

Intuitionistic fuzzy estimation of the liquidity of the banks. A generalized net model

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Abstract: A generalized net is used to construct a model which describes the typical banking activities. The model can be used to simulate some processes in the bank sector. For better understanding here we use intuitionistic fuzzy estimation.

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1 Introduction

Banks play an important and crucial role in each country as they are main participants as well as key enablers of the financial intermediation process. This position, combined with exposure to internal and external risks and shocks as well as information asymmetry, market failures, make strict control and regulation a much needed requirement. The regulatory regime is primarily placed at the country level but more and more becomes a matter of pan-regional or international interest due to cross border relations that the banks are involved in. The regulations are implemented and enforced as a prevention against the system risk to market participants and to the economy. There are three main motives mentioned in the specialized literature for the need of strict regulation of banking sector:

- Significant share of the banks’ activities are actually inwardly vulnerable because the banks tend to have longer term assets versus liabilities (in simple terms: banks tend to extend credits with terms longer than the terms of the deposits necessary to finance the credits extended);

- Banks are prone to infection effects where small and distant disturbances may have a snow ball effect on the bank's balance sheet, its credibility in the market and its ability to find counter parts to market transactions (for example see [7]); and
- Banks are the main provider of specific services to the society – a substantial share of population's liquid assets is held at banks, banks secure and guarantee the payments between the various economic agents in the country and abroad, banks provide transfer of liquidity between parties with excess liquidity (depositors) and parties that require liquidity (loan takers), etc.

In our analysis, for simplicity and as an introduction to a much broader topic, we have focused on the historically principal banking activity, commercial banking, which involves taking deposits and extending loans. For simplicity we narrow down our analysis of the bank activities to the environment of single currency and stable interest rates.

Each bank operating needs to carefully manage three key areas of its activities: liquidity management, credit risk management, and reserve requirements.

1.1 *Liquidity management*

In order to meet their planned and incidental expenses, the credit institutions must maintain a certain level of liquidity. By definition liquidity is the capability of an asset to be transformed into money in a short period of time at minimum transaction costs to meet bank's obligations. From this point of view, the liquidity is not only a necessary, but also an obligatory condition accompanying all banks' activities.

In Bulgaria, as well as in majority of advanced economies, the management and the supervision of banks' liquidity is carried out by a specially designated Bank Supervision Administration, which is located a part of central bank or as a specially designated regulatory body (in Bulgaria it is a part of the Bulgarian National Bank's structure [5]). Each commercial bank has in its structures a liquidity management body, named ALCO (Assets and Liabilities Committee). This department monitors daily liquidity, plans for liquidity events as well as drafts action plans in cases of unforeseen outflow of funds and performs periodical performance checks (often called *stress test*¹). This body also maintains an informational system and procedures for reporting of bank obligations and obligations of third parties to the bank which are later aggregated into maturity structure tables for the balance sheet and off-balance sheet positions; based on this information it performs assessment of cash inflows and outflows and calculation of the necessary amount of liquid assets and liquidity buffers.

It is common practice that the maturity table allocates the each individual position (being inflow or outflow) based on term into one of the following categories:

- up to seven days;
- from eight days to one month;
- over one month up to three months;
- over three months up to six months;
- over six months up to a year;
- over a year.

¹ Stress tests are a mandatory component of the risk management strategy. In their essence, they are various scenarios simulating certain adverse or positive shocks, aimed at exposing areas of bank's vulnerability. Based on those results management can build a preventive action plan or adjust current strategy to minimize impact of those risks.

Banks evaluate the cash inflows and outflows according to the maturity of each balance sheet position category individually. Where the cash inflows within one maturity interval exceed the cash outflows of the same maturity interval, the so called long mismatch is reviewed as an additional cash inflow for the next maturity interval, and in case of short mismatch – as potential alert for liquidity need.

Pursuant to the regulatory requirements, banks' liquid assets are: the moneys maintained in accounts with the central bank; assets in settlement accounts with other credit institutions, as well as depositing funds with other banks with a term up to 7 days; the issued by central governments or central banks marketable debt securities, including treasury bonds/bills of the local government (e.g. Bulgarian Government); marketable debt securities issued by international banks and organizations; investment gold.

The liquidity of the credit institutions is calculated using the two key ratios:

- *Liquid asset ratio* – amount of available liquid assets / the amount of deposits and other bank's liabilities
- *Liquidity coefficient by maturity time bands* – this represent the ratio of the amount of assets (cash inflow) for the relevant maturity time band plus the excess of the net cash flow from the preceding time band to deposits and other credit institution's liabilities (cash outflow) for the same maturity time band; where a shortage of the net cash flow from the preceding time band is established in calculating the ratio, the shortage should be added to deposits and other bank's liabilities (cash outflow).

According to Ordinance 11, bank's liquidity is deemed acceptable if the liquidity ratio by maturity time bands is not under 1 at least for the first two maturity time bands. Central banks monitor bank's liquidity and if necessary it can intervene and impose minimum values of coefficients for each bank individually under which they may not go down.

1.2 Credit risk management

Another area of bank's activity that involves taking risk and careful management of risk is credit risk. According to banking legislation, risk exposures are all loans and other claims of a bank, regardless of the ground for their occurrence and the financial instrument used, for which there is a risk of losses [6]. Each bank assesses and classifies its risk exposures of each loan or basket of loans when there are signs of deterioration in the financial condition of the obligor or circumstances that can lead to future losses. The risk exposures are evaluated and classified based on the period of delay in amounts due, the assessment of the debtor's financial state and the main sources for repayment of the debtor's obligation. The estimated risk exposures are arranged in four groups as follows.

- *Regular exposition*. This group includes expositions whose principals and interests are repaid with the terms of the contract or delay up to 30 days.
- *Supervised expositions*. They include expositions in which are accumulated overdued principal or interest from 31 to 90 days.
- *Non-performing expositions*. Non-performing expositions are risk exposures where there are significant weaknesses in their service. The principal or interest arrears payments have been past-due 91 to 180 days.
- *Lost*. The principal or interest arrears payments have been past-due over 180 days.

1.3 Reserve requirements

And finally banks' activities involve management of reserve requirements which involves setting money aside for identified or possible risk as well as money for undefined risks. Reserve requirement involves on one side setting aside a portion of client deposits as a liquidity reserve (those funds are typically placed as deposits with central banks). This typically applies to current deposits and ranges from as low as 1% in Eurozone countries (lowered on January 18, 2012 from 2%) to 10% in Bulgaria (lowered from 12% in November 28, 2008 and subject to exceptions). On the other it involves lowering expected amounts due from creditors, which are late with their repayments, by incurring reserve costs that lower the financial result of the bank. The required reserves are arranged in three groups following the risk exposure groups:

- *Supervised exposition* – reserve of 20% of outstanding principal and interest
- *Non-performing exposition* – reserve of 50% of outstanding principal and interest
- *Lost* – reserve for 100% of outstanding principal and interest

These reserves need to be kept for the whole period of time that the exposure is present and they need to be periodically evaluated and adjusted to the current status of the credit exposure of the bank.

For better understanding here we will use intuitionistic fuzzy estimation.

2 Intuitionistic fuzzy logic

Intuitionistic Fuzzy Sets (IFSs) [3, 4] are defined as an extension of ordinary fuzzy sets. All results which are valid for fuzzy sets can be transformed here too. Also, all research, for which the apparatus of fuzzy sets can be used, can be used to describe the details of IFSs.

On the other hand, there have been defined over IFSs not only operations similar to those of ordinary fuzzy sets, but also operators that cannot be defined in the case of ordinary fuzzy sets.

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E\},$$

where functions $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

For every $x \in E$, let

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, the function π determines the degree of uncertainty. Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ and for each $x \in E$, these sets have the form:

$$\{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in E\}.$$

3 The generalized net model

The GN-model [1, 2] for this section (Figure 1) contains 8 transitions and 29 places, collected in four groups and related to the four types of the tokens that will enter respective types of places:

- α -tokens and a -places represent the bank clerks and their activities,
- β -tokens and b -places represent a data base with list of banking,
- γ -tokens and c -places represent the clients and their activity,
- φ -tokens and d -places represent the money and activities with them,
- δ -tokens and f -places represent the IFS estimations.

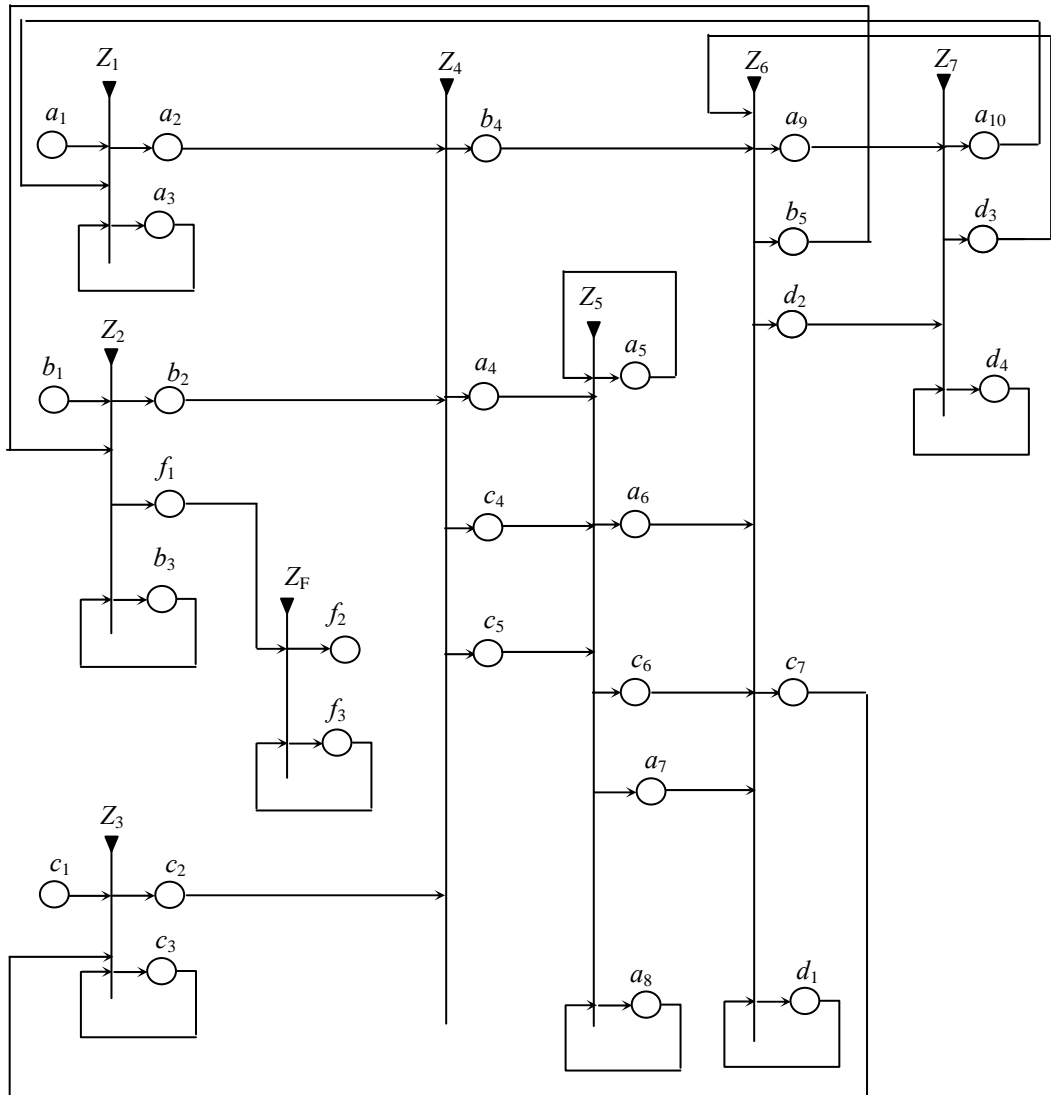


Figure 1: Generalized net model for describing typical banking activities

For brevity, we shall use the notation α -, β -, γ -, φ -, and δ -tokens instead of α_s -, β_j -, γ_i -, φ_l -, and δ_f -tokens, where s, j, i, l, f are numerations of the respective tokens.

Initially the α -, β -, γ -, φ -, and δ -tokens remain, respectively, in places a_3, b_3, c_3, d_4, d_7 and f_3 with initial characteristics:

$$\begin{aligned} x_0^\alpha &= \text{“name and activity of an accountant”}, \\ x_0^\beta &= \text{“banking services and criteria for offering credits”}, \\ x_0^\gamma &= \text{“name of a client”}, \\ x_0^{\varphi_1} &= \text{“current status of the pay-desk”}, \\ x_0^{\varphi_2} &= \text{“current financial status of the Bulgarian National Bank”}, \\ x_0^\delta &= \text{“current estimation of the liquidity of the bank”}. \end{aligned}$$

All α -tokens, all β -tokens, all γ -tokens, and all φ -tokens have equal priorities, but the priority of α -tokens is higher than the priority of β -tokens, that is higher than the priority of γ -tokens, that is higher than the priority of φ -tokens.

Let $x_{cu}^\alpha, x_{cu}^\beta, x_{cu}^\gamma, x_{cu}^\varphi$ and x_{cu}^δ be the current characteristics of the α -, β -, γ -, φ and δ -tokens' respectively. The forms of the transitions are the following.

$$Z_1 = \langle \{a_1, a_3, a_{10}\}, \{a_2, a_3\}, \begin{array}{c|cc} & a_2 & a_3 \\ \hline a_1 & false & true \\ a_3 & W_{3,2}^a & W_{3,3}^a \\ a_{10} & false & true \end{array} \rangle,$$

where the predicates in the index matrix have the following meaning:

- $W_{3,2}^a = \text{“The clerk has a client”}$,
- $W_{3,3}^a = \neg W_{3,2}^a$.

The α -tokens do not obtain new characteristic in place a_3 and they obtain in place a_2 the characteristic:

$$x_{cu}^\alpha = \text{“Bank clerk, the requirements of banking service”}.$$

$$Z_2 = \langle \{b_1, b_3, b_5\}, \{b_2, b_3, f_1\}, \begin{array}{c|ccc} & b_2 & b_3 & f_1 \\ \hline b_1 & false & true & true \\ b_3 & W_{3,2}^b & W_{3,3}^b & false \\ b_5 & false & true & true \end{array} \rangle,$$

where the predicates in the index matrix are defined, as follows:

- $W_{3,2}^b = \text{“The banking service is included in } x_{cu}^\alpha \text{”}$,
- $W_{3,3}^b = \neg W_{3,2}^b$.

The β -tokens do not have new characteristic in place b_3 and in place b_2 they obtain the characteristic:

$$x_{cu}^\beta = \text{“banking service, current status of the bank resources and inflow funds”}.$$

The δ -tokens entering in place f_1 obtain the characteristic:

$$x_{cu}^\delta = \text{“current status of the bank resources and inflow funds”}.$$

$$Z_3 = \langle \{c_1, c_3, c_7\}, \{c_2, c_3\}, \begin{array}{c|cc} & c_2 & c_3 \\ \hline c_1 & false & true \\ c_3 & W_{3,2}^c & W_{3,3}^c \\ c_7 & false & true \end{array} \rangle,$$

where the predicates in the index matrix have the following meaning:

- $W_{3,2}^c =$ “The client has to complete documents for chosen banking service”,
- $W_{3,3}^c = \neg W_{3,2}^c$.

The γ -tokens do not obtain new characteristics in places c_3 and c_2 .

$$Z_F = \langle \{f_1, f_3\}, \{f_2, f_3\}, \begin{array}{c|cc} & f_2 & f_3 \\ \hline f_1 & false & true \\ f_3 & true & true \end{array} \rangle,$$

The tokens in place f_3 obtain the characteristic “intuitionistic fuzzy estimation of the liquidity of the bank” according to the formula

$$\langle \bar{\mu}_1, \bar{\nu}_1 \rangle = \left\langle \frac{n\mu + m}{n + m}, \frac{n\nu}{n + m} \right\rangle$$

for the process for the cash inflow, where n represents the bank resources, while m stands for inflow funds.

The degree of uncertainty for this process is calculated by the formula:

$$\bar{\pi}_1 = \frac{n(1 - \mu - \nu)}{n + m} = \frac{n\pi}{n + m}$$

and

$$\langle \bar{\mu}_2, \bar{\nu}_2 \rangle = \left\langle \frac{n\mu - m}{n - m}, \frac{n\nu}{n - m} \right\rangle$$

for the process where funds are withdrawn. The degree of uncertainty for this process is:

$$\bar{\pi}_2 = \frac{n(1 - \mu - \nu)}{n - m} = \frac{n\pi}{n - m}$$

The tokens in place f_2 do not obtain new characteristics .

$$Z_4 = \langle \{a_2, b_2, c_2\}, \{a_4, b_4, c_4, c_5\}, \begin{array}{c|cccc} & a_4 & b_4 & c_4 & c_5 \\ \hline a_2 & true & false & false & false \\ b_2 & false & true & false & false \\ c_2 & false & false & W_{2,4}^c & W_{2,5}^c \end{array} \rangle.$$

where the predicates in the index matrix are defined, as follows:

- $W_{2,4}^c =$ “The client has chosen to make deposit”,
- $W_{2,5}^c =$ “The client has chosen to take credit”.

The α - and β -tokens do not have new characteristic in places a_4 and b_4 , respectively, while γ -tokens obtain in places c_4 and c_5 the following characteristics, respectively:

“Client, banking service: deposit”,

“Client, banking service: credit”.

$$Z_5 = \langle \{a_4, a_5, a_8, c_4, c_5\}, \{a_5, a_6, a_7, a_8, c_6\},$$

$$R_5 = \left. \begin{array}{c|ccccc} & a_5 & a_6 & a_7 & a_8 & c_6 \\ \hline a_4 & W_{4,5}^a & false & false & W_{4,8}^a & false \\ a_5 & W_{5,5}^a & W_{5,6}^a & false & false & false \\ a_8 & false & false & W_{8,7}^a & W_{8,8}^a & false \\ c_4 & false & false & false & false & true \\ c_5 & false & false & false & false & true \end{array} \right\},$$

where the predicates in the index matrix have the following meaning:

- $W_{4,5}^a = W_{5,5}^a =$ “There are clients whose documents for a deposit must be processed by the clerk”,
- $W_{4,8}^a = W_{8,8}^a =$ “There are clients whose documents for a credit must be processed by the clerk”,
- $W_{5,6}^a = \neg W_{4,5}^a$, $W_{8,7}^a = \neg W_{4,8}^a$.

The α -tokens do not obtain new characteristic in places a_5 and a_8 . The α -tokens that enter places a_6 and a_7 obtain in places a_6 and a_7 the characteristics, respectively,

“Clerk, banking service, documents for a deposit”

“Clerk, banking service, documents for a credit”

The γ -tokens do not obtain new characteristic in place c_6 .

$$Z_6 = \langle \{a_6, a_7, b_4, c_6, d_1, d_3\}, \{a_9, b_5, c_7, d_1, d_2\},$$

	a_9	b_5	c_7	d_1	d_2
a_6	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
a_7	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>
b_4	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>
c_6	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
d_1	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	$W_{1,2}^d$
d_3	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>

$$\rangle,$$

where the predicate $W_{1,2}^d =$ “There are financial resources for the Bulgarian National Bank”.

The α -, β and γ -tokens do not obtain new characteristic in places a_9 , b_5 and c_7 . The φ -tokens that enter place d_2 obtain characteristics:

“Clerk, client, banking service: credit, financial resource”.

$$Z_7 = \langle \{a_9, d_2, d_4\}, \{a_{10}, d_3, d_4\},$$

	a_{10}	d_3	d_4
a_1	<i>true</i>	<i>false</i>	<i>false</i>
d_2	<i>false</i>	<i>false</i>	<i>true</i>
d_4	<i>false</i>	$W_{4,3}^d$	<i>true</i>

$$\rangle,$$

where the predicate $W_{4,3}^d =$ “The clerk has a client”.

The α -tokens do not obtain new characteristic in places a_{10} . The φ -tokens obtain in place d_3 the characteristic:

$$x_{cu}^{\varphi} = \text{“Financial resources for the commercial bank”}.$$

Conclusions

The so-constructed GN-model gives possibility to simulate some processes, related with typical banking activities. The present model is an element of a more general model describing different processes, flowing in a bank. The author, together with some colleagues has been preparing an extensive research on this theme.

Here we use the theory of the intuitionistic fuzzy set for the estimating the degree of the bank's liquidity.

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