $v_i$ is the summation of all $\left\langle \mu^c_{ij}, \nu^c_{ij} \right\rangle$ values of $E_{h_j}$, where $(v_i, v_j)$ are consecutively adjacent vertices. It is denoted by $d_{ca}(v_i) = \sum \left\langle \mu^c_{ij}, \nu^c_{ij} \right\rangle (v_i, v_j)$.

**Note.** For any vertex $v_i$ in $\mathfrak{I}_{sh}$, these degrees have a linear ordering:

$$d_a(v_i) \geq d_{ca}(v_i) \geq d(v_i) \geq d_e(v_i).$$

**Theorem 3.1.** Let $\mathfrak{I}_{sh}$ be an IFSHG. Then

$(a)$ $\sum\limits_{v_i \in V} d_e(v_i) = 2 \sum\limits_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$

$(b)$ $\sum\limits_{v_i \in V} d(v_i) = \sum\limits_{E_{h_j} \in E_h} |E_{h_j}| \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$

*Proof.* $(a)$. Let $\mathfrak{I}_{sh}$ be an IFSHG. Every hyperedges in IFSHG consists of two end vertices. Thus,

$$\sum_{v_i \in V} d_e(v_i) = 2 \sum_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j}) \tag{1}$$

Thus, proof of $(a)$ is obtained.

$(b)$. By using the definition of degree of $\mathfrak{I}_{sh}$, it is obtained by adding the $\langle \mu_{ij}, \nu_{ij} \rangle$ values of all hyperedges $E_{h_j}$ that contains $v_i$. Here, $v_i$ may be end vertex, middle vertex or middle-end vertex.

Degree of an end vertices can be calculated as $\sum_{v_i \in V} d_e(v_i)$ and the number of middle vertices on the corresponding hyperedges is obtained by $|E_{h_j}| - 2$.

Thus,

$$\begin{aligned}\sum_{v_i\in V} d(v_i) &= \sum_{v_i\in V} d_e(v_i) + \sum_{E_{h_j}\in E_h} (|E_{h_j}|-2)\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j})\\ &= \sum_{v_i\in V} d_e(v_i) + \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j}) - 2\sum_{E_{h_j}\in E_h}\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j})\\ &= \sum_{v_i\in V} d_e(v_i) + \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j}) - \sum_{v_i\in V} d_e(v_i) \qquad \text{[from (1)]}\\ &= \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j}).\end{aligned}$$

This completes the proof. □

**Theorem 3.2.** *Let $\mathcal{I}_{sh}$ be an IFSHG. Let $\mu^c_{ij} : V\times V \to [0,1]$ and $\nu^c_{ij} : V\times V\to[0,1]$ be a constant function for the set of all consectively adjacent pair of vertices. Then:*

(i) $\sum_{v_i\in V} d^c_a(v_i) + \sum_{v_i\in V} d^c(v_i) \le \sum_{E_{h_j}\in E_h} |E_{h_j}|^2\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j})$,

(ii) $\sum_{v_i\in V} d^c_{ca}(v_i) + \sum_{v_i\in V} d^c_e(v_i) \le 2\sum_{v_i\in V} d^c(v_i)$.

*Proof.* (i) Let $\mathcal{I}_{sh}$ be an IFSHG. Let $\mu^c_{ij} : V\times V\to[0,1]$ and $\nu^c_{ij} : V\times V\to[0,1]$ be any constant function, say $\langle C_1, C_2\rangle$ for any pair of consective pair of vertices and $\mu_{ij} : V\times V\to[0,1]$ and $\nu_{ij} : V\times V\to[0,1]$ also a constant function for every hyperedge.

For every vertex $v_i \in E_{h_j}$, there are $|E_{h_j}|-1$ partial hyperedges where $v_i$ is an end vertex. Thus every hyperedge in $\mathcal{I}_{sh}$ contains $|E_{h_j}|(|E_{h_j}|-1)\langle C_1, C_2\rangle$ to the sum of adjacent degree of the vertex $v_i$, thus

$$\begin{aligned}\sum_{v_i\in V} d^c_a(v_i) &\le \sum_{E_{h_j}\in E_h} |E_{h_j}|(|E_{h_j}|-1)\langle C_1,C_2\rangle\\ &\le \sum_{E_{h_j}\in E_h} |E_{h_j}|^2\langle C_1,C_2\rangle - \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle C_1,C_2\rangle\end{aligned}$$

$$\sum_{v_i\in V} d^c_a(v_i) + \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle C_1,C_2\rangle \le \sum_{E_{h_j}\in E_h} |E_{h_j}|^2\langle C_1,C_2\rangle$$

$$\sum_{v_i\in V} d^c_a(v_i) + \sum_{E_{h_j}\in E_h} |E_{h_j}|\langle\mu_{ij},\nu_{ij}\rangle \le \sum_{E_{h_j}\in E_h} |E_{h_j}|^2\langle\mu_{ij},\nu_{ij}\rangle.$$

From Theorem 3.1, $\sum_{v_i\in V} d^c_a(v_i) + \sum_{v_i\in V} d^c(v_i) \le \sum_{E_{h_j}\in E_h} |E_{h_j}|^2\langle\mu_{ij},\nu_{ij}\rangle(E_{h_j})$.

Hence the result $(i)$ is obtained.

$(ii)$. To prove: $\sum_{v_i\in V} d^c_{ca}(v_i) + \sum_{v_i\in V} d^c_e(v_i) \le 2\sum_{v_i\in V} d^c(v_i)$.

In any hyperedge of IFSHG, for any end vertex there is only one consecutive vertex and for every middle vertex there are exactly two consecutive vertices. Therefore, a hyperedge in $\mathcal{I}_{sh}$ consists of $(2|E_{h_j}|-2)\langle C_1, C_2\rangle$ to the sum of the consecutive adjacent degree of every vertex in $\mathcal{I}_{sh}$.

$$\sum_{v_i \in V} d^c_{ca}(v_i) \leq \sum_{E_{h_j} \in E} (2|E_{h_j}| - 2)\langle C_1, C_2\rangle$$
$$\leq 2 \sum_{E_{h_j} \in E_h} |E_{h_j}|\langle C_1, C_2\rangle - 2 \sum_{E_{h_j} \in E_h} \langle C_1, C_2\rangle$$
$$\leq 2 \sum_{E_{h_j} \in E_h} |E_{h_j}|\langle \mu_{ij}, \nu_{ij}\rangle - 2 \sum_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij}\rangle.$$

From the Theorem 3.1,

$$\sum_{v_i \in V} d^c_{ca}(v_i) \leq 2 \sum_{v_i \in V} d^c(v_i) - \sum_{v_i \in V} d^c_e(v_i).$$

Hence,

$$\sum_{v_i \in V} d^c_{ca}(v_i) + \sum_{v_i \in V} d^c_e(v_i) \leq 2 \sum_{v_i \in V} d^c(v_i).$$

Hence $(ii)$ is proved. □

**Corollary 3.1.** *Let $\mathfrak{I}_{sh}$ be an IFSHG. If the computation of consecutive adjacent degree of all vertices $v_i$, each partial hyperedge contributes exactly two counts. Then,*

$$d_{ca}(v_i) = 2 * N(C_{v_{ij}})\langle C_1, C_2\rangle,$$

*where $N(C_{v_{ij}})$ = Number of consecutive adjacent pairs.*

**Note.** The above corollary exists only if a partial hyperedge contributes exactly two counts, otherwise it fails.

**Example 3.1.** Consider an IFSHG where $v_1, v_5, v_7$ and $v_{10}$ are the end vertices, $v_3, v_4, v_6$ and $v_9$ are the middle vertices, $v_2$ and $v_8$ are the middle end vertices. Let $E_h = \{E_{h_1}, E_{h_2}, E_{h_3}, E_{h_4}, E_{h_5}\}$ be the set of all intuitionistic fuzzy hyperedges in $\mathfrak{I}_{sh}$. The following graph (Figure 1) depicts the given condition.

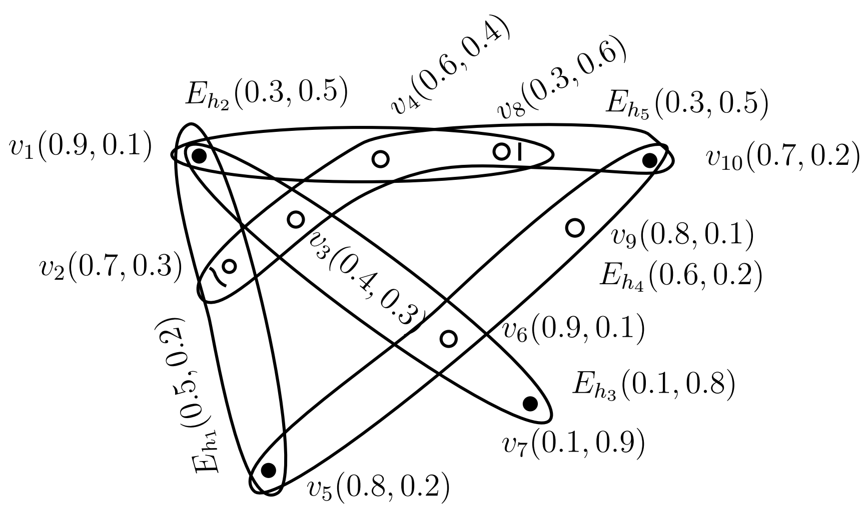


Figure 1. An example IFSHG

In Table 1, consecutive adjacent values of a given IFSHG are calculated. In Table 2, the different types of degrees are displayed. The $d_e(v_i)$ and $d(v_i)$ represent the end vertex degree and degree of the given IFSHG, while $d_e^c(v_i)$, $d^c(v_i)$, $d_a^c(v_i)$, $d_{ca}^c(v_i)$ represent the various degrees, where $\langle \mu_i, \nu_i \rangle$ are all constant.
Let $\langle \mu_i, \nu_i \rangle (v_i) = \langle 0.7, 0.2 \rangle$ whereas $\langle \mu_{ij}^c, \nu_{ij}^c \rangle (v_i, v_j) = \langle 0.4, 0.3 \rangle$ and $\langle \mu_{ij}, \nu_{ij} \rangle = \langle 0.4, 0.3 \rangle$.

Table 1. Consecutive adjacent values of IFSHG

| | | | |
|---|---|---|---|
| $C_{v_{1,2}}(0.6, 0.2)$ | $C_{v_{1,4}}(0.5, 0.3)$ | $C_{v_{1,3}}(0.1, 0.4)$ | $C_{v_{2,3}}(0.3, 0.2)$ |
| $C_{v_{2,5}}(0.5, 0.2)$ | $C_{v_{3,4}}(0.3, 0.3)$ | $C_{v_{3,6}}(0.3, 0.3)$ | $C_{v_{4,8}}(0.3, 0.5)$ |
| $C_{v_{6,7}}(0.1, 0.8)$ | $C_{v_{6,9}}(0.6, 0.2)$ | $C_{v_{8,10}}(0.3, 0.4)$ | $C_{v_{9,10}}(0.6, 0.2)$ |

Table 2. Degrees of IFSHG

| $v_i$ | $d_e(v_i)$ | $d(v_i)$ | $d_e^c(v_i)$ | $d^c(v_i)$ | $d_a^c(v_i)$ | $d_{ca}^c(v_i)$ |
|---|---|---|---|---|---|---|
| $v_1$ | $\langle 0.9, 1.5 \rangle$ | $\langle 0.9, 1.5 \rangle$ | $\langle 1.2, 0.9 \rangle$ | $\langle 1.2, 0.9 \rangle$ | $\langle 2.8, 2.1 \rangle$ | $\langle 1.2, 0.9 \rangle$ |
| $v_2$ | $\langle 0.3, 0.5 \rangle$ | $\langle 0.8, 0.7 \rangle$ | $\langle 0.4, 0.3 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.4, 1.8 \rangle$ | $\langle 1.2, 0.9 \rangle$ |
| $v_3$ | $\langle 0, 0 \rangle$ | $\langle 0.4, 1.3 \rangle$ | $\langle 0, 0 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.8, 2.1 \rangle$ | $\langle 1.6, 1.2 \rangle$ |
| $v_4$ | $\langle 0, 0 \rangle$ | $\langle 0.6, 1.0 \rangle$ | $\langle 0, 0 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.0, 1.5 \rangle$ | $\langle 1.2, 0.9 \rangle$ |
| $v_5$ | $\langle 1.1, 0.4 \rangle$ | $\langle 1.1, 0.4 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.0, 1.5 \rangle$ | $\langle 0.8, 0.6 \rangle$ |
| $v_6$ | $\langle 0, 0 \rangle$ | $\langle 0.7, 1.0 \rangle$ | $\langle 0, 0 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.4, 1.8 \rangle$ | $\langle 1.6, 1.2 \rangle$ |
| $v_7$ | $\langle 0.1, 0.8 \rangle$ | $\langle 0.1, 0.8 \rangle$ | $\langle 0.4, 0.3 \rangle$ | $\langle 0.4, 0.3 \rangle$ | $\langle 1.2, 0.9 \rangle$ | $\langle 0.4, 0.3 \rangle$ |
| $v_8$ | $\langle 0.3, 0.5 \rangle$ | $\langle 0.6, 1.0 \rangle$ | $\langle 0.4, 0.3 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.0, 1.5 \rangle$ | $\langle 0.8, 0.6 \rangle$ |
| $v_9$ | $\langle 0, 0 \rangle$ | $\langle 0.6, 0.2 \rangle$ | $\langle 0, 0 \rangle$ | $\langle 0.4, 0.3 \rangle$ | $\langle 1.2, 0.9 \rangle$ | $\langle 0.8, 0.6 \rangle$ |
| $v_{10}$ | $\langle 0.9, 0.7 \rangle$ | $\langle 0.9, 0.7 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 0.8, 0.6 \rangle$ | $\langle 2.8, 2.1 \rangle$ | $\langle 0.8, 0.6 \rangle$ |
| $\sum$ | $\langle 3.6, 4.4 \rangle$ | $\langle 6.7, 8.6 \rangle$ | $\langle 4.0, 3.0 \rangle$ | $\langle 7.6, 5.7 \rangle$ | $\langle 31.4, 14.2 \rangle$ | $\langle 10.4, 7.8 \rangle$ |

**Definition 3.2.** Let $\mathfrak{I}_{sh}$ be an IFSHG. If every hyperedge has the same cardinality $r$, then $\mathfrak{I}_{sh}$ is said to be an $r$-*uniform* IFSHG.

**Theorem 3.3.** *Let $\mathfrak{I}_{sh}$ be an $r$-uniform IFSHG. Let $\mu_i : V \to [0, 1]$ and $\nu_i : V \to [0, 1]$ be a constant function say $\langle C_1, C_2 \rangle$. Then:*

*(i)* $\sum_{v_i \in V} d_e(v_i) = 2|E_h|\langle C_1, C_2 \rangle$;

*(ii)* $\sum_{v_i \in V} d(v_i) = r|E_h|\langle C_1, C_1 \rangle$;

*(iii)* $\sum_{v_i \in V} d_a(v_i) \leq r(r-1)|E_h|\langle C_1, C_2 \rangle$;

*(vi)* $\sum_{v_i \in V} d_{ca}(v_i) \leq 2(r-1)|E_h|\langle C_1, C_2 \rangle$.

*Proof.* Let $\mathfrak{I}_{sh}$ be an $r$-uniform IFSHG.
$(i)$: To prove $\sum_{v_i \in V} d_e(v_i) = 2|E_h|\langle C_1, C_2 \rangle$.

In an $r$-uniform IFSHG, each hyperedge contains $r$ vertices. We know that every hyperedge consists of two end vertices, $\sum\limits_{v_i \in V} d_e(v_i) = 2 \sum\limits_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$. Since $\langle \mu_i, \nu_i \rangle (v_i) = \langle C_1, C_2 \rangle$ is a constant, then $\sum\limits_{v_i \in V} d_e(v_i) = 2|E_h| \langle C_1, C_2 \rangle$.

Hence case $(i)$ is proved. Similarly, case $(ii)$ follows.

$(iii)$: To prove Case $(iii)$, in general, for every vertex $v_i \in E_{h_j}$, there are $|E_{h_j}| - 1$ partial hyperedges where $v_i$ is an end vertex.

Thus every hyperedge in $\mathfrak{I}_{sh}$ contains $|E_{h_j}|(|E_{h_j}| - 1)\langle C_1, C_2 \rangle$ to the sum of adjacent degree of the vertex, thus

$$\sum_{v_i \in V} d_a^c(v_i) \leq \sum_{E_{h_j} \in E_h} |E_{h_j}|(|E_{h_j}| - 1)\langle C_1, C_2 \rangle$$

Since it is $r$- uniform, each vertex can be adjacent to at most $r-1$ other vertices in each hyperedge, which implies $\sum\limits_{v_i \in V} d_a^c(v_i) \leq r(r-1)|E_h| \langle C_1, C_2 \rangle$

Thus case $(iii)$ is proved. Similarly, case $(iv)$ follows obviously. □

**Note.** In an IFSHG, for any pair of hyperedges if there is only one common vertex, the inequality in case $(iii)$ and $(iv)$ will result in equality condition, suppose if any pair of hyperedge have more than one common vertex, then inequality holds true.

**Example 3.2.** *Consider an IFSHG $\mathfrak{I}_{sh}$ with $V = \{v_1, v_2, \ldots, v_{10}\}$ and $E_h = \{E_{h_1}, E_{h_2}, E_{h_3}, E_{h_4}\}$ where $E_{h_1} = \{v_1, v_2, v_5, v_8\}$, $E_{h_2} = \{v_1, v_3, v_6, v_7\}$, $E_{h_3} = \{v_4, v_5, v_6, v_7\}$ and $E_{h_4} = \{v_4, v_8, v_9, v_{10}\}$. The following Figure 2 is an example of an $r$-uniform IFSHG.*

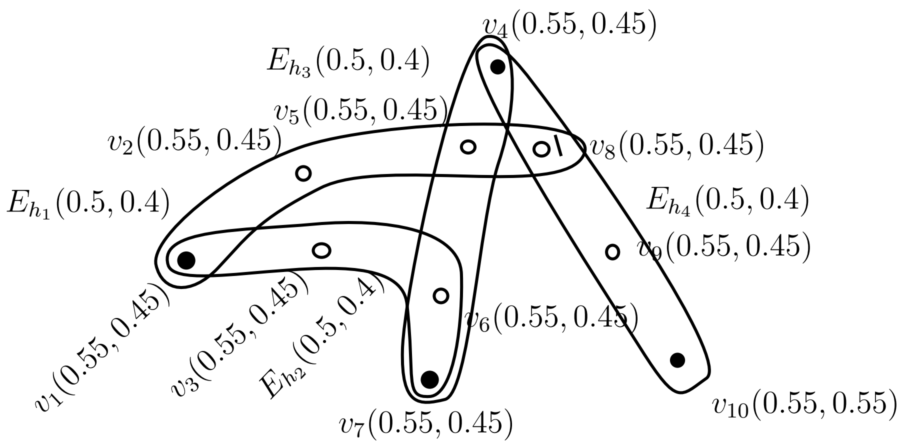


Figure 2. Example of 4-uniform IFSHG

**Corollary 3.2.** (1) *Let $\mathfrak{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ be an IFSHG such that the functions $\mu_i : V \to [0, 1]$ and $\nu_i : V \to [0, 1]$ are constant. Then for each $v_i \in V$:*

$(i)$ $d_e(v_i) \leq \langle \mu_i, \nu_i \rangle (v_i). \deg_e(v_i)$

$(ii)$ $d(v_i) \leq \langle \mu_i, \nu_i \rangle (v_i). \deg(v_i)$

$(iii)$ $d_a(v_i) \leq \langle \mu_i, \nu_i \rangle (v_i). \deg_a(v_i)$

$(iv)$ $d_{ca}(v_i) \leq \langle \mu_i, \nu_i \rangle (v_i). \deg_{ca}(v_i)$

(2) *Let* $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ *be an effective IFSHG such that the functions* $\mu_i : V \to [0,1]$ *and* $\nu_i : V \to [0,1]$ *are constant. Then for each* $v_i \in V$*:*

$(i)$ $d_e(v_i) = \langle \mu_i, \nu_i \rangle (v_i). \deg_e(v_i)$;

$(ii)$ $d(v_i) = \langle \mu_i, \nu_i \rangle (v_i). \deg(v_i)$;

$(iii)$ $d_a(v_i) = \langle \mu_i, \nu_i \rangle (v_i). \deg_a(v_i)$;

$(iv)$ $d_{ca}(v_i) = \langle \mu_i, \nu_i \rangle (v_i). \deg_{ca}(v_i)$.

## 3.1 Order and size of IFSHGs

**Definition 3.3.** Consider a IFSHG $\mathfrak{I}_{sh}$, the *Order* of $\mathfrak{I}_{sh}$ is $O(\mathfrak{I}_{sh}) = \sum_{v_i \in V} \langle \mu_i, \nu_i \rangle (v_i), \forall v_i \in V$. Different adjacencies result in three types of *sizes*, which are defined as follows:

$i)$ by considering the hyperedge values, the *Crisp Size* of $\mathfrak{I}_{sh}$ is defined as

$$CS(\mathfrak{I}_{sh}) = \sum_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j});$$

$ii)$ by considering the consecutive adjacency of vertices, the *Size* of $\mathfrak{I}_{sh}$ is given by

$$S(\mathfrak{I}_{sh}) = \sum_{v_i, v_j \in C_v} \left\langle \mu^c_{ij}, \nu^c_{ij} \right\rangle (v_i, v_j);$$

$iii)$ by considering the cardinality of hyperedges, the *Pseudo Size* of $\mathfrak{I}_{sh}$ is defined as,

$$PS(\mathfrak{I}_{sh}) = \sum_{E_{h_j} \in E_h} |E_{h_j}|.$$

**Example 3.3.** From Example 3.1,

- *Order:* $O(\mathfrak{I}_{sh}) = \langle 6.2, 3.2 \rangle$;
- *Size:* $S(\mathfrak{I}_{sh}) = \langle 11.7, 3.8 \rangle$;
- *Crisp size:* $CS(\mathfrak{I}_{sh}) = \langle 1.9, 2.2 \rangle$;
- *Pseudo size:* $PS(\mathfrak{I}_{sh}) = 19$.

**Theorem 3.4.** *Let* $\mathfrak{I}_{sh}$ *be an effective IFSHG where* $\mu_i : V \to [0,1]$ *and* $\nu_i : V \to [0,1]$ *are constant functions. Then:*

$(a)$ $CS(\mathfrak{I}_{sh}) = \dfrac{|E_h|}{|V|} O(\mathfrak{I}_{sh})$,

$(b)$ $S(\mathfrak{I}_{sh}) \leq CS(\mathfrak{I}_{sh}) \left[ \dfrac{PS(\mathfrak{I}_{sh})}{|E_h|} - 1 \right]$.

*Proof.* $(a)$: Given that $\mathfrak{I}_{sh}$ is an effective IFSHG. For each vertex $v_i$ in $V$, let $\langle \mu_i, \nu_i \rangle (v_i) = \langle C_1, C_2 \rangle$ be a constant, that needs not be an integer. Then the order of $\mathfrak{I}_{sh}$,

$$O(\mathfrak{I}_{sh}) = \sum_{v_i \in V} \langle \mu_i, \nu_i \rangle (v_i) = |V| \langle C_1, C_2 \rangle$$

$$\langle C_1, C_2 \rangle = \frac{O(\mathfrak{I}_{sh})}{|V|} \tag{2}$$

By the Definition of crisp size,

$$CS(\mathfrak{I}_{sh}) = \sum_{E_{h_j} \in E_h} \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j}).$$

$$CS(\mathfrak{I}_{sh}) = |E_h| \langle C_1, C_2 \rangle = |E_h| \frac{O(\mathfrak{I}_{sh})}{|V|} \qquad \text{[from (2)]} \tag{3}$$

Hence $(a)$ is proved.

$(b)$: By the Definition of size,

$$S(\mathfrak{I}_{sh}) = \sum_{v_i, v_j \in C_v} \left\langle \mu_{ij}^c, \nu_{ij}^c \right\rangle (v_i, v_j).$$

By the consecutive adjacency,

$$\begin{aligned}
S(\mathfrak{I}_{sh}) &\leq 2[\sum_{E_{h_j} \in E_h} (|E_{h_j}| - 1) \langle C_1, C_2 \rangle \\
&\leq 2[\langle C_1, C_2 \rangle \sum_{E_{h_j} \in E_h} |E_{h_j}| - \langle C_1, C_2 \rangle \sum_{E_{h_j} \in E_h} 1] \\
&\leq 2[\langle C_1, C_2 \rangle \sum_{E_{h_j} \in E_h} |E_{h_j}| - \langle C_1, C_2 \rangle |E_h|] \\
&\leq 2[\frac{CS(\mathfrak{I}_{sh})}{|E_h|} \sum_{E_{h_j} \in E_h} |E_{h_j}| - CS(\mathfrak{I}_{sh})] \qquad \text{[from (3)]} \\
&\leq 2[\frac{CS(\mathfrak{I}_{sh})}{|E_h|} PS(\mathfrak{I}_{sh}) - CS(\mathfrak{I}_{sh})] \quad \text{[by the definition of pseudo size]} \\
&\leq 2CS(\mathfrak{I}_{sh})[\frac{PS(\mathfrak{I}_{sh})}{|E_h|} - 1].
\end{aligned} \tag{4}$$

Hence $(b)$ is also obtained. □

**Corollary 3.3.** *The following statements are obvious for IFSHGs:*

$(i)$ $CS(\mathfrak{I}_{sh}) \leq S(\mathfrak{I}_{sh})$;

$(ii)$ $|v_i| \min\langle \mu_i, \nu_i \rangle \leq O(\mathfrak{I}_{sh}) \leq |v_i| \max(\langle \mu_i, \nu_i \rangle, \forall v_i \in V$;

$(iii)$ $0 \leq PS(\mathfrak{I}_{sh}) \leq |V|(|V| - 1)$;

$(iv)$ $\sum_{v_i \in V} \deg(v_i) = PS(\mathfrak{I}_{sh})$.

# 4 Regularity of intuitionistic fuzzy semihypergraphs

In this section, the study concentrates on various degrees that are associated with a vertex in an IFSHGs results in different regularity concepts.

**Definition 4.1.** Let $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be an IFSHG. Then for any real number, if each vertex has same degree then $\mathfrak{I}_{sh}$ is said to be *regular*.

In particular, $\mathfrak{I}_{sh}$ is said to be an *e-regular* if each vertex has same end vertex degree, if each vertex has equal adjacent degree, then $\mathfrak{I}_{sh}$ is said to be *a-regular,* and if each vertex has same consecutive adjacent degree, then $\mathfrak{I}_{sh}$ is said to be *ca-regular*.

**Definition 4.2.** Let $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be an IFSHG. The *total degree* of $v_i$ is defined as:

(1) The *total $e$-degree* of vertex $v_i$ is given by $td_e(v_i) = d_e(v_i) + \langle \mu_i, \nu_i \rangle(v_i)$.

(2) The *total degree* of vertex $v_i$ is given by $td(v_i) = d(v_i) + \langle \mu_i, \nu_i \rangle(v_i)$.

(3) The *total adjacent degree* of vertex $v_i$ is given by $td_a(v_i) = d_a(v_i) + \langle \mu_i, \nu_i \rangle(v_i)$.

(4) The *total consecutive adjacent degree* of vertex $v_i$ is given by $td_{ca}(v_i) = d_{ca}(v_i) + \langle \mu_i, \nu_i \rangle(v_i)$.

**Definition 4.3.** Let $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be an IFSHG. If all $v_i \in V$ in $\mathfrak{I}_{sh}$ have the same:

(1) total end vertex degree $td_e(v_i)$, then $\mathfrak{I}_{sh}$ is said to be *totally $e$-regular* IFSHGs.

(2) total degree $td(v_i)$, then $\mathfrak{I}_{sh}$ is said to be *totally regular* IFSHGs.

(3) total adjacent degree $td_a(v_i)$, then $\mathfrak{I}_{sh}$ is said to be *totally $a$-regular* IFSHGs.

(4) total consecutive adjacent degree $td_{ca}(v_i)$, then $\mathfrak{I}_{sh}$ is said to be *totally $ca$-regular* IFSHGs.

**Theorem 4.1.** *Let $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be an IFSHG. If $\mu_i : V \to [0,1], \nu_i : V \to [0,1]$ are constant functions, then the following statements are equivalent:*

*(1) $\mathfrak{I}_{sh}$ is an $e$-regular IFSHG,*

*(2) $\mathfrak{I}_{sh}$ is a totally $e$-regular IFSHG.*

*Conversely, if (1) and (2) are equivalent, then $\langle \mu_i, \nu_i \rangle$ is a constant.*

*Proof.* For all $v_i \in V$, let $\mu_i : V \to [0,1], \nu_i : V \to [0,1]$ be a constant function, say $\langle C_1, C_2 \rangle$.

(1) ⇒ (2). Suppose that $\mathfrak{I}_{sh}$ is an $e$-regular IFSHG. By Definition 3.1, end vertex degree will be constant for all vertex $v_i$, say $\langle s_1, s_2 \rangle$, where $s_1$ represents membership and $s_2$ represents non-membership values. Then,

$$d_e(v_i) = \langle s_1, s_2 \rangle, \forall v. \tag{5}$$

The total $e$-degree is given by,

$$\begin{aligned} td_e(v_i) &= d_e(v_i) + \langle \mu_i, \nu_i \rangle(v_i) \\ &= \langle s_1, s_2 \rangle + \langle \mu_i, \nu_i \rangle(v_i), \quad \text{[from (5)]} \\ &= \langle s_1, s_2 \rangle + \langle C_1, C_2 \rangle \\ &= \langle s_1 + C_1, s_2 + C_2 \rangle, \end{aligned}$$

which proves that $\mathfrak{I}_{sh}$ is a totally $e$-regular IFSHG.

(2) ⇒ (1). Assume that $\mathfrak{I}_{sh}$ is a totally $e$-regular IFSHG. Let the total degree of each vertex be $\langle l_1, l_2 \rangle$, where $l_1$ represents membership and $l_2$ represents non-membership values. Then, $td_e(v_i) = \langle l_1, l_2 \rangle, \forall v_i$. Since $td_e(v_i) = d_e(v_i) + \langle \mu_i, \nu_i \rangle(v_i)$, then for all $v_i \in V$,

$$\begin{aligned} \langle l_1, l_2 \rangle &= d_e(v_i) + \langle \mu_i, \nu_i \rangle(v_i), \\ d_e(v_i) &= \langle l_1, l_2 \rangle - \langle \mu_i, \nu_i \rangle(v_i) \\ &= \langle l_1, l_2 \rangle - \langle C_1, C_2 \rangle \\ &= \langle l_1 - C_1, l_2 - C_2 \rangle. \end{aligned}$$

Hence $\mathfrak{I}_{sh}$ is an $e$-regular IFSHG.

Hence, if $\mathfrak{I}_{sh}$ is $e$-regular, then it is totally $e$-regular.

Conversely, assume the both (1) and (2) are equivalent, and $\mathfrak{I}_{sh}$ is $e$-regular and totally $e$-regular. Let $v_m$ and $v_n$ be any two vertices where $\langle \mu_i, \nu_i \rangle(v_m) \neq \langle \mu_i, \nu_i \rangle(v_n)$.

Since $\mathfrak{I}_{sh}$ is $e$-regular and totally $e$-regular, $d_e(v_m) = \langle s_1, s_2 \rangle = d_e(v_n)$ and

$$td_e(v_m) = \langle s_1, s_2 \rangle + \langle \mu_i, \nu_i \rangle(v_m) = td_e(v_n) = \langle s_1, s_2 \rangle + \langle \mu_i, \nu_i \rangle(v_n),$$

hence, $\langle \mu_i, \nu_i \rangle(v_m) = \langle \mu_i, \nu_i \rangle(v_n)$, which is a contradiction to our assumption that vertices have different values. □

**Theorem 4.2.** *Let $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be both an $e$-regular and totally $e$-regular IFSHG. Then $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ for each element $v_i \in V$, are constant functions.*

*Proof.* Suppose that $\mathfrak{I}_{sh}$ is an $e$-regular and totally $e$-regular IFSHG for some constants $\langle s_1, s_2 \rangle$ and $\langle l_1, l_2 \rangle$. Then for every vertex $v_i$, $d_e(v_i) = \langle s_1, s_2 \rangle$ and $td_e(v_i) = (l_1, l_2)$.

For all $v_i \in V$,

$$\begin{aligned} d_e(v_i) + \langle \mu_i, \nu_i \rangle(v_i) &= \langle l_1, l_2 \rangle \\ \langle s_1, s_2 \rangle + \langle \mu_i, \nu_i \rangle(v_i) &= \langle l_1, l_2 \rangle \\ \langle \mu_i, \nu_i \rangle(v_i) &= \langle l_1, l_2 \rangle - \langle s_1, s_2 \rangle. \end{aligned}$$

Since $\langle l_1, l_2 \rangle$ and $\langle s_1, s_2 \rangle$ are constants, $\langle \mu_i, \nu_i \rangle(v_i)$ is also constant. Hence proved. □

**Example 4.1.** Consider an IFSHG where $V = \{v_1, v_2, \ldots, v_8\}$ and $E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}, E_{h_4}\}$ are hyperedges in which $\{v_1, v_6, v_8\}$ are end vertices, $\{v_2, v_3, v_4, v_5, v_7\}$ are middle end vertices.

In Figure 3, for each vertex, the end vertex degree is $d_e(v_i) = \langle 0.2, 0.4 \rangle$ and total $e$-degree is $td_e(v_i) = \langle 0.5, 0.9 \rangle$. Thus, it is an $e$-regular and totally $e$-regular IFSHG.

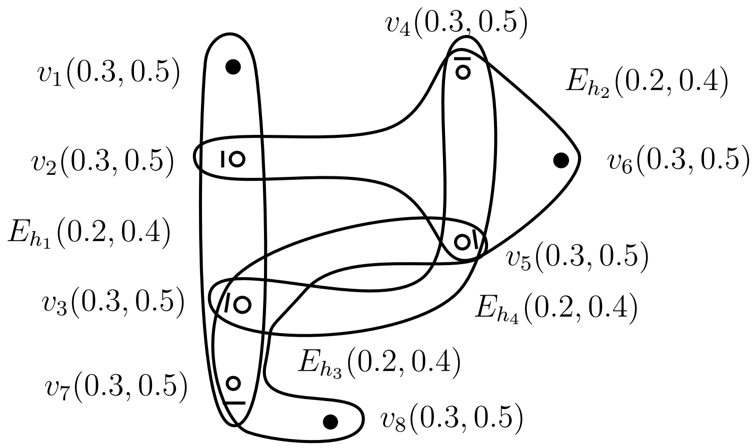


Figure 3. $e$-Regular and Totally $e$-regular

**Corollary 4.1** (Converse of Theorem 4.2)**.** *Suppose $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ for each element $v_i \in V$, is a constant function then $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ will be both an e-regular and totally e-regular IFSHG, if the IFSHG is effective; however, the converse might not be accurate in other cases.*

<u>Case I:</u> *Consider an effective IFSHG (Figure 4) with the vertex set $V = \{v_1, v_2, \ldots, v_7\}$ where $\langle \mu_i, \nu_i \rangle(v_i) = \langle 0.4, 0.5 \rangle$, for every $v_i$. Then by Definition of Effective IFSHG, for each hyperedge $E_{h_j}$, the values are $\langle 0.4, 0.5 \rangle$.*

*For every vertex $v_i$ in $V$, the degree is $d_e(v_i) = \langle 0.4, 0.5\rangle$, and the total degree is $td_e(v_i) = \langle 0.8, 0.1\rangle$. Thus, given IFSHG is both an $e$-regular and totally $e$-regular IFSHG.*

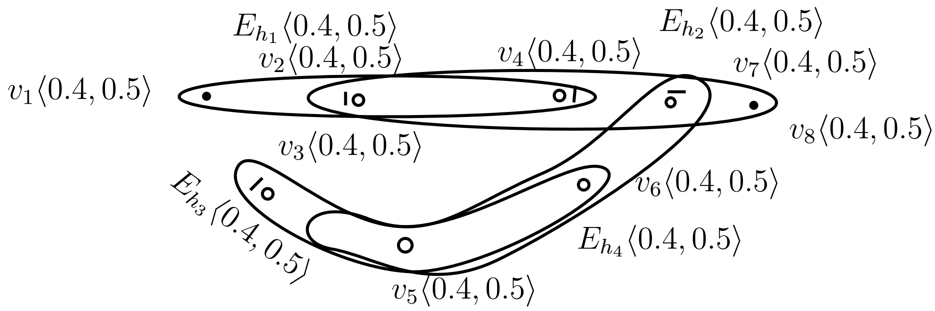


Figure 4. Effective IFSHG which is $e$-regular and totally $e$-regular

*<u>Case II:</u> Consider an IFSHG which is not effective, with the vertex set $V = \{v_1, v_2, \dots, v_7\}$ where $\langle\mu_i, \nu_i\rangle(v_i) = \langle 0.4, 0.5\rangle$ and the hyperedges be $E_{h_1} = \langle 0.2, 0.4\rangle$, $E_{h_2} = \langle 0.3, 0.5\rangle$, $E_{h_3} = \langle 0.1, 0.3\rangle, E_{h_4} = \langle 0.3, 0.3\rangle$ (see Figure 5). The values of $\langle\mu_i, \nu_i\rangle(v_i)$ are constant for every vertex, but $d_e(v_i)$ is not same for all $v_i \in V$. For instance, $d_e(v_1) = \langle 0.3, 0.3\rangle$, $d_e(v_7) = \langle 0.1, 0.3\rangle$.*

*Thus, $\mathfrak{I}_{sh}$ is neither $e$-regular nor totally $e$-regular intuitionistic fuzzy semihypergraph.*

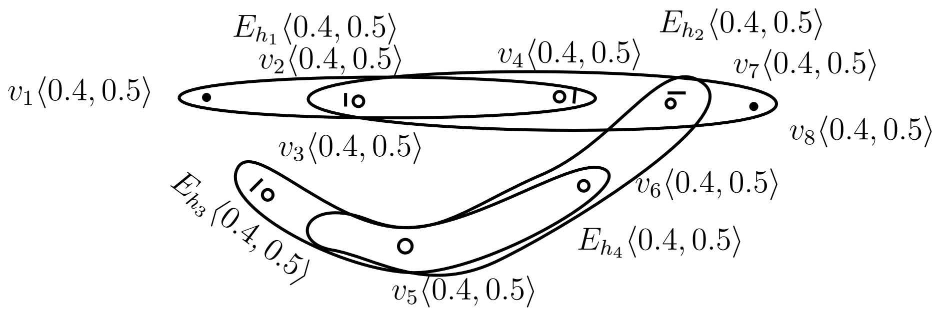


Figure 5. Counter example for the converse of Theorem 4.2.

**Theorem 4.3.** *The crisp size of an $e$-regular IFSHG is $\frac{|V|\langle s_1, s_2\rangle}{2}$.*

*Proof.* Suppose that $\mathfrak{I}_{sh}$ is an $e$-regular IFSHG. Then,

$$d_e(v_i) = \langle s_1, s_2\rangle\,, \text{ for all } v_i. \tag{6}$$

The crisp size of $\mathfrak{I}_{sh}$, $CS(\mathfrak{I}_{sh}) = \sum\limits_{E_{h_j}\in E_h} \langle\mu_{ij}, \nu_{ij}\rangle(E_{h_j})$. From Theorem 3.1,

$$\sum_{v_i\in V} d_e(v_i) = 2\sum_{E_{h_j}\in E_h} \langle\mu_{ij}, \nu_{ij}\rangle(E_{h_j}) = 2CS(\mathfrak{I}_{sh}) \tag{7}$$

From Equation (6) it follows that

$$\sum_{v_i\in V} \langle s_1, s_2\rangle = 2CS(\mathfrak{I}_{sh}),$$
$$|V|\,\langle s_1, s_2\rangle = 2CS(\mathfrak{I}_{sh}).$$

Hence,

$$CS(\mathcal{I}_{sh}) = \frac{|V| \langle s_1, s_2 \rangle}{2}.$$

Hence proved. □

**Theorem 4.4.** *Let* $\mathcal{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ *be a totally* $e$*-regular IFSHG. Then,* $2CS(\mathcal{I}_{sh}) + O(\mathcal{I}_{sh}) = |V| \langle l_1, l_2 \rangle$.

*Proof.* Assume $\mathcal{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ is a totally $e$-regular IFSHG, such that, $td_e(v_i) = \langle l_1, l_2 \rangle$, for all $v_i \in V$ it follows that:

$$td_e(v_i) = d_e(v_i) + \langle \mu_i, \nu_i \rangle (v_i) = \langle l_1, l_2 \rangle .$$

If summation is performed over all the vertices $v_i \in V$,

$$\sum_{v_i \in V} d_e(v_i) + \sum_{v_i \in V} \langle \mu_i, \nu_i \rangle (v_i) = \sum_{v_i \in V} \langle l_1, l_2 \rangle .$$

From Equation (7) of Theorem 4.3 and Definition 3.3,

$$2CS(\mathcal{I}_{sh}) + O(\mathcal{I}_{sh}) = |V| \langle l_1, l_2 \rangle .$$ □

**Corollary 4.2.** Let $\mathcal{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ be an $e$-regular and totally $e$-regular IFSHG. Then $O(H) = |V| (\langle l_1, l_2 \rangle - \langle s_1, s_2 \rangle)$.

*Proof.* The result is obvious from Theorem 4.3 and Theorem 4.4. □

**Example 4.2.** Consider Example 3.1, where $|V| = 8$, $CS(\mathcal{I}_{sh}) = \langle 0.8, 1.6 \rangle$, $O(\mathcal{I}_{sh}) = \langle 2.4, 4.0 \rangle$, $\langle s_1, s_2 \rangle = \langle 0.2, 0.4 \rangle$, $\langle l_1, l_2 \rangle = \langle 0.5, 0.9 \rangle$. Thus,

$(a)$: $CS(\mathcal{I}_{sh}) = \langle 0.8, 1.6 \rangle = \frac{|V| \langle s_1, s_2 \rangle}{2}$.

$(b)$: $2CS(\mathcal{I}_{sh}) + O(\mathcal{I}_{sh}) = \langle 4.0, 7.2 \rangle = |V| \langle l_1, l_2 \rangle$.

**Definition 4.4.** A *cycle* in an IFSHG $\mathcal{I}_{sh} = \langle V, C_v, E_{h_j} \rangle$ is a sequence of hyperedges such that the hyperedges in the cycle must connect a series of vertices in a closed loop. Specifically, for a sequence of hyperedges $E_{h_1}, E_{h_2}, \ldots, E_{h_k}$ with $(k \geq 3)$, there exists a set of vertices $\{v_1, v_2, \ldots, v_k\}$ such that, each hyperedge $E_{h_j}$ includes the vertices $v_i$ and $v_{i+1}$ with $v_{k+1} = v_1$, forming a closed path.

A cycle is said to be even (odd) if $k$ is even (odd).

**Definition 4.5.** An IFSHG is defined to be $me$*-regular* if all middle vertices have the same degree and all end vertices have the same degree. Suppose for all middle vertices, $d(v_i) = \mathcal{M}$ and for all end vertices, $d(v_i) = \mathcal{N}$, then is denoted as $\langle \mathcal{M}, \mathcal{N} \rangle$-$me$*-regular IFSHG.*

**Note.** Here, $\mathcal{M} = \langle m_1, m_2 \rangle$ represents the membership and non-membership degree of middle vertices while $\mathcal{N} = \langle n_1, n_2 \rangle$ represents the membership and non-membership degree of end vertices.

**Theorem 4.5.** *Suppose that* $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ *is an IFSHG, where the underlying IFSHG is an odd cycle. Then* $\mathfrak{I}_{sh}$ *is a* $me$*-regular IFSHG if and only if* $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$ *is constant for all hyperedges.*

*Proof.* Suppose that $\mathfrak{I}_{sh}$ is a $\langle \mathcal{M}, \mathcal{N} \rangle -$ $me$-regular IFSHG for some constants, implies uniformity in the degrees of vertices, where $\mathcal{M} = \langle m_1, m_2 \rangle$ and $\mathcal{N} = \langle n_1, n_2 \rangle$.
Let $E_{h_1}, E_{h_2}, \ldots, E_{h_{2r+1}}, r = 0, 1, 2, \ldots$ be the hyperedges in an underlying IFSHG.

Let $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_1})$ be any constant, say $\langle k_1, k_2 \rangle$. Then $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_2}) = \langle n_1, n_2 \rangle - \langle k_1, k_2 \rangle$, $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_3}) = \langle n_1, n_2 \rangle - (\langle n_1, n_2 \rangle - \langle k_1, k_2 \rangle)$, and so on. Hence continuing on the same way,

$$\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_{2r+1}}) = \begin{cases} \langle k_1, k_2 \rangle, & \text{if } r \text{ is even} \\ \langle n_1, n_2 \rangle - \langle k_1, k_2 \rangle, & \text{if } r \text{ is odd} \end{cases}$$

Since it is an odd cycle, $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_1}) = \langle \mu_{ij}, \nu_{ij} \rangle (E_{h_{2r+1}}) = \langle k_1, k_2 \rangle$. Also, as $v_i$ is incident in both the hyperedges $E_{h_1}$ and $E_{h_{2r+1}}$, the end vertex degree condition implies:

$$d(v_i) = 2\langle k_1, k_2 \rangle \quad \Longrightarrow \quad \langle n_1, n_2 \rangle = 2\langle k_1, k_2 \rangle \quad \Longrightarrow \quad \langle k_1, k_2 \rangle = \frac{\langle n_1, n_2 \rangle}{2}.$$

Thus,$\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_{2r+1}}) = \frac{\langle n_1, n_2 \rangle}{2}$, for all $r$. This shows that $\langle \mu_{ij}, \nu_{ij} \rangle$ is a constant function.

Conversely, assume that $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j}) = \langle C_1, C_2 \rangle$ is a constant, for all hyperedges. Then:

$$d(v_i) = \begin{cases} \langle 0, 0 \rangle, & \text{if } v_i \text{ is not an end vertex of any hyperedge in } \mathfrak{I}_{sh}, \\ 2\langle C_1, C_2 \rangle, & \text{if } v_i \text{ is an end vertex of any hyperedge in } \mathfrak{I}_{sh}. \end{cases}$$

Thus, $\mathfrak{I}_{sh}$ is a $\langle 0, 0 \rangle, 2\langle c_1, c_2 \rangle$-$me$-regular IFSHG. □

**Corollary 4.3.** *Suppose that* $\mathfrak{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ *is an IFSHG, where an underlying semihypergraph is an even cycle and* $E_{h_1}, E_{h_2}, \ldots, E_{h_{2r}}, r = 0, 1, 2, \ldots$ *are the hyperedges in an underlying semihypergraph in that order. Then* $\mathfrak{I}_{sh}$ *is a* $me$*-regular IFSHG if and only if either* $\langle \mu_{ij}, \nu_{ij} \rangle (E_{h_j})$ *is constant, or the alternative hyperedges have the same membership and non-membership values.*

# 5 Conclusion

In this article, various types of degrees of vertex and three types of sizes of IFSHGs are defined. It has been proved that for any vertex $v_i$ in $\mathfrak{I}_{sh}$, degrees have a linear ordering:

$$d_a(v_i) \geq d_{ca}(v_i) \geq d(v_i) \geq d_e(v_i).$$

An $r$-uniform IFSHG has been defined and it has been proved that

$$\sum_{v_i \in V} d_a(v_i) \leq r(r-1)|E_h|\langle C_1, C_2 \rangle.$$

Types of regular and totally IFSHGs based on degrees are also defined with its properties. IFSHGs can be used to model uncertainity relationships in various fields like financial modeling, data mining, image processing and decision-making systems to analyse vaguness and predict risk management strategies. Further, as an extension, the authors have an idea to work on operations and domination of IFSHGs.

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