Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy hemiring structure spaces

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Abstract: The classical notion of hemirings through Atanassov's intuitionistic fuzzy sets by extending the concepts of intuitionistic fuzzy hemiring structure spaces. Further we explore the relationships between intuitionistic $\mathfrak{h}$ open sets, intuitionistic fuzzy $\mathfrak{h}\alpha$ open sets and their corresponding closure and interior operators within hemiring structure spaces. Also intuitionistic fuzzy hemiring shrinking and an intuitionistic fuzzy hemiring swelling in fuzzy hemiring spaces are introduced and some of their characteristics have studied.

Keywords: Intuitionistic fuzzy hemiring, Intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring, Intuitionistic fuzzy hemiring shrinking, Intuitionistic fuzzy hemiring swelling.
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1 Introduction

The idea of fuzzy sets is essential to many scientific disciplines, including computer science, engineering and pure mathematics. Almost every branch of mathematics has been influenced by fuzzy sets since Zadeh [18] introduced the concept in 1965. Fuzzy topological space theory was first proposed and explored by Chang [11] in 1968. Njåsted [15] introduced α-open set, Abbas [1] provided h-open set. The generalized fuzzy open set has gained considerable attention, as open sets from the foundation of topological structures. Singal, Rajvansi and Bin Shahna presented the idea of fuzzy α-open sets in [9, 17]. As a significant extension of fuzzy open sets, fuzzy α-open sets have been proposed. The fuzzy α-open sets are a helpful bridge in the study of sensitive topology since they are weaker than some other generalized sets but stronger than fuzzy open sets. Numerous academics have expanded and examined different generalizations of fuzzy sets, such as Pu and Liu [16], Azad [7]. The concepts of fuzzy α-open sets further enriches this line of research by establishing a bridge between classical open sets and other generalized open sets, enabling more comprehensive understanding of fuzzy topological structures.

Ali *et al.* [2, 3] investigated the concepts of hemirings and fuzzy hemirings. The concept of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [5], in which each element has a membership degree and a non-membership degree, with the sum of these being limited to be less than or equal to one. Hutton [10] was the first to suggest the idea of fuzzy normal spaces. Kubiak [14] established many interesting properties of fuzzy normal spaces.

This paper introduces and establishes some properties of intuitionistic fuzzy hemiring structure spaces, intuitionistic fuzzy $\mathfrak{h}\alpha$ open and closed hemirings. The notions of intuitionistic fuzzy hemiring shrinking, intuitionistic fuzzy hemiring swelling and intuitionistic fuzzy hemiring $\mathfrak{h}\alpha$ normal spaces are presented in this paper, along with some of its characteristics.

2 Preliminaries

In this section some basic definitions are given.

Definition 2.1. [6] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \to [0,1]$ define, respectively, the degree of membership and the degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2. [12] An intuitionistic fuzzy topology (IFT for short) on a non-empty set X is a family τ of IFSs in X satisfying:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_j \in \tau$ for any family $\{G_j \mid j \in \tau\} \subset \tau$.

In this case, the pair (X, τ) is called an IFTS and any IFS in τ is called an intuitionistic fuzzy open set (IFOS for short) in X. The complement of intuitionistic fuzzy open set is intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.3. [12] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$Cl(A) = \cap\{K : K \text{ is an IFCS in } X \text{ and } A \subseteq X\},$$

$$Int(A) = \cup\{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\}.$$

Note that *Cl*(*A*) is an IFCS and *Int*(*A*) is an IFOS in X. Further,

(a) A is an IFCS in X if and only if $Cl(A) = A$;

(b) A is an IFOS an X if and only if $Int(A) = A$.

Definition 2.4. [13] The intersection of all fuzzy open subsets of a fuzzy topological space (X, τ) containing A is called the kernel of A (briefly *Ker*(*A*)), this means that

$$Ker(A) = \cap\{G \in T : A \subseteq G\}.$$

Definition 2.5. [13] A fuzzy topological space (X, τ) is called fuzzy R_0-space (FR_0-space, for short) if for each fuzzy open set U and $x_\lambda \in U$, $Cl\{x_\lambda\} \leq U$.

Definition 2.6. [13] A fuzzy topological space (X, τ) is called fuzzy R_1-space (FR_1-space, for short) if for each two distinct fuzzy points x_λ and y_α of X with $Cl\{x_\lambda\} \neq Cl\{y_\alpha\}$, there exist disjoint fuzzy open sets U, V such that $cl\{x_\lambda\} \leq U$ and $cl\{y_\alpha\} \leq V$.

Definition 2.7. [2] Let R be a hemiring. A fuzzy set μ of R is said to be fuzzy hemiring of R if it satisfies the following conditions:

(FH_1): $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$;

(FH_2): $\mu(xy) \geq \min\{\mu(x), \mu(y)\}; \forall x, y \in R$.

Definition 2.8. [4] Let (X, T) be a fuzzy topological space. For $\lambda_i \in I^X$, $i \in \mathfrak{I}$ the swelling of the family $\mathcal{A} = \{\lambda_i : i \in \mathfrak{I}\}$ is the family of fuzzy sets $\mathcal{B} = \mu_i \in I^X : i \in \mathfrak{I}$ such that $\lambda_i \leq \mu_i$ for every $i \in \mathfrak{I}$ and for every finite set of indices $i_1, i_2, i_3, \dots, i_m \in \mathfrak{I}$, the family $\{\lambda_{i_1}, \lambda_{i_2}, \lambda_{i_3}, \dots, \lambda_{i_m}\}$ is an overlapping family if and only if $\{\mu_{i_1}, \mu_{i_2}, \mu_{i_3}, \dots, \mu_{i_m}\}$ is an overlapping family.

A fuzzy swelling is said to be a fuzzy open (closed) swelling if all its members are fuzzy open (closed) sets.

Clearly, every fuzzy swelling $\mathcal{B}$ of a family $\mathcal{A}$ satisfies the equality $Ord_f\mathcal{A} = Ord_f\mathcal{B}$.

Definition 2.9. [4] Let (X, T) be a fuzzy topological space. The fuzzy shrinking of the fuzzy cover $\mathcal{A} = \{\lambda_i : i \in \mathfrak{I}\}$ is defined to be the fuzzy cover $\mathcal{B} = \mu_i \in I^X : i \in \mathfrak{I}$ such that $\mu_j \leq \lambda_i$ where $j \leq i$ for every $i \in \mathfrak{I}$. A fuzzy shrinking is open (closed) if all its members are fuzzy open (fuzzy closed) sets.

Clearly, every fuzzy shrinking $\mathcal{B}$ of a fuzzy cover $\mathcal{A}$ is a fuzzy refinement of $\mathcal{A}$ and satisfies the inequality $Ord_f\mathcal{B} \leq Ord_f\mathcal{A}$.

Definition 2.10. [3] Let R be a hemiring. A fuzzy set μ in R is defined as a mapping from R to [0,1], the usual interval of real numbers. We denote by I^R the set of all fuzzy sets in R.

Definition 2.11. [12] Let X be a non-empty set. A family $\mathfrak{B} = \{\mu_\lambda\}_{\lambda \in \Lambda}$ of fuzzy sets in X is said to be an overlapping family if there exists $x \in X$ such that $\mu_\alpha(x) + \mu_\beta(x) > 1$, for all $\alpha, \beta \in \Lambda$.

A family $\{\mu_\lambda\}_{\lambda\in\Lambda}$ is non-overlapping if it is not overlapping, that is, for every $x \in X$ there exist $\alpha, \beta \in \Lambda$ such that $\mu_\alpha(x) + \mu_\beta(x) \leq 1$.

Definition 2.12. [8] A space X is called fuzzy normal if for each pair of fuzzy closed sets A and B of X with A is non-quasi-coincidence with B, there exist fuzzy open sets U and V such that $A \leq U$ and $B \leq V$ and U is non-quasi-coincidence with V.

Definition 2.13. [11] A family $\mathcal{A}$ of fuzzy sets is a cover of a fuzzy set $\mathcal{B}$ if and only if $\mathcal{B} \subset \cup \{A \mid A \in \mathcal{A}\}$. It is an open cover if and only if each member of $\mathcal{A}$ is an open fuzzy set. A subcover of $\mathcal{A}$ is a subfamily of $\mathcal{A}$ which is also a cover.

3 Intuitionistic fuzzy hemiring structure space

In this section, the concept of an intuitionistic fuzzy hemiring R_i $(i = 1,2)$ spaces are initiated and some of their attributes are studied. Throughout this paper $\mathfrak{R}$ indicates a hemiring, the collection of intuitionistic fuzzy hemirings in $\mathfrak{R}$ is indicated by $I^{\mathfrak{R}}$, where $I = [0,1]$.

Definition 3.1. Let $\mathfrak{R}$ be a hemiring. Let $\mathcal{H}$ be a family of intuitionistic fuzzy hemirings over $\mathfrak{R}$ which satisfies the following axioms:

(i) $0_{\mathfrak{R}}, 1_{\mathfrak{R}} \in \mathcal{H}$;

(ii) If $\gamma_1, \gamma_2 \in \mathcal{H}$, then $\gamma_1 \wedge \gamma_2 \in \mathcal{H}$;

(iii) If $\gamma_i \in \mathcal{H}$ for each $i \in J$, then $\vee\ \gamma_i \in \mathcal{H}$.

Then $\mathcal{H}$ is said to be an intuitionistic fuzzy hemiring structure ($IF\mathcal{H}S$) on $\mathfrak{R}$ and the ordered pair $(\mathfrak{R}, \mathcal{H})$ is said to be an intuitionistic fuzzy hemiring structure space ($IF\mathcal{H}SS$). Every member of $\mathcal{H}$ is said to be an intuitionistic fuzzy open hemiring. The complement of an intuitionistic fuzzy open hemiring is called as an intuitionistic fuzzy closed hemiring.

Example 3.1. Let $\mathfrak{R}$ = {0, 1, 2} be a set of integers modulo 3 with binary operations as follows:

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Then ($\mathfrak{R}$, · , +) is a hemiring. Let the membership values of two intuitionistic fuzzy hemirings $\gamma_1, \gamma_2 \in I^{\mathfrak{R}}$ be defined by $\gamma_{\mu_1}, \gamma_{\mu_2} : \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{\mu_1}(0) = 0.2$	$\gamma_{\mu_2}(0) = 0.4$
$\gamma_{\mu_1}(1) = 0.4$	$\gamma_{\mu_2}(1) = 0.4$
$\gamma_{\mu_1}(2) = 0.5$	$\gamma_{\mu_2}(2) = 0.5$

Let the non-membership values of the intuitionistic fuzzy hemirings $\gamma_1, \gamma_2 \in I^{\mathfrak{R}}$ be defined by $\gamma_{\nu_1}, \gamma_{\nu_2} : \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{v_1}(0) = 0.6$	$\gamma_{v_2}(0) = 0.5$
$\gamma_{v_1}(1) = 0.3$	$\gamma_{v_2}(1) = 0.3$
$\gamma_{v_1}(2) = 0.4$	$\gamma_{v_2}(2) = 0.3$

Clearly $\mathcal{H} = \{0_{\mathfrak{R}}, \gamma_1, \gamma_2, 1_{\mathfrak{R}}\}$ is an $IF\mathcal{H}S$ on $\mathfrak{R}$. Then the ordered pair $(\mathfrak{R}, \mathcal{H})$ is an $IF\mathcal{H}SS$.

Definition 3.2. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then for any $\lambda \in I^{\mathfrak{R}}$, the intuitionistic fuzzy interior of λ denoted by $IF\mathcal{H}Int(\lambda)$ and is defined as $IF\mathcal{H}Int(\lambda) = \vee\{\gamma \in I^{\mathfrak{R}}: \gamma \leq \lambda \text{ and } \gamma \in IFO\mathcal{H}\}$.

Definition 3.3. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then for any $\lambda \in I^{\mathfrak{R}}$, the intuitionistic fuzzy closure of λ denoted by $IF\mathcal{H}Cl(\lambda)$ and is defined as $IF\mathcal{H}Cl(\lambda) = \wedge\{\gamma \in I^{\mathfrak{R}}: \lambda \leq \gamma \text{ and } \gamma \in IFC\mathcal{H}\}$.

Definition 3.4. Let $\mathfrak{R}$ be a hemiring and $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Any $\lambda \in I^{\mathfrak{R}}$ in $(\mathfrak{R}, \mathcal{H})$ is said to be an Intuitionistic Fuzzy α-open hemiring ($IF\alpha O\mathcal{H}$), if $\lambda \leq IF\mathcal{H}Int\left(IF\mathcal{H}Cl(IF\mathcal{H}Int(\lambda))\right)$.

Example 3.2. As in Example 3.1, $(\mathfrak{R}, \cdot, +)$ is a hemiring. Let the membership values of the intuitionistic fuzzy hemirings $\gamma_1, \gamma_2, \gamma_3 \in I^{\mathfrak{R}}$ be defined by $\gamma_{\mu_1}, \gamma_{\mu_2}, \gamma_{\mu_3}: \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{\mu_1}(0) = 0.4$	$\gamma_{\mu_2}(0) = 0.4$	$\gamma_{\mu_3}(0) = 0.4$
$\gamma_{\mu_1}(1) = 0.5$	$\gamma_{\mu_2}(1) = 0.5$	$\gamma_{\mu_3}(1) = 0.4$
$\gamma_{\mu_1}(2) = 0.3$	$\gamma_{\mu_2}(2) = 0.4$	$\gamma_{\mu_3}(2) = 0.3$

Let the non-membership values of the intuitionistic fuzzy hemirings $\gamma_1, \gamma_2, \gamma_3 \in I^{\mathfrak{R}}$ be defined by $\gamma_{v_1}, \gamma_{v_2}, \gamma_{v_3}: \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{v_1}(0) = 0.6$	$\gamma_{v_2}(0) = 0.5$	$\gamma_{v_3}(0) = 0.6$
$\gamma_{v_1}(1) = 0.4$	$\gamma_{v_2}(1) = 0.4$	$\gamma_{v_3}(1) = 0.5$
$\gamma_{v_1}(2) = 0.6$	$\gamma_{v_2}(2) = 0.5$	$\gamma_{v_3}(2) = 0.6$

Clearly $\mathcal{H} = \{0_{\mathfrak{R}}, \gamma_1, \gamma_2, 1_{\mathfrak{R}}\}$ is an $IF\mathcal{H}S$ on $\mathfrak{R}$. Then the ordered pair $(\mathfrak{R}, \mathcal{H})$ is an $IF\mathcal{H}SS$. Then γ_3 is $IF\alpha O\mathcal{H}$.

Definition 3.5. Any $\lambda \in I^{\mathfrak{R}}$ in $(\mathfrak{R}, \mathcal{H})$ is referred to as an intuitionistic fuzzy $\mathfrak{h}$-open hemiring ($IF\mathfrak{h}O\mathcal{H}$), if $\lambda \leq IF\mathcal{H}Int(\lambda \vee \mu)$ such that $\mu \in \mathcal{H}$ and $\mu \neq 1_{\mathfrak{R}}$.

Example 3.3. As defined in Example 3.2, $(\mathfrak{R}, \cdot, +)$ is a hemiring and $\gamma_3 \in I^{\mathfrak{R}}$. Then γ_3 is an $IF\mathfrak{h}O\mathcal{H}$.

Definition 3.6. Any intuitionistic fuzzy hemiring λ is said to be intuitionistic fuzzy $\mathfrak{h}\alpha$-open hemiring ($IF\mathfrak{h}\alpha O\mathcal{H}$) if $\lambda \leq IF\mathcal{H}int(\lambda \vee \mu)$ such that $\mu \in \mathcal{H}^{\alpha}$ and $\mu \neq 1_{\mathfrak{R}}$ where $\mathcal{H}^{\alpha}$ is a collection of all $IF\alpha O\mathcal{H}$.

Example 3.4. As defined in Example 3.2, $(\mathfrak{R}, \cdot, +)$ is a hemiring and $\gamma_3 \in I^{\mathfrak{R}}$. Then γ_3 is an $IF\mathfrak{h}\alpha O\mathcal{H}$.

Definition 3.7. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then for any $\lambda \in I^{\mathfrak{R}}$, the intuitionistic fuzzy $\mathfrak{h}\alpha$ interior of λ is denoted by $IF\mathfrak{h}\alpha\mathcal{H}Int(\lambda)$ and is defined as

$$IF\mathfrak{h}\alpha\mathcal{H}Int(\lambda) = \vee\{\gamma \in I^{\mathfrak{R}}: \gamma \leq \lambda \text{ and } \gamma \in IF\mathfrak{h}\alpha O\mathcal{H}\}.$$

Example 3.5. As in Example 3.2, let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$ and $\gamma_3 \in I^{\mathfrak{R}}$. Let the membership value of an intuitionistic fuzzy hemiring $\lambda \in I^{\mathfrak{R}}$ be defined by $\lambda_\mu : \mathfrak{R} \longrightarrow [0,1]$ such that

$\lambda_\mu(0) = 0.4$
$\lambda_\mu(1) = 0.5$
$\lambda_\mu(2) = 0.6$

Let the non-membership value of the intuitionistic fuzzy hemiring $\lambda \in I^{\mathfrak{R}}$ be defined by $\lambda_\nu : \mathfrak{R} \longrightarrow [0,1]$ such that

$\lambda_\nu(0) = 0.5$
$\lambda_\nu(1) = 0.5$
$\lambda_\nu(2) = 0.4$

Then $IF\mathfrak{h}\alpha\mathcal{H}Int(\lambda) = \gamma_3$.

Definition 3.8. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then for any $\lambda \in I^{\mathfrak{R}}$, the intuitionistic fuzzy $\mathfrak{h}\alpha$ closure of λ is denoted by $IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Cl}(\lambda)$ and is defined as

$$IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Cl}(\lambda) = \wedge\left\{\gamma \in I^{\mathfrak{R}} : \lambda \leq \gamma \text{ } and \text{ } \gamma \in IF\mathfrak{h}\alpha C\mathcal{H}\right\}.$$

Example 3.6. As in Example 3.2, let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Let the membership value of an intuitionistic fuzzy hemiring $\lambda \in I^{\mathfrak{R}}$ be defined by $\lambda_\mu : \mathfrak{R} \longrightarrow [0,1]$ such that

$\lambda_\mu(0) = 0.3$
$\lambda_\mu(1) = 0.4$
$\lambda_\mu(2) = 0.5$

Let the non-membership value of the intuitionistic fuzzy hemiring $\lambda \in I^{\mathfrak{R}}$ be defined by $\lambda_\nu : \mathfrak{R} \longrightarrow [0,1]$ such that

$\lambda_\nu(0) = 0.5$
$\lambda_\nu(1) = 0.6$
$\lambda_\nu(2) = 0.4$

Then $IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda) = \gamma_2$.

Proposition 3.1. Every $IFO\mathcal{H}$ is an $IF\mathfrak{h}O\mathcal{H}$.

Proof. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$ and $\lambda \in I^{\mathfrak{R}}$ be any $IFO\mathcal{H}$. Then, $\lambda = Int(\lambda) \leq Int(\lambda \vee \mu)$ for every non-empty hemiring $\mu \neq 1_{\mathfrak{R}}$ and $\mu \in \mathcal{H}^\alpha$. Thus λ is an $IF\mathfrak{h}O\mathcal{H}$. ☐

Proposition 3.2. Every $IFO\mathcal{H}$ is an $IF\alpha O\mathcal{H}$.

Proof. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$ and let $\lambda \in I^{\mathfrak{R}}$ be any $IFO\mathcal{H}$. Therefore, $\lambda = Int(\lambda) \leq Int\left(Cl\left(Int(\lambda)\right)\right)$. Thus λ is an $IF\alpha O\mathcal{H}$. ☐

Proposition 3.3. Each $IF\mathfrak{h}O\mathcal{H}$ in any IF$\mathcal{H}$SS is an $IF\mathfrak{h}\alpha O\mathcal{H}$.

Proof. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}$SS and let $\lambda \in I^{\mathfrak{R}}$ be any IF$\mathfrak{h}O\mathcal{H}$. Then, for each IF$\mathcal{H}$ that is $\mu \neq 0_{\mathfrak{R}}$, $\mu \neq 1_{\mathfrak{R}}$ and $\mu \in \mathcal{H}$, $\lambda \leq Int(\lambda \vee \mu)$. Also, every $IFO\mathcal{H}$ is an $IF\alpha O\mathcal{H}$ gives μ is $IF\alpha O\mathcal{H}$. Therefore, λ is an $IF\mathfrak{h}\alpha O\mathcal{H}$. ☐

Proposition 3.4. Any $IFO\mathcal{H}$ in any IF$\mathcal{H}$SS is an $IF\mathfrak{h}\alpha O\mathcal{H}$.

Proof. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and let $\lambda \in I^{\mathfrak{R}}$ be any $IFO\mathcal{H}$. Since each $IFO\mathcal{H}$ is $IF\mathfrak{h}O\mathcal{H}$, then λ is $IF\mathfrak{h}O\mathcal{H}$. By Proposition 3.3, we get that λ is an $IF\mathfrak{h}\alpha O\mathcal{H}$. □

Definition 3.9. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and $\lambda \in I^{\mathfrak{R}}$. The intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring kernel of λ *(*$IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$*)* is defined as $IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda) = \wedge\{\mu : \lambda \leq \mu \text{ and } \mu \in IF\mathfrak{h}\alpha O\mathcal{H}\}$.

Proposition 3.5. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and let $\gamma,\eta \in I^{\mathfrak{R}}$ be any two intuitionistic fuzzy hemirings. If $\gamma \leq \eta$, then $IF\mathfrak{h}\alpha\mathcal{H}Ker(\gamma) \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$.
Proof. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and let $\gamma,\eta \in I^{\mathfrak{R}}$ be any two intuitionistic fuzzy hemirings. Let $\gamma \leq \eta$. Then $IF\mathfrak{h}\alpha\mathcal{H}Ker(\gamma) = \wedge\{\mu_i : \gamma \leq \mu_i \text{ and } \mu_i \in IF\mathfrak{h}\alpha O\mathcal{H}\}$ and $IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta) = \wedge\{\alpha_i : \eta \leq \alpha_i \text{ and } \alpha_i \in IF\mathfrak{h}\alpha O\mathcal{H}\}, i \in J$, where J is an indexed set. Since $\gamma \leq \eta$, each $\mu_i \leq \alpha_i$. Therefore, $\wedge\mu_i \leq \wedge\alpha_i$. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Ker(\gamma) \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. □

Proposition 3.6. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$. Then the following statements hold true:

(i) For any intuitionistic fuzzy hemiring $\lambda \in I^{\mathfrak{R}}$, $\lambda \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$;

(ii) $\lambda \in I^{\mathfrak{R}}$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring then $\lambda = IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$.

Proof. (i) Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and $\lambda \in I^{\mathfrak{R}}$ be any intuitionistic fuzzy hemiring. By definition $IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda) = \wedge\{\mu_i : \lambda \leq \mu_i \text{ and } \mu_i \in IF\mathfrak{h}\alpha O\mathcal{H}\}, i \in J$, where J is an indexed set. Since $\lambda \leq \mu_i$ for every μ_i, $\lambda \leq \wedge\mu_i$. Thus $\lambda \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$.

(ii) Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and $\lambda \in I^{\mathfrak{R}}$ be any $IF\mathfrak{h}\alpha O\mathcal{H}$. By (i), $\lambda \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$. Also, $IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda) = \wedge\{\mu_i : \lambda \leq \mu_i \text{ and } \mu_i \in IF\mathfrak{h}\alpha O\mathcal{H}\}, i \in J$, where J is an indexed set. Since $\lambda \in I^{\mathfrak{R}}$ is $IF\mathfrak{h}\alpha O\mathcal{H}$, $\wedge\mu_i \leq \lambda$. Therefore, $IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda) \leq \lambda$. Hence, $\lambda = IF\mathfrak{h}\alpha\mathcal{H}Ker(\lambda)$. □

Definition 3.10. Any $IF\mathcal{H}SS$ $(\mathfrak{R},\mathcal{H})$ is called an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space if for every intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\lambda \in I^{\mathfrak{R}}$ in $(\mathfrak{R},\mathcal{H})$ and for every $\gamma \in I^{\mathfrak{R}}$ with $\gamma \leq \lambda$, $IF\mathfrak{h}\alpha O\mathcal{H}Cl(\gamma) \leq \lambda$.

Example 3.7. As in Example 3.1, $(\mathfrak{R}, \cdot, +)$ is a hemiring. Let the membership values of two intuitionistic fuzzy hemirings $\eta_1, \eta_2 \in I^{\mathfrak{R}}$ be defined by $\eta_{\mu_1}, \eta_{\mu_2} : \mathfrak{R} \longrightarrow [0,1]$ such that

$\eta_{\mu_1}(0) = 0.4$	$\eta_{\mu_2}(0) = 0.3$
$\eta_{\mu_1}(1) = 0.5$	$\eta_{\mu_2}(1) = 0.5$
$\eta_{\mu_1}(2) = 0.2$	$\eta_{\mu_2}(2) = 0.2$

Let the non-membership values of the intuitionistic fuzzy hemirings $\eta_1, \eta_2 \in I^{\mathfrak{R}}$ be defined by $\eta_{v_1}, \eta_{v_2} : \mathfrak{R} \longrightarrow [0,1]$ such that

$\eta_{v_1}(0) = 0.4$	$\eta_{v_2}(0) = 0.5$
$\eta_{v_1}(1) = 0.3$	$\eta_{v_2}(1) = 0.4$
$\eta_{v_1}(2) = 0.5$	$\eta_{v_2}(2) = 0.6$

Clearly, $\mathcal{H} = \{0_{\mathfrak{R}}, \eta_1, \eta_2, 1_{\mathfrak{R}}\}$ is an $IF\mathcal{H}S$ on $\mathfrak{R}$. Then the ordered pair $(\mathfrak{R},\mathcal{H})$ is an $IF\mathcal{H}SS$.

Let $\eta_1 \in \mathcal{H}$ and for any intuitionistic fuzzy hemiring $\gamma \in I^{\mathfrak{R}}$ with $\gamma \leq \eta_1$. Then $IF\,\mathfrak{h}\alpha\mathcal{H}Cl(\gamma) = (1_{\mathfrak{R}} - \eta_2)$. This implies that $(1_{\mathfrak{R}} - \eta_2) = \eta_1$. Thus $IF\mathfrak{h}\alpha\mathcal{H}Cl(\gamma) = \eta_1$. Similarly, for every $\eta_i \in \mathcal{H}$ and $\gamma \leq \eta_i$, $i \in$ J and $\gamma \in \mathfrak{R}$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\gamma) = \eta_i$ is true. Hence $(\mathfrak{R},\mathcal{H})$ is an intutionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space.

Definition 3.11. Let $(\mathfrak{R}, \mathcal{H})$ be an *IF$\mathcal{H}$SS*. Any of two intutionistic fuzzy hemirings $\lambda, \eta \in I^{\mathfrak{R}}$ is said to be an:

(i) Intuitionistic fuzzy quasi-coincidence if $\lambda(a) + \eta(a) > 1, \forall a \in \mathfrak{R}$

(ii) Intuitionistic fuzzy non-quasi-coincidence if $\lambda(a) + \eta(a) \leq 1, \forall a \in \mathfrak{R}$

Proposition 3.7. Let $(\mathfrak{R}, \mathcal{H})$ be an *IF$\mathcal{H}$SS*. Then the following statements are equivalent:

(i) $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space ;

(ii) For any intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring $\lambda \in I^{\mathfrak{R}}$ in $(\mathfrak{R}, \mathcal{H})$ and an intuitionistic fuzzy hemiring $\mu \in I^{\mathfrak{R}}$ with μ being non-quasi-coincidence with λ, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\gamma \in I^{\mathfrak{R}}$ in $(\mathfrak{R}, \mathcal{H})$ such that $\lambda \leq \gamma$ and μ is non-quasi-coincidence with γ;

(iii) For any intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring $\lambda \in I^{\mathfrak{R}}$ in $(\mathfrak{R}, \mathcal{H})$ and μ is non-quasi-coincidence with λ, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with λ.

Proof. (i) ⇒ (ii) Let $\lambda, \mu \in I^{\mathfrak{R}}$ where λ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring in $(\mathfrak{R}, \mathcal{H})$ and μ is non-quasi-coincidence with λ.

This implies that $\mu \leq (1_{\mathfrak{R}} - \lambda)$. Then by (i), $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq (1_{\mathfrak{R}} - \lambda)$.

Let $\gamma = (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu))$, then γ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R}, \mathcal{H})$ and also $\lambda \leq (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu))$ implies $\lambda \leq \gamma$.

Since $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$, $\mu \leq (1_{\mathfrak{R}} - (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)))$. This implies that $\mu \leq (1_{\mathfrak{R}} - \gamma)$. Then μ is non-quasi-coincidence with γ. Hence $\lambda \leq \gamma$ and μ is non-quasi-coincidence with γ.

(ii) ⇒ (iii) Let $\lambda, \mu \in I^{\mathfrak{R}}$ and λ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring in $(\mathfrak{R}, \mathcal{H})$ and μ is non-quasi-coincidence with λ. Then by (ii), there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\gamma \in I^{\mathfrak{R}}$ such that $\lambda \leq \gamma$ and μ is non-quasi-coincidence with γ. This implies that $\mu \leq (1_{\mathfrak{R}} - \gamma)$. Since γ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R}, \mathcal{H})$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(1_{\mathfrak{R}} - \gamma) = (1_{\mathfrak{R}} - \gamma)$ which implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with γ. Since $\lambda \leq \gamma$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with λ.

(iii) ⇒ (i) Let $\lambda \in I^{\mathfrak{R}}$ be an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R}, \mathcal{H})$ and let μ be non-quasi-coincidence with λ. Now, $(1_{\mathfrak{R}} - \lambda)$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring in $(\mathfrak{R}, \mathcal{H})$ such that μ is non-quasi-coincidence with $(1_{\mathfrak{R}} - \lambda)$. By (iii), $IF\,\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with $(1_{\mathfrak{R}} - \lambda)$ and hence $IF\,\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq \lambda$. Therefore, $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space. □

Proposition 3.8. Let $(\mathfrak{R}, \mathcal{H})$ be an *IF$\mathcal{H}$SS*. Then $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space if and only if for every pair of intuitionistic fuzzy hemirings $\mu, \eta \in I^{\mathfrak{R}}$ with μ being non-quasi-coincidence with η and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Proof. Suppose that an *IF$\mathcal{H}$SS* $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space. Let $\mu, \eta \in I^{\mathfrak{R}}$ with μ being non-quasi-coincidence with η and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Then there exists an intuitionistic fuzzy hemiring $\delta \in I^{\mathfrak{R}}$ such that $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ and $\delta \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. If $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, then $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Hence $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, but this is a contradiction.

Thus $\nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, that is, $\mu \leq (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta))$. Since $(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta))$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R}, \mathcal{H})$ and $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$

hemiring R_0 space, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta))$. Hence $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Conversely, assume that if for every pair $\mu,\ \eta \in I^{\mathfrak{R}}$ with μ is non-quasi-coincidence with η and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Let $\eta \in I^{\mathfrak{R}}$ be an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R},\mathcal{H})$. Let $\mu \leq \eta$ and $\nu \nleq \eta$. Then $\nu \leq (1_{\mathfrak{R}} - \eta)$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\nu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(1_{\mathfrak{R}} - \eta) = (1_{\mathfrak{R}} - \eta)$. Since $\mu \leq \eta,\ \mu$ is non-quasi-coincidence with $(1_{\mathfrak{R}} - \eta)$ and also $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\nu) \leq (1_{\mathfrak{R}} - \eta)$ implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq (1_{\mathfrak{R}} - \eta)$. By hypothesis, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ is non-quasi-coincidence with $(1_{\mathfrak{R}} - \eta)$. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq (1_{\mathfrak{R}} - 1_{\mathfrak{R}} + \eta) = \eta$. Therefore, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq \eta$. Hence $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space. □

Proposition 3.9. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and $\mu,\eta \in I^{\mathfrak{R}}$ be an intuitionistic fuzzy hemirings. Then $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ if and only if $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Proof. Suppose that $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$ and $\eta \nleq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$. By the definition of $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\lambda \in I^{\mathfrak{R}}$ such that $\mu \leq \lambda$ and $\eta \nleq \lambda$. Hence $\eta \leq (1_{\mathfrak{R}} - \lambda)$ which implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(1_{\mathfrak{R}} - \lambda) = (1_{\mathfrak{R}} - \lambda)$. Then $\lambda \leq \left(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)\right)$. Since $\mu \leq \lambda, \mu \leq \left(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)\right)$. But this is a contradiction to $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Hence $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker\left(x_\mu\right)$.

Conversely, suppose that $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ and $\mu \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Then $\mu \leq \left(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)\right)$ and |then $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \leq \left(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)\right)$. Since $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)) \leq (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)), \eta \leq (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu))$. But this is a contradiction to $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$. Hence $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. □

Proposition 3.10. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$ and $\mu\,,\eta \in I^{\mathfrak{R}}$ be intuitionistic fuzzy hemirings. Then $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$ if and only if $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Proof. Suppose $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. Then there exists an intuitionistic fuzzy hemiring $\delta \in I^{\mathfrak{R}}$ such that $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ and $\delta \nleq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. Now consider $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$. By Proposition 3.9, $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$. On the other hand, $\delta \nleq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$, by Proposition 3.9, $\eta \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$ which implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$. Since $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Conversely, suppose that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Then there exists an intuitionistic fuzzy hemiring $\delta \in I^{\mathfrak{R}}$ such that $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu)$ and $\delta \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. If $\mu \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$, then $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Hence $\delta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. But this is contradiction. Then $\mu \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. By Proposition 3.9, $\eta \nleq IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ and $\eta \leq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. Hence $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. □

Proposition 3.11. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$. If $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space, then for each pair of an intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\mathfrak{R}}$ with α is non-quasi-coincidence with β, either $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$ or $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$.

Proof. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$. Suppose that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$.

Since $\alpha \le IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha)$ and $\alpha \nleq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$, thus $\alpha \le (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta))$, $(1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta))$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring in $(\mathfrak{R},\mathcal{H})$. Since $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \le (1_{\mathfrak{R}} - IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta))$. This is a contradiction to our assumption. Hence either $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$ or $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$. ☐

Proposition 3.12. Let $(\mathfrak{R},\mathcal{H})$ be an $IF\mathcal{H}SS$. If $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space, then for each pair of intuitionistic fuzzy hemirings $\mu,\eta \in I^{\mathfrak{R}}$ with μ non-quasi-coincidence with η and $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$, $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$.

Proof. Suppose that an $IF\mathcal{H}SS$ $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space. Let $\mu,\eta \in I^{\mathfrak{R}}$ with μ be non-quasi-coincidence with η and $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. By Proposition **3**.10, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Suppose that $IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\mu)$ is quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\eta)$ for some $z \in \mathfrak{R}$. Let $\lambda = IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\mu)(z) \wedge IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\eta)(z) \in (0,1]$. Then $\lambda \le IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\mu)$, $\lambda \le IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\eta)$. Consider $\lambda \le IF\mathfrak{h}\alpha\mathcal{H}\mathrm{Ker}(\mu)$, by Proposition 3.9 $\mu \le IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda)$ which implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \le IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda)$. Then by Proposition 3.11, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda)$.

Similarly, on the other hand, $\lambda \le IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$ implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda)$. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) = IF\mathfrak{h}\alpha\mathcal{H}Cl(\lambda)$. This is a contradiction to $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$. Therefore, $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Ker(\eta)$. ☐

Definition 3.12. If an $IF\mathcal{H}SS$ $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space, then for each pair of intuitionistic fuzzy hemirings η_1, $\eta_2 \in I^{\mathfrak{R}}$ such that η_1 is non-quasi-coincidence with η_2 and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) \neq 1_{\mathfrak{R}}$, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\gamma_1,\gamma_2 \neq 1_{\mathfrak{R}}$ in $(\mathfrak{R},\mathcal{H})$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \le \gamma_1$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) \le \gamma_2$ and γ_1 is non-quasi-coincidence with γ_2.

Example 3.8. As defined in Example 3.1, $(\mathfrak{R}, \cdot, +)$ is a hemiring. Let the membership values of two intuitionistic fuzzy hemirings $\gamma_1,\gamma_2 \in I^{\mathfrak{R}}$ be defined by $\gamma_{\mu_1},\gamma_{\mu_2}: \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{\mu_1}(0) = 0.3$	$\gamma_{\mu_2}(0) = 0.7$
$\gamma_{\mu_1}(1) = 0.3$	$\gamma_{\mu_2}(1) = 0.7$
$\gamma_{\mu_1}(2) = 0.3$	$\gamma_{\mu_2}(2) = 0.7$

Let the non-membership values of the two intuitionistic fuzzy hemirings $\gamma_1,\gamma_2 \in I^{\mathfrak{R}}$ be defined by $\gamma_{\nu_1},\gamma_{\nu_2}: \mathfrak{R} \longrightarrow [0,1]$ such that

$\gamma_{\nu_1}(0) = 0.7$	$\gamma_{\nu_2}(0) = 0.3$
$\gamma_{\nu_1}(1) = 0.7$	$\gamma_{\nu_2}(1) = 0.3$
$\gamma_{\nu_1}(2) = 0.7$	$\gamma_{\nu_2}(2) = 0.3$

Clearly $\mathcal{H} = \{0_{\mathfrak{R}}, \gamma_1, \gamma_2, 1_{\mathfrak{R}}\}$ is an $IF\mathcal{H}S$ on $\mathfrak{R}$. Then the ordered pair $(\mathfrak{R},\mathcal{H})$ is an $IF\mathcal{H}SS$. Then $0_{\mathfrak{R}}, \gamma_1, \gamma_2, 1_{\mathfrak{R}}$ are $IF\mathfrak{h}\alpha\mathcal{H}Clopens$.

Let the membership values of two intuitionistic fuzzy hemirings $\eta_1, \eta_2 \in I^{\Re}$ be defined by $\eta_{\mu_1}, \eta_{\mu_2}: \Re \longrightarrow [0,1]$ such that

$\eta_{\mu_1}(0) = 0.1$	$\eta_{\mu_2}(0) = 0.2$
$\eta_{\mu_1}(1) = 0.1$	$\eta_{\mu_2}(1) = 0.2$
$\eta_{\mu_1}(2) = 0.1$	$\eta_{\mu_2}(2) = 0.2$

Let the non-membership values of the two intuitionistic fuzzy hemirings $\eta_1, \eta_2 \in I^{\Re}$ be defined by $\eta_{\nu_1}, \eta_{\nu_2}: \Re \longrightarrow [0,1]$ such that

$\eta_{\nu_1}(0) = 0.8$	$\eta_{\nu_2}(0) = 0.6$
$\eta_{\nu_1}(1) = 0.8$	$\eta_{\nu_2}(1) = 0.6$
$\eta_{\nu_1}(2) = 0.8$	$\eta_{\nu_2}(2) = 0.6$

Then η_1 is non-quasi-coincidence with η_2 if there exist $\gamma_1, \gamma_2 \in I^{\Re}$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) = \gamma_1$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) = \gamma_2$ and γ_1 is non-quasi-coincidence with γ_2. Thus $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2)$. Similarly, for every pair of intuitionistic fuzzy hemirings η_1, $\eta_2 \in I^{\Re}$ with η_1 being non-quasi-coincidence with η_2 and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) \neq 1_{\Re}$, there exist $\gamma_1, \gamma_2 \in \mathcal{H}$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) = \gamma_1$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) = \gamma_2$ and γ_1 is non-quasi-coincidence with γ_2 true. Hence $(\Re, \mathcal{H})$ is an intuitionistic fuzzy hemiring R_1 space. ☐

Proposition 3.13. Let $(\Re, \mathcal{H})$ be an $IF\mathcal{H}SS$. If $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space, then $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space.

Proof. Suppose that $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space. Then for every pair η_1, $\eta_2 \in I^{\Re}$ such that η_1 is non-quasi-coincidence with η_2 and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) \neq 1_{\Re}$, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $1_{\Re} \neq \gamma_1, \gamma_2$ in $(\Re, \mathcal{H})$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \leq \gamma_1$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) \leq \gamma_2$ and γ_1 is non-quasi-coincidence with γ_2. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2)$. By Proposition 3.8, $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space. ☐

Remark 3.1. The following Example 3.9 shows that the reverse assertion of Proposition 3.13 needs not be true.

Example 3.9. As defined in Example 3.1, $(\Re, \cdot, +)$ is a hemiring. Let the membership values of four intuitionistic fuzzy hemirings $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in I^{\Re}$ be defined by $\gamma_{\mu_1}, \gamma_{\mu_2}, \gamma_{\mu_3}, \gamma_{\mu_4}: \Re \longrightarrow [0,1]$ such that

$\gamma_{\mu_1}(0) = 0.3$	$\gamma_{\mu_2}(0) = 0.6$	$\gamma_{\mu_3}(0) = 0.7$	$\gamma_{\mu_4}(0) = 0.4$
$\gamma_{\mu_1}(1) = 0.3$	$\gamma_{\mu_2}(1) = 0.6$	$\gamma_{\mu_3}(1) = 0.7$	$\gamma_{\mu_4}(1) = 0.4$
$\gamma_{\mu_1}(2) = 0.3$	$\gamma_{\mu_2}(2) = 0.6$	$\gamma_{\mu_3}(2) = 0.7$	$\gamma_{\mu_4}(2) = 0.4$

Let the non-membership values of the four intuitionistic fuzzy hemirings $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in I^{\Re}$ be defined by $\gamma_{\nu_1}, \gamma_{\nu_2}, \gamma_{\nu_3}, \gamma_{\nu_4}: \Re \longrightarrow [0,1]$ such that

$\gamma_{\nu_1}(0) = 0.7$	$\gamma_{\nu_2}(0) = 0.4$	$\gamma_{\nu_3}(0) = 0.3$	$\gamma_{\nu_4}(0) = 0.6$
$\gamma_{\nu_1}(1) = 0.7$	$\gamma_{\nu_2}(1) = 0.4$	$\gamma_{\nu_3}(1) = 0.3$	$\gamma_{\nu_4}(1) = 0.6$
$\gamma_{\nu_1}(2) = 0.7$	$\gamma_{\nu_2}(2) = 0.4$	$\gamma_{\nu_3}(2) = 0.3$	$\gamma_{\nu_4}(2) = 0.6$

Clearly, $\mathcal{H} = \{0_{\Re}\ \gamma_1,\ \gamma_{2,}\ \gamma_3,\ \gamma_4\ ,1_{\Re}\}$ is an $IF\mathcal{H}S$ on $\Re$. Then the ordered pair $(\Re,\mathcal{H})$ is an $IF\mathcal{H}SS$. Clearly, $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space.

Let the membership values of two intuitionistic fuzzy hemirings $\eta_1,\eta_2\ \in I^{\Re}$ be defined by $\eta_{\mu_1},\eta_{\mu_2}: \Re \longrightarrow [0,1]$ such that

$\eta_{\mu_1}(0) = 0.6$	$\eta_{\mu_2}(0) = 0.2$
$\eta_{\mu_1}(1) = 0.6$	$\eta_{\mu_2}(1) = 0.2$
$\eta_{\mu_1}(2) = 0.6$	$\eta_{\mu_2}(2) = 0.2$

Let the non-membership values of the two intuitionistic fuzzy hemirings $\eta_1,\eta_2\ \in I^{\Re}$ be defined by $\eta_{\nu_1},\eta_{\nu_2}: \Re \longrightarrow [0,1]$ such that

$\eta_{\nu_1}(0) = 0.3$	$\eta_{\nu_2}(0) = 0.6$
$\eta_{\nu_1}(1) = 0.3$	$\eta_{\nu_2}(1) = 0.6$
$\eta_{\nu_1}(2) = 0.3$	$\eta_{\nu_2}(2) = 0.6$

Then η_1 is non-quasi-coincidence with η_2 if there exist $\gamma_3,\ \gamma_4 \in I^{\Re}$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) = \gamma_3$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2) = \gamma_4$. Thus $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_1) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta_2)$. But $(\gamma_3 + \gamma_4) > I^{\Re}$. Hence, γ_3 is quasi-coincidence with γ_4. Hence $(\Re,\mathcal{H})$ is not an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space.

Proposition 3.14. Let $(\Re,\mathcal{H})$ be an $IF\mathcal{H}SS$. Then $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space if and only if for every pair of an intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\Re}$ such that α is non-quasi-coincidence with β and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$, there exist $1_{\Re} \neq \lambda_1$, $\lambda_2 \in \mathcal{H}$ such that $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2 and $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space.

Proof. Suppose that $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space. Let $\alpha,\ \beta \in I^{\Re}$ be two intuitionistic fuzzy hemirings in $\Re$ such that α is non-quasi-coincidence with β and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$. Then there exist $1_{\Re}\ \neq\ \lambda_1,\lambda_2 \in \mathcal{H}$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \leq \lambda_1$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2. Since $\alpha \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha)$ and $\beta \leq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$, $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2.

Conversely, assume that for every pair of intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\Re}$ such that α is non-quasi-coincidence with β and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$ there exist $1_{\Re}\ \neq\ \lambda_1,\lambda_2 \in \mathcal{H}$ such that $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2. Since $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_0 space, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \leq \lambda_1, IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \leq \lambda_2$. Thus $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \leq \lambda_1$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2. Hence $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space. □

Proposition 3.15. Let $(\Re,\mathcal{H})$ be an $IF\mathcal{H}SS$. Then $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space if and only if for every pair of intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\Re}$ where α is non-quasi-coincidence with β such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta)$, there exist $1_{\Re} \neq \lambda_1,\lambda_2 \in \mathcal{H}$ such that $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2.

Proof. The proof follows immediately from Definition 3.9 and Proposition 3.11. □

Proposition 3.16. Let $(\Re,\mathcal{H})$ be an $IF\mathcal{H}SS$. Then the following statements are equivalent:

(i) $(\Re,\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space;

(ii) For any two intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\Re}$ where α is non-quasi-coincidence with β, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$ implies that there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings $\lambda_1, \lambda_2 \in I^{\Re}$ in $(\Re, \mathcal{H})$ such that $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$, $\alpha \nleq \lambda_2$, $\beta \nleq \lambda_1$ and λ_1 is non-quasi-coincidence with λ_2.

Proof. (i) ⇒ (ii) Suppose that $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space. Let $\alpha,\ \beta \in I^{\Re}$ with α being non-quasi-coincidence with β and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$. By Proposition 3.14, there exist two intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\lambda_1, \lambda_2 \in I^{\Re}$ in $(\Re, \mathcal{H})$ such that $\alpha \leq \lambda_1$, $\beta \leq \lambda_2$ and λ_1 is non-quasi-coincidence with λ_2. Take $\eta = (1_{\Re} - \lambda_2)$, $\nu = (1_{\Re} - \lambda_1)$. Then η and ν are intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings in $(\Re, \mathcal{H})$, $\alpha \leq \eta, \alpha \nleq \nu, \beta \nleq \eta,\ \beta \leq \nu$ and η is quasi-coincidence with ν.

(ii) ⇒ (i) Let $\alpha,\ \beta \in I^{\Re}$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\alpha) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\beta) \neq 1_{\Re}$. By (ii), there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings $\eta, \nu \in I^{\Re}$ in $(\Re, \mathcal{H})$ such that $\alpha \leq \eta,\ \beta \leq \nu, \alpha \nleq \nu$, $\beta \nleq \eta$ and η is quasi-coincidence with ν. This implies $\alpha \leq (1_{\Re} - \nu),\ \beta \leq (1_{\Re} - \eta)$. Therefore $(1_{\Re} - \nu)$ and $(1_{\Re} - \eta)$ are intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings in $(\Re, \mathcal{H})$ and $(1_{\Re} - \nu)$ is non-quasi-coincidence with $(1_{\Re} - \eta)$. By Proposition 3.14, $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space. □

Proposition 3.17. Let $(\Re, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then $(\Re, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring R_1 space if and only if for every pair of an intuitionistic fuzzy hemirings $\alpha,\ \beta \in I^{\Re}$ with α being non-quasi-coincidence with β and $IF\mathfrak{h}\alpha\mathcal{H}Ker(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Ker(\delta)$, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\lambda, \gamma \in I^{\Re}$ in $(\Re, \mathcal{H})$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \leq \lambda$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta) \leq \gamma$ and λ is non-quasi-coincidence with γ.

Proof. The proof follows immediately from Proposition 3.10 since $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu) \neq IF\mathfrak{h}\alpha\mathcal{H}Cl(\delta)$. □

4 Properties of intuitionistic fuzzy hemiring swelling and intuitionistic fuzzy hemiring shrinking via IFHSS

In this section, the concepts of intuitionistic fuzzy hemiring shrinking and intuitionistic fuzzy hemiring swelling are introduced. Also, some propositions of them are studied.

Definition 4.1. Let $(\Re, \mathcal{H})$ be an $IF\mathcal{H}SS$. A family $\mathcal{B} = \{\mu_\lambda\}_{\lambda \in \Lambda}$ of intuitionistic fuzzy hemirings in $\Re$ is said to be an intuitionistic fuzzy overlapping family if there exists $x \in \Re$ such that $\mu_\alpha(x) + \mu_\beta(x) > 1$, for all $\alpha, \beta \in \Lambda$.

A family $\{\mu_\lambda\}_{\lambda \in \Lambda}$ is intuitionistic fuzzy non-overlapping if it is not intuitionistic fuzzy overlapping that is for every $x \in \Re$ there exist $\alpha, \beta \in \Lambda$ such that $\mu_\alpha(x) + \mu_\beta(x) \leq 1$.

Definition 4.2. Let $(\Re, \mathcal{H})$ be an $IF\mathcal{H}SS$. For $\lambda_i \in I^{\Re}$, $i \in J$ where J is an indexed set, the intuitionistic fuzzy hemiring swelling of the family $\mathcal{A} = \{\lambda_i : i \in J\}$ is the family of intuitionistic fuzzy hemirings $\mathcal{B} = \{\mu_i \in I^{\Re} : i \in J\}$ such that $\lambda_i \leq \mu_i$ for every $i \in J$ and for every finite set of indices $i_1, i_2, i_3, \dots, i_m \in J$, the family $\{\lambda_{i_1}, \lambda_{i_2}, \lambda_{i_3}, \dots, \lambda_{i_m}\}$ is an overlapping family if and only if $\{\mu_{i_1}, \mu_{i_2}, \mu_{i_3}, \dots, \mu_{i_m}\}$ is an overlapping family.

An intuitionistic fuzzy hemiring swelling is said to be an intuitionistic fuzzy $\mathfrak{h}\alpha$ open (respectively, intuitionistic fuzzy $\mathfrak{h}\alpha$ closed) hemiring swelling if all of its members are intuitionistic fuzzy $\mathfrak{h}\alpha$ open (respectively, intuitionistic fuzzy $\mathfrak{h}\alpha$ closed) hemirings.

Clearly, every intuitionistic fuzzy hemiring swelling $\mathcal{B}$ of a family $\mathcal{A}$ satisfies the equality $Ord_{IF\mathcal{H}}\mathcal{A} = Ord_{IF\mathcal{H}}\mathcal{B}$.

Definition 4.3. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. The intuitionistic fuzzy hemiring shrinking of the fuzzy cover $\mathcal{A} = \{\lambda_i \in I^{\mathfrak{R}} : i \in J\}$ of $(\mathfrak{R}, \mathcal{H})$ is defined to be the intuitionistic fuzzy cover $\mathcal{B} = \{\mu_i \in I^{\mathfrak{R}} : i \in J\}$ of $(\mathfrak{R}, \mathcal{H})$ such that $\mu_j \le \lambda_i$ where $j \le i$ for every $i \in J$. An intuitionistic fuzzy hemiring shrinking is said to be an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring (respectively, intuitionistic fuzzy $\mathfrak{h}\alpha$ closed) shrinking if all of its members are intuitionistic fuzzy $\mathfrak{h}\alpha$ open (respectively, intuitionistic fuzzy $\mathfrak{h}\alpha$ closed) hemirings.

Clearly, every intuitionistic fuzzy hemiring shrinking $\mathcal{B}$ of an intuitionistic fuzzy cover $\mathcal{A}$ is an intuitionistic fuzzy refinement of $\mathcal{A}$ and satisfies the inequality $Ord_{IF\mathcal{H}}\mathcal{B} \le Ord_{IF\mathcal{H}}\mathcal{A}$.

Definition 4.4. A hemiring $(\mathfrak{R}, \mathcal{H})$ is called an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal if for each pair of intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings λ and μ of $(\mathfrak{R}, \mathcal{H})$ with λ being non-quasi-coincidence with μ, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings γ and η such that $\lambda \le \gamma$ and $\mu \le \eta$ and γ is non-quasi-coincidence with η.

Proposition 4.1. Let $(\mathfrak{R}, \mathcal{H})$ be an $IF\mathcal{H}SS$. Then the following statements are equivalent:

(i) $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space;

(ii) For each intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring $\gamma \in I^{\mathfrak{R}}$ and for each intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\lambda \in I^{\mathfrak{R}}$ with $\gamma \le \lambda$, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\eta \in I^{\mathfrak{R}}$ where $\gamma \le \eta$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \le \lambda$;

(iii) For each pair of intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings $\gamma,\ \mu \in I^{\mathfrak{R}}$ with γ is non-quasi-coincidence with μ, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\eta \in I^{\mathfrak{R}}$ where $\gamma \le \eta$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$ is non-quasi-coincidence with μ;

(iv) For each pair of an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings $\gamma,\ \mu \in I^{\mathfrak{R}}$ with γ being non-quasi-coincidence with μ, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\eta,\ \nu \in I^{\mathfrak{R}}$ where $\gamma \le \eta$ and $\mu \le \nu$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl\,(\nu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

Proof. (i) ⇒ (ii) Let $(\mathfrak{R}, \mathcal{H})$ be an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space. Let $\gamma \in I^{\mathfrak{R}}$ be an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring and $\lambda \in I^{\mathfrak{R}}$ be any intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring with $\gamma \le \lambda$. This implies that γ is non-quasi-coincidence with $(1_{\mathfrak{R}} - \lambda)$. Also, γ and $(1_{\mathfrak{R}} - \lambda)$ are intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings in $(\mathfrak{R}, \mathcal{H})$. Since $(\mathfrak{R}, \mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space, there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\eta,\ \nu \in I^{\mathfrak{R}}$ with $\gamma \le \eta$ and $(1_{\mathfrak{R}} - \lambda) \le \nu$ such that η is non-quasi-coincidence with ν. This implies that $\eta \le (1_{\mathfrak{R}} - \nu)$ and hence $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \le IF\mathfrak{h}\alpha\mathcal{H}Cl(1_{\mathfrak{R}} - \nu) = (1_{\mathfrak{R}} - \nu)$, since $(1_{\mathfrak{R}} - \nu)$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring. Since $(1_{\mathfrak{R}} - \lambda) \le \nu$, $(1_{\mathfrak{R}} - \nu) \le \lambda$. Therefore, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \le (1_{\mathfrak{R}} - \nu) \le \lambda$. Hence, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \le \lambda$.

(ii) ⇒ (iii) Let $\gamma,\ \mu \in I^{\mathfrak{R}}$ be two intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings with γ being non-quasi-coincidence with μ. Then $\gamma \le (1_{\mathfrak{R}} - \mu)$**,** where $(1_{\mathfrak{R}} - \mu)$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring. Therefore by (ii) there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\eta \in I^{\mathfrak{R}}$ with $\gamma \leq \eta$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta) \leq (1_{\mathfrak{R}} - \mu)$. This implies that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$ is non-quasi-coincidence with μ.

(iii) ⇒ (iv) Let $\gamma,\ \mu \in I^{\mathfrak{R}}$ be intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings with γ being non-quasi-coincidence with μ. Then by (iii), there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\eta \in I^{\mathfrak{R}}$ with $\gamma \leq \eta$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$ is non-quasi-coincidence with μ. Again by (iii), there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\nu \in I^{\mathfrak{R}}$ where $\mu \leq \nu$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl\,(\nu)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\eta)$.

(iv) ⇒ (i) Let $\gamma,\ \mu \in I^{\mathfrak{R}}$ be two intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings with γ being non-quasi-coincidence with μ. Then by (iv), there exist intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\eta,\ \nu \in I^{\mathfrak{R}}$ where $\gamma \leq \eta$ and $\mu \leq \nu$ such that $IF\mathfrak{h}\alpha\mathcal{H}Cl\,(\eta)$ is non-quasi-coincidence with $IF\mathfrak{h}\alpha\mathcal{H}Cl(\nu)$. This implies that η is non-quasi-coincidence with ν. Hence $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space.

This completes the proof. □

Proposition 4.2. Let $(\mathfrak{R},\mathcal{H})$ be an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space and for every finite family $\left\{\lambda_i \in I^{\mathfrak{R}}\right\}_{i=1}^{k}$ of intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring swelling $\left\{\mu_i \in I^{\mathfrak{R}}\right\}_{i=1}^{k}$ such that $\lambda_i \leq \mu_i$. Moreover, if a family $\left\{\gamma_i \in I^{\mathfrak{R}}\right\}_{i=1}^{k}$ of intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings satisfies the inequality $\lambda_i \leq \gamma_i$ for $i = 1,2,3,\dots,k$, then for an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring swelling $\left\{\gamma_i \in I^{\mathfrak{R}}\right\}_{i=1}^{k}$, $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_i) \leq \gamma_i$ for $i = 1,2,3,\dots,k$.

Proof. Let $\left\{\lambda_{i_j} \in I^{\mathfrak{R}},\ j = 1,2,\dots,m,\ i = 1,2,\dots,k\right\}$ be an overlapping family of intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings in $(\mathfrak{R},\mathcal{H})$ which satisfies the condition that λ_1 is non-quasi-coincidence with λ_{i_j}, $j = 1,2,\dots,m,\ i = 1,2,\dots,k$ where $\lambda_1 \in I^{\mathfrak{R}}$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring in $(\mathfrak{R},\mathcal{H})$. Let $\eta_1 = \vee_{i=1}^{k}\left(\wedge_{j=1}^{m} \lambda_{i_j}\right)$, $j = 1,2,\dots,m,\ i = 1,2,\dots,k$. Then η_1 is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring in $(\mathfrak{R},\mathcal{H})$ such that η_1 is non-quasi-coincidence with λ_1. Since $(\mathfrak{R},\mathcal{H})$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ hemiring normal space and η_1 is non-quasi-coincidence with λ_1, by Proposition 4.1, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring μ_1 such that $\lambda_1 \leq \mu_1$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_1)$ is non-quasi-coincidence with η_1. This implies that the family $\{IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_1),\ \lambda_2,\ \lambda_3,\dots,\lambda_k\}$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring swelling of the family $\{\lambda_i\}_{i=1}^{k}$.

Assume that for $i = 1,2,\dots,n-1$ and $n < k$, there is an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring μ_i such that $\lambda_i \leq \mu_i$ and the family $\{IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_1),\ IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_2),\dots,$ $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_{n-1}),\ \lambda_n,\dots,\lambda_k\}$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring swelling of the family $\{\lambda_i\}_{i=1}^{k}$. Let

$$\mathfrak{F} = \left\{ IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_1)_j, IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_2)_j,\dots, IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_{n-1})_j, \lambda_{n_j},\dots,\lambda_{k_j}, j = 1,2,\dots,m \right\}$$

be an overlapping family of an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemirings in $(\mathfrak{R},\mathcal{H})$ and for any j, α_{i_j} be the members of $\mathfrak{F}$ which satisfies the condition α_{i_j} is non-quasi-coincidence with λ_n, $j = 1,2,\dots,m,\ i = 1,2,\dots,k$. Let $\eta_n = \vee_{i=1}^{k}\,(\wedge_{j=1}^{m} \alpha_{i_j})$. Then η_n is an intuitionistic fuzzy $\mathfrak{h}\alpha$

closed hemiring in $(\mathfrak{R}, \mathcal{H})$ such that η_n is non-quasi-coincidence with λ_n. Again by Proposition 4.1, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring μ_n such that $\lambda_n \leq \mu_n$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_n)$ is non-quasi-coincidence with η_n. Hence the family

$$\{IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_1), IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_2), IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_3), \dots, IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_n), \lambda_{n+1}, \dots, \lambda_k\}$$

is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring swelling of the family $\{\lambda_i\}_{i=1}^{k}$. Similarly, there are some intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings $\mu_1, \mu_2, \dots, \mu_k$ such that $\lambda_i \leq \mu_i$ for $i = 1, 2, \dots, k$ and the family $\{IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_i)\}_{i=1}^{k}$ is an intuitionistic fuzzy $\mathfrak{h}\alpha$ closed hemiring swelling of the family $\{\lambda_i\}_{i=1}^{k}$. Clearly, the family $\{\mu_i\}_{i=1}^{k}$ is the required intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring swelling of the family $\{\lambda_i\}_{i=1}^{k}$.

Assume that a family $\{\gamma_i \in I^{\mathfrak{R}}\}_{I=1}^{k}$ of intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemirings of $(\mathfrak{R}, \mathcal{H})$ satisfies $\lambda_i \leq \gamma_i$ for $i = 1, 2, \dots, k$. Then by Proposition 4.1, there exists an intuitionistic fuzzy $\mathfrak{h}\alpha$ open hemiring $\mu_i \in I^{\mathfrak{R}}$ such that $\lambda_i \leq \mu_i$ and $IF\mathfrak{h}\alpha\mathcal{H}Cl(\mu_i) \leq \gamma_i$ for $i = 1, 2, \dots, k$. □

5 Conclusion

In this paper intuitionistic fuzzy hemiring structure spaces axioms are studied and established with examples. Also, the concept of an intuitionistic fuzzy hemiring shrinking and an intuitionistic fuzzy hemiring swelling and an intuitionistic fuzzy hemiring $\mathfrak{h}\alpha$ normal spaces are introduced and some of its properties are studied. In further study intuitionistic fuzzy hemiring may be extended in covering dimension of intuitionistic fuzzy hemiring $\mathfrak{h}\alpha$ normal spaces.

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