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REMARK AND OPEN PROBLEMS ON INTUITIONISTIC FUZZY SETS AND COMPLEX ANALYSIS

Krassimir Atanassov

Centre of Biomedical Engineering "Prof. Ivan Daskalov"
Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
e-mail: *krat@bas.bg*

Over Intuitionistic Fuzzy Sets (IFSs, see [1]) different operations, relations and operators are defined. Operation "negation" has important place among other operations. One of the reasons is that there are different ways for defining it. Here, for brevity, we shall discuss only the standard negation, marked in [2, 3] by $\neg_1$. For a given IFS

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

it has the form

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

where E is a fixed universe and functions μ_A and ν_A are degrees of membership and non-membership of the IFS A.

For two elements x and y of E we shall call that they are in relation *negation* if

$$\mu_A(x) = \nu_A(y) \text{ and } \nu_A(x) = \mu_A(y).$$

Their geometrical interpretation in the intuitionistic fuzzy interpretation triangle is shown on Fig. 1.

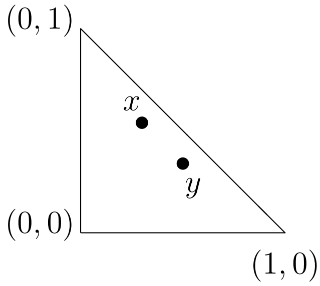

Fig. 1.

It is well-known that numbers $a + \mathbf{i}b$ and $a - \mathbf{i}b$ are complex conjugate in the complex plane, where $\mathbf{i}$ is the imaginary unit. Their geometical representation in Euclidean metric is shown on Fig. 2 and in Hamming metric – on Fig. 3. Let the radius of the circle from Fig. 2 (only the right hand of the circle is shown for brevity) and sections OA, OB and OC from Fig. 3 have length 1 (unit length).

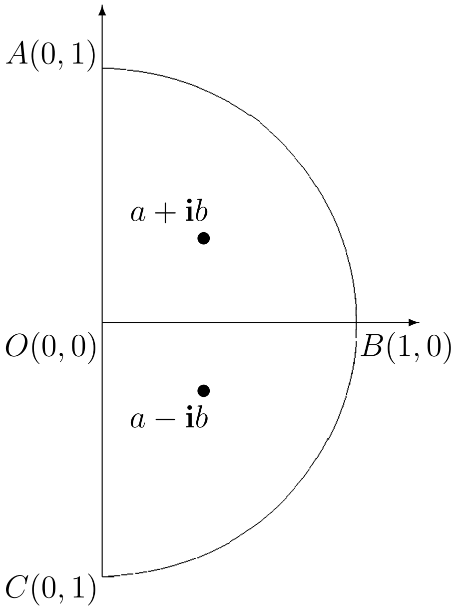

Fig. 2.

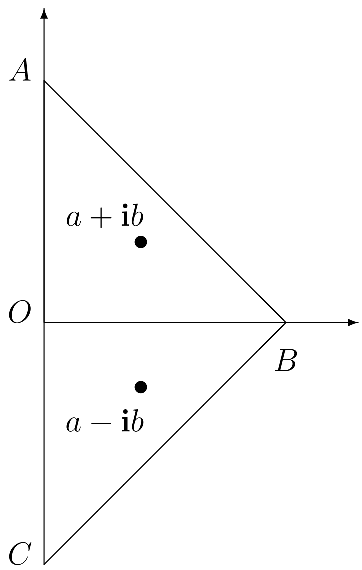

Fig. 3.

We shall introduce formulae that transform the points of triangle ABC in triangle ABO. One of the possible forms of these formulae is:

$$f(a,b) = \begin{cases} (\frac{a}{2}, \frac{a}{2} + b), & \text{if } b \geq 0 \\ \\ (\frac{a}{2} - b, \frac{a}{2}), & \text{if } b \leq 0 \end{cases},$$

where (a, b) are the coordinates of complex number $a + \mathbf{i}b$ and $a \in [0, 1], b \in [-1, 1]$.

We can see immediately, that

$$f(0,0) = (0,0),$$

$$f(0,1) = (0,1),$$

$$f(1,0) = (\frac{1}{2}, \frac{1}{2}),$$

$$f(0,-1) = (1,0).$$

It can be easily checked that function f is continuous and bijective.

From the above formulae we directly see that

$$f^{-1}(a,b) = (2a, b-a).$$

Now, we see that for the arbitrary complex conjugate numbers $a + \mathbf{i}b$ and $a - \mathbf{i}b$ (here $a, b, a+b \in [0,1]$):

$$f(a,b) = (\frac{a}{2}, \frac{a}{2} + b),$$
$$f(a,-b) = (\frac{a}{2} + b, \frac{a}{2}).$$

Therefore, if IFS-element x has degrees of membership and non-membership $\frac{a}{2}$ and $\frac{a}{2} + b$, then IFS-element y that has degrees of membership and non-membership $\frac{a}{2} + b$ and $\frac{a}{2}$) is in a relation *negation* with x.

So, we proved the validity of the following assertion

Two numbers are complex conjugate if and only if their f-transform coincide with the degrees of two elements of a given IFS, which are in relation *negation* and of its dual assertion

Two elements of a given IFS are in relation *negation* **if and only if their f^{-1}-transform coincide with two complex conjugate numbers**.

The above construction generates a lot of interesting (and at the moment open) problems. Some of them are the following:

Problem 1: To develop a complex analysis for the intuitionistic fuzzy interpretation triangle.

Problem 2: To find transformations from the complex plane to the intuitionistic fuzzy interpretation triangle and vice versa in order to acquire more convenient methods for studying different properties of given IFSs or of given objects in the complex plane.

Problem 3: To find interpretations of the other intuitionistic fuzzy negations in complex plane and to study their behaviour.

Problem 4: To construct complex analysis interpretations of the IFS-operators from modal, and topological and level types.

References

[1] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] Atanassov, K. On some intuitionistic fuzzy implications. *Comptes Rendus de l'Academie bulgare des Sciences,* Tome 59, 2006 , No. 1, 19-24.

[3] Atanassov, K., On some types of intuitionistic fuzzy negations. *Notes on Intuitionistic Fuzzy Sets,* Vol. 11, 2005, No. 4, 170-172.