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# Regularity and duality of intuitionistic fuzzy *k*-partite hypergraphs

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Abstract: A graph in which the edge can connect more than two vertices is called a Hypergraph. A k-partite hypergraph is a hypergraph whose vertices can be split into k different independent sets. In this paper, regular, totally regular, totally irregular, totally neighborly irregular Intuitionistic Fuzzy k-Partite Hypergraphs (IFk-PHGs) are defined. Also order and size along with the properties of regular and totally regular IFk-PHGs are discussed. It has been proved that the size  $S(\mathcal{H})$  of a r-regular IFk-PHG is  $\frac{tr}{2}$  where t = |V|. The dual IFk-PHG has also been defined with example.

**Keywords:** Total degree, Regular IF*k*-PHG, Totally regular IF*k*-PHG, Totally irregular IF*k*-PHG, Totally neighborly irregular IF*k*-PHG, Dual IF*k*-PHG.

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## **1** Introduction

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. The history of graph theory may be specifically traced

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to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. In several real-world applications, relationships are more problematic among the objects, then the graph theory doesn't work to handle such relationships when we consider more than two objects. In this case, we use hypergraphs to represent complex relationships among objects.

In 1976, Berge [2] putforth the ideas of graph and hypergraph theory. The authors in [3] developed the concepts of fuzzy graph and fuzzy hypergraph. As a generalization of fuzzy sets, Atanassov [1] introduced the concepts of intuitionistic fuzzy set in 1983. In [7], regular fuzzy graphs and totally regular fuzzy graphs are compared through various examples. Also, a necessary and sufficient condition under which they are equivalent is provided. The authors I. Pradeepa, and S. Vimala in [9, 10] discussed some of the properties of regular and irregular intuitionistic fuzzy hypergraphs. Certain ideas like single-valued neutrosophic hypergraph are discussed by the authors in [4]. The concepts of Intuitionistic fuzzy hypergraph(IFHG), strength of an edge, dual IFHG are developed in [8].

In [5, 6] the authors K. K. Myithili and R. Keerthika proposed the concepts of intuitionistic fuzzy k-partite hypergraphs. Also they discussed isomorphic relation between two IFk-PHGs. In this paper, an attempt had been made to study the regular, totally regular, totally irregular, dual IFk-PHGs. The necessary and sufficient condition for which regular and totally regular IFk-PHGs are equivalent is provided in this article.

### 2 Preliminaries

Basic definitions relating to intuitionistic fuzzy set, intuitionistic fuzzy hypergraph and intuitionistic fuzzy *k*-partite hypergraph are discussed in this section.

**Definition 2.1** ([1]). Let a set E be fixed. An intuitionistic fuzzy set (IFS) V in E is an object of the form  $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i) \rangle | v_i \in E\}$ , where the function  $\mu_i : E \to [0, 1]$  and  $\nu_i : E \to [0, 1]$  determine the degree of membership and the degree of non-membership of the element  $v_i \in E$ , respectively and for every  $v_i \in E$ ,  $0 \le \mu_i(v_i) + \nu_i(v_i) \le 1$ .

**Definition 2.2** ([8]). An intuitionistic fuzzy hypergraph (IFHG) is an ordered pair  $H = \langle V, E \rangle$  where

(i)  $V = \{v_1, v_2, \dots, v_n\}$ , is a finite set of intuitionistic fuzzy vertices (ii)  $E = \{E_1, E_2, \dots, E_m\}$  is a family of crisp subsets of V(iii)  $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)) : \mu_j(v_i), \nu_j(v_i) \ge 0 \text{ and } \mu_j(v_i) + \nu_j(v_i) \le 1\}, \forall v_i \in V, j = 1, 2, \dots, m$ (iv)  $E_j \ne \emptyset, j = 1, 2, \dots, m$ (v)  $\bigcup_j \operatorname{supp}(E_j) = V, j = 1, 2, \dots, m$ 

Here, the hyperedges  $E_j$  are crisp sets of intuitionistic fuzzy vertices,  $\mu_j(v_i)$  and  $\nu_j(v_i)$  denote the degrees of membership and non-membership of vertex  $v_i$  to edge  $E_j$ . Thus, the elements of the incidence matrix of IFHG are of the form  $\langle v_{ij}, \mu_j(v_i), \nu_j(v_j) \rangle$ . The sets (V, E) are crisp sets. **Definition 2.3** ([5]). *The IFk-PHG is an ordered triple* H *where* 

- $V = \{v_1, v_2, \dots, v_n\}$  is a finite set of vertices
- $E = \{E_1, E_2, E_3, \dots, E_m\}$  is a family of intuitionistic fuzzy subsets of V
- $E_j = \{ \langle v_i, \mu_j(v_i), \nu_j(v_i) \rangle : \mu_j(v_i), \nu_j(v_i) \ge 0 \text{ and } \mu_j(v_i) + \nu_j(v_i) \le 1 \}, 1 \le j \le m, \forall v_i \in V$
- $E_j \neq \emptyset$ ,  $1 \leq j \leq m$
- $\bigcup_{j} \operatorname{supp}(E_j) = V, 1 \le j \le m$
- $(\forall v_i \in E) \exists k$ -disjoint sets  $\psi_i$ , i = 1, 2, ..., k and no two vertices in the same set are adjacent such that  $E_k = \bigcap_{i=1}^k \psi_i = \emptyset$

**Definition 2.4** ([5]). Let an IFk-PHG be  $\mathcal{H} = (V, E, \psi)$ . The height of IFk-PHG is defined by  $h(\mathcal{H}) = \{\max(\min(\mu_{k_{ij}})), \max(\max(\nu_{k_{ij}}))\} \forall 1 \leq i \leq m \text{ and } 1 \leq j \leq n. \text{ Also } \mu_{k_{ij}} \& \nu_{k_{ij}} \text{ are membership and non-membership values of the k-partite hyperedge } \psi_{ij}.$ 

**Definition 2.5** ([6]). The order of an IFk-PHG,  $\mathcal{H} = (V, E, \psi)$  is defined to be  $O(\mathcal{H}) = O_{\langle \mu_k, \nu_k \rangle}(\mathcal{H}) = \sum_{v_i \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v_i).$ 

**Definition 2.6** ([6]). The size of an IFk-PHG,  $\mathcal{H} = (V, E, \psi)$  is defined to be  $S(\mathcal{H}) = S_{\langle \mu_k, \nu_k \rangle}(\mathcal{H}) = \sum_{v_i, v_j \in V} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j).$ 

**Definition 2.7** ([6]). The degree of a vertex v in an IFk-PHG,  $\mathcal{H}$  is denoted by  $d_{\mathcal{H}}(v)$  and defined by  $d_{\mathcal{H}}(v) = \langle d_{\mu}(v), d_{\nu}(v) \rangle$  where  $d_{\mu}(v) = \mu_{k_{ij}}, d_{\nu}(v) = \nu_{k_{ij}}$ .

*Note:* The degree of each vertex in a k-partite hyperedge is nothing but the membership and non-membership values of the corresponding k-partite hyperedge.

**Example 1.** For an IF*k*-PHG  $\mathcal{H}$ , let  $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ ,  $E = \{E_1, E_2, E_3\}$  and  $\psi = \{\psi_1, \psi_2, \psi_3, \psi_4\}$  where  $v_1 = \langle 0.2, 0.6 \rangle, v_2 = \langle 0.3, 0.4 \rangle, v_3 = \langle 0.4, 0.5 \rangle, v_4 = \langle 0.2, 0.5 \rangle,$   $v_5 = \langle 0.1, 0.8 \rangle, v_6 = \langle 0.6, 0.2 \rangle, v_7 = \langle 0.4, 0.3 \rangle, v_8 = \langle 0.5, 0.2 \rangle,$   $E_1 = \{v_1 \langle 0.2, 0.6 \rangle, v_2 \langle 0.3, 0.4 \rangle, v_3 \langle 0.4, 0.5 \rangle, v_4 \langle 0.2, 0.5 \rangle\},$   $E_2 = \{v_1 \langle 0.2, 0.6 \rangle, v_2 \langle 0.3, 0.4 \rangle, v_5 \langle 0.1, 0.8 \rangle, v_6 \langle 0.6, 0.2 \rangle\},$   $E_3 = \{v_5 \langle 0.1, 0.8 \rangle, v_6 \langle 0.6, 0.2 \rangle, v_7 \langle 0.4, 0.3 \rangle, v_8 \langle 0.5, 0.2 \rangle\}$  with  $\psi_1 = \{v_1 \langle 0.2, 0.6 \rangle, v_7 \langle 0.4, 0.3 \rangle\}, \psi_2 = \{v_2 \langle 0.3, 0.4 \rangle, v_8 \langle 0.5, 0.2 \rangle\},$  $\psi_3 = \{v_3 \langle 0.4, 0.5 \rangle, v_5 \langle 0.1, 0.8 \rangle\}, \psi_4 = \{v_4 \langle 0.2, 0.5 \rangle, v_6 \langle 0.6, 0.2 \rangle\}.$ 

Then

$$d_{\mathcal{H}}(v_1) = \langle 0.2, 0.6 \rangle, d_{\mathcal{H}}(v_2) = \langle 0.3, 0.4 \rangle, d_{\mathcal{H}}(v_3) = \langle 0.1, 0.8 \rangle, d_{\mathcal{H}}(v_4) = \langle 0.2, 0.5 \rangle, \\ d_{\mathcal{H}}(v_5) = \langle 0.1, 0.8 \rangle, d_{\mathcal{H}}(v_6) = \langle 0.2, 0.5 \rangle, d_{\mathcal{H}}(v_7) = \langle 0.2, 0.6 \rangle, d_{\mathcal{H}}(v_8) = \langle 0.3, 0.4 \rangle.$$

# 3 Regular and irregular intuitionistic fuzzy *k*-partite hypergraphs

**Definition 3.1.** The open neighborhood degree  $d_{N_k}(v)$  of a vertex v in an intuitionistic fuzzy k-partite hypergraph,  $\mathcal{H} = (V, E, \psi)$  is defined by  $d_{N_k}(v) = (d^{\mu}(v), d^{\nu}(v))$  where  $d^{\mu}(v) = \sum_{v \in N_k(v)} \mu_{k_i}(v), d^{\nu}(v) = \sum_{v \in N_k(v)} \nu_{k_i}(v).$ 

**Example 2.** For an intuitionistic fuzzy k-partite hypergraph  $\mathcal{H}$ , define  $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, E = \{E_1, E_2, E_3, E_4\}, \psi = \{\psi_1, \psi_2, \psi_3\} \text{ where}$   $E_1 = \{v_1 \langle 0.4, 0.3 \rangle, v_2 \langle 0.5, 0.1 \rangle, v_3 \langle 0.7, 0.2 \rangle\},$   $E_2 = \{v_1 \langle 0.4, 0.3 \rangle, v_2 \langle 0.5, 0.1 \rangle, v_4 \langle 0.6, 0.2 \rangle\},$   $E_3 = \{v_2 \langle 0.5, 0.1 \rangle, v_3 \langle 0.7, 0.2 \rangle, v_5 \langle 0.1, 0.7 \rangle\},$   $E_4 = \{v_5 \langle 0.1, 0.7 \rangle, v_6 \langle 0.2, 0.6 \rangle, v_7 \langle 0.3, 0.4 \rangle\} \text{ with}$   $\psi_1 = \{v_1 \langle 0.4, 0.3 \rangle, v_5 \langle 0.1, 0.7 \rangle\},$   $\psi_2 = \{v_2 \langle 0.5, 0.1 \rangle, v_6 \langle 0.2, 0.6 \rangle\},$   $\psi_3 = \{v_3 \langle 0.7, 0.2 \rangle, v_4 \langle 0.6, 0.2 \rangle, v_7 \langle 0.3, 0.4 \rangle\}.$ 

The vertex  $v_3$  has the open neighborhood degree of (0.9, 0.6).

**Definition 3.2.** The closed neighborhood degree  $d_{N_k}[v]$  of a vertex v in an intuitionistic fuzzy k-partite hypergraph,  $\mathcal{H} = (V, E, \psi)$  is defined by  $d_{N_k}[v] = (d^{\mu}[v], d^{\nu}[v])$ , where  $d^{\mu}[v] = d^{\mu}(v) + \mu_{k_i}(v), d^{\nu}[v] = d^{\nu}(v) + \nu_{k_i}(v)$ .

**Example 3.** From the above example, it is known that the vertex  $v_3$  has the closed neighborhood degree of  $\langle 1.6, 0.8 \rangle$ .

**Definition 3.3.** The *total degree* of a vertex v in an intuitionistic fuzzy k-partite hypergraph  $\mathcal{H}$ , denoted by  $td_{\mathcal{H}}(v)$  is defined as  $td_{\mathcal{H}}(v) = \langle td_{\mu}(v), td_{\nu}(v) \rangle$  where

$$td_{\mu}(v) = d_{\mu}(v) + \mu_{k_i}(v), td_{\nu}(v) = d_{\nu}(v) + \nu_{k_i}(v).$$

Example 4. From Example 1, it is clear that the total degree of each of the vertex is

$$\begin{aligned} td_{\mathcal{H}}(v_1) &= \langle 0.4, 1.2 \rangle, td_{\mathcal{H}}(v_2) = \langle 0.6, 0.8 \rangle, td_{\mathcal{H}}(v_3) = \langle 0.5, 1.3 \rangle, td_{\mathcal{H}}(v_4) = \langle 0.4, 1.0 \rangle, \\ td_{\mathcal{H}}(v_5) &= \langle 0.2, 1.6 \rangle, td_{\mathcal{H}}(v_6) = \langle 0.8, 0.7 \rangle, td_{\mathcal{H}}(v_7) = \langle 0.6, 0.9 \rangle, td_{\mathcal{H}}(v_8) = \langle 0.8, 0.6 \rangle. \end{aligned}$$

**Definition 3.4.** Let  $\mathcal{H} = (V, E, \psi)$  be an IF*k*-PHG. If all the vertices in *V* have the same degree *r*, then  $\mathcal{H}$  is said to be an *r*-regular intuitionistic fuzzy *k*-partite hypergraph.

**Definition 3.5.** Let  $\mathcal{H} = (V, E, \psi)$  be an IFk-PHG. If all the vertices in V have the same total degree s, then  $\mathcal{H}$  is said to be an s-totally regular intuitionistic fuzzy k-partite hypergraph.

**Definition 3.6.** If  $\mathcal{H} = (V, E, \psi)$  is both *r*-regular and *s*-totally regular IF*k*-PHG, then it is called *perfectly regular* IF*k*-PHG.

**Example 5.** Consider an IF*k*-PHG,  $\mathcal{H}$  with  $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}, E = \{E_1, E_2, E_3, E_4, E_5\}, \psi = \{\psi_1, \psi_2, \psi_3\}$  which is shown below.



Fig. 1. IFk-PHG  $\mathcal{H}$ 

Here  $d_{\mathcal{H}}(v_1) = d_{\mathcal{H}}(v_2) = d_{\mathcal{H}}(v_3) = d_{\mathcal{H}}(v_4) = d_{\mathcal{H}}(v_5) = d_{\mathcal{H}}(v_6) = \langle 0.4, 0.3 \rangle$ . Hence,  $\mathcal{H}$  is a  $\langle 0.4, 0.3 \rangle$ -regular intuitionistic fuzzy k-partite hypergraph.

Also,  $td_{\mathcal{H}}(v_1) = td_{\mathcal{H}}(v_2) = td_{\mathcal{H}}(v_3) = td_{\mathcal{H}}(v_4) = td_{\mathcal{H}}(v_5) = td_{\mathcal{H}}(v_6) = \langle 0.8, 0.6 \rangle$ . Hence,  $\mathcal{H}$  is a  $\langle 0.8, 0.6 \rangle$ -totally regular intuitionistic fuzzy k-partite hypergraph.

The above IF*k*-PHG,  $\mathcal{H}$  is both regular and totally regular, hence it is called perfectly regular IF*k*-PHG.

**Theorem 3.1.** Let  $\mathcal{H}$  be an intuitionistic fuzzy k-partite hypergraph. If  $\mathcal{H}$  is a regular IFk-PHG and  $\mu_{k_i} : V \to [0, 1]$ ,  $\nu_{k_i} : V \to [0, 1]$  is a constant function, then  $\mathcal{H}$  is a totally regular IFk-PHG.

*Proof.* Let  $\mathcal{H}$  be a regular IF*k*-PHG and  $\mu_{k_i}$ ,  $\nu_{k_i}$  be a constant function, then  $\mu_{k_i}(v) = N_1$  and  $\nu_{k_i}(v) = N_2$  for all  $v \in \psi_i$ ,  $N_1$ ,  $N_2$  are constant,  $(N_1, N_2) \in [0, 1]$  and  $d_{\mu}(v) = m_1$ ,  $d_{\nu}(v) = m_2$  for all  $v \in \psi_i$ , i = 1, 2, ..., k. Since,  $td_{\mathcal{H}}(v) = \langle d_{\mu}(v) + \mu_{k_i}(v), d_{\nu}(v) + \nu_{k_i}(v) \rangle$  this implies that  $td_{\mathcal{H}}(v) = \langle m_1 + N_1, m_2 + N_2 \rangle$  for all  $v \in \psi_i$ . Hence,  $\mathcal{H}$  is a totally regular IF*k*-PHG.

**Theorem 3.2.** Let  $\mathcal{H}$  be an IFk-PHG. If  $\mathcal{H}$  is a totally regular IFk-PHG and  $\mu_{k_i} : V \to [0,1]$ and  $\nu_{k_i} : V \to [0,1]$  is a constant function then  $\mathcal{H}$  is a regular IFk-PHG.

*Proof.* Let  $\mathcal{H}$  be a totally regular IFk-PHG and  $\langle \mu_{k_i}, \nu_{k_i} \rangle$  be a constant function, then  $\mu_{k_i}(v) = N_1$ and  $\nu_{k_i}(v) = N_2$  for all  $v \in \psi_i, N_1, N_2$  are constant,  $(N_1, N_2) \in [0, 1]$  and  $td_{\mu}(v) = r_1, td_{\nu}(v) = r_2$ for all  $v \in \psi_i, i = 1, 2, ..., k$ . Since,  $td_{\mathcal{H}}(v) = \langle d_{\mu}(v) + \mu_{k_i}(v), d_{\nu}(v) + \nu_{k_i}(v) \rangle$  this implies that

 $d_{\mathcal{H}}(v) = \langle td_{\mu}(v) - \mu_{k_i}(v), td_{\nu}(v) - \nu_{k_i}(v) \rangle$  $\Rightarrow d_{\mathcal{H}}(v) = \langle r_1 - N_1, r_2 - N_2 \rangle \text{ for all } v \in \psi_i.$ 

Hence  $\mathcal{H}$  is a regular IF*k*-PHG.

**Theorem 3.3.** If  $\mathcal{H}$  is both regular and totally regular IFk-PHG, then  $\langle \mu_{k_i}, \nu_{k_i} \rangle$  is a constant function.

*Proof.* Let  $\mathcal{H}$  be both regular and totally regular IF*k*-PHG, then  $d_{\mu}(v) = m_1, d_{\nu}(v) = m_2$  for all  $v \in \psi_i, i = 1, 2, ..., k$ . This implies that  $d_{\mu}(v) + \mu_{k_i}(v) = r_1 \quad \forall v \in \psi_i$   $\Rightarrow m_1 + \mu_{k_i}(v) = r_1$   $\Rightarrow \mu_{k_i}(v) = r_1 - m_1$ Also,  $d_{\nu}(v) + \nu_{k_i}(v) = r_2$   $\Rightarrow m_2 + \nu_{k_i}(v) = r_2$   $\Rightarrow \nu_{k_i}(v) = r_2 - m_2 \quad \forall v \in \psi_i$ Hence,  $\langle \mu_{k_i}, \nu_{k_i} \rangle$  is a constant function.

**Remark 1.** The converse of the above theorem is also true. Consider Figure 1, in which  $\langle \mu_{k_i}, \nu_{k_i} \rangle$  are constant. Here  $d_{\mathcal{H}}(v_1) = d_{\mathcal{H}}(v_2) = d_{\mathcal{H}}(v_3) = d_{\mathcal{H}}(v_4) = d_{\mathcal{H}}(v_5) = d_{\mathcal{H}}(v_6) = \langle 0.4, 0.3 \rangle$ . Also,  $td_{\mathcal{H}}(v_1) = td_{\mathcal{H}}(v_2) = td_{\mathcal{H}}(v_3) = td_{\mathcal{H}}(v_4) = td_{\mathcal{H}}(v_5) = td_{\mathcal{H}}(v_6) = \langle 0.8, 0.6 \rangle$ . Hence,  $\mathcal{H}$  is both regular and totally regular IF*k*-PHG.

**Definition 3.7.** An IF*k*-PHG,  $\mathcal{H}$  is said to be *totally irregular* if there exists a vertex which is adjacent to the vertices with distinct  $td_{\mathcal{H}}(v)$ , i.e., total degree.

**Definition 3.8.** If the total degree, i.e.,  $td_{\mathcal{H}}(v)$  of every pair of adjacent vertices of  $\mathcal{H}$  are distinct then  $\mathcal{H} = (V, E, \psi)$  is said to be *totally neighborly irregular*.

**Example 6.** Assume an IFk-PHG,  $\mathcal{H}$  as the one shown below.



Fig. 2. Totally neighborly irregular  $\mathcal{H}$ 

Here  $v_1$  adjacent to  $v_5$  and  $v_9$  have distinct  $td_{\mathcal{H}}(v)$ , since  $td_{\mathcal{H}}(v_1) = \langle 0.5, 1.0 \rangle$ ,  $td_{\mathcal{H}}(v_5) = \langle 0.8, 0.8 \rangle$ ,  $td_{\mathcal{H}}(v_9) = \langle 0.4, 1.2 \rangle$ .

**Theorem 3.4.** The sufficient condition for a regular and totally regular IFk-PHG to be perfectly regular IFk-PHG is that each vertex of  $\psi_i$ , i = 1, 2, ..., k is linked through an hyperedge.

*Proof.* Assume that  $\mathcal{H}$  is both regular and totally regular IFk-PHG and each vertex of  $\psi_i$ ,  $i = 1, 2, \ldots, k$  is linked through an hyperedge. Since  $\mathcal{H}$  is regular IFk-PHG,  $d_{\mathcal{H}}(v_1) = d_{\mathcal{H}}(v_2) = \ldots d_{\mathcal{H}}(v_r)$  for all vertices  $v_1, v_2, \ldots, v_r \in V$ .

As each vertex of  $\psi_i$ , i = 1, 2, ..., k is linked through the hyperedge, which means that all vertices of  $\psi_i$ , i = 1, 2, ..., k are adjacent.

From the above results we have  $d_{\mathcal{H}}(a) = d_{\mathcal{H}}(b) \forall v_1, v_2, \dots, v_r \in V$ . Similarly it can be shown that  $td_{\mathcal{H}}(a) = td_{\mathcal{H}}(b) \forall v_1, v_2, \dots, v_r \in V$ .

Hence, the degree and total degree of all the vertices of  $\psi_i$  are the same. So,  $\mathcal{H}$  is perfectly regular IFk-PHG.

**Theorem 3.5.** The size  $S(\mathcal{H})$  of a r-regular IFk-PHG is  $\frac{tr}{2}$  where t = |V|.

*Proof.* The size of  $\mathcal{H}$  is  $S(\mathcal{H}) = \sum_{v_i, v_j \in V} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j)$ . Since  $\mathcal{H}$  is *r*-regular,  $d_{\mathcal{H}}(v) = r \ \forall \ v \in V$ . We have

 $\sum_{v \in V} d_{\mathcal{H}}(v) = 2 \sum_{v_i, v_j \in V} \left\langle \mu_{k_{ij}}, \nu_{k_{ij}} \right\rangle (v_i, v_j)$  $= 2S(\mathcal{H}).$ 

So,

$$2S(\mathcal{H}) = \sum_{v \in V} d_{\mathcal{H}}(v) = \sum_{v \in V} r = tr$$
(3.1)

Hence,  $S(\mathcal{H}) = \frac{tr}{2}$ .

**Theorem 3.6.** If  $\mathcal{H} = (V, E, \psi)$  is a s-totally regular intuitionistic fuzzy k-partite hypergraph, then  $2S(\mathcal{H}) + O(\mathcal{H}) = ts$ , where t = |V|.

*Proof.* Since  $\mathcal{H}$  is an *s*-totally regular IF*k*-PHG,

$$s = td_{\mathcal{H}}(v) = \langle td_{\mu}(v), td_{\nu}(v) \rangle = \langle d_{\mu}(v) + \mu_{k_i}(v), d_{\nu}(v) + \nu_{k_i}(v) \rangle \, \forall v \in V$$

So

$$\sum_{v \in V} s = \sum_{v \in V} \langle d_{\mu}(v), d_{\nu}(v) \rangle + \sum_{v \in V} \langle \mu_{k_i}(v), \nu_{k_i}(v) \rangle$$
$$= \sum_{v \in V} d_{\mathcal{H}}(v) + \sum_{v \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v)$$

This implies that from equation (3.1) and from Definition 2.5,  $ts = 2S(\mathcal{H}) + O(\mathcal{H})$  which completes the proof of the theorem.

**Corollary 3.1** If  $\mathcal{H}$  is *r*-regular and *s*-totally regular intuitionistic fuzzy *k*-partite hypergraph, then  $O(\mathcal{H}) = t(s - r)$ .

*Proof.* From Theorem 3.5,  $S(\mathcal{H}) = \frac{tr}{2} \Rightarrow 2S(\mathcal{H}) = tr$ . From Theorem 3.6,  $2S(\mathcal{H}) + O(\mathcal{H}) = ts$ . So,

$$O(\mathcal{H}) = ts - 2S(\mathcal{H})$$
  
=  $ts - tr$   
=  $t(s - r)$ .

**Definition 3.9.** An IF*k*-PHG  $\mathcal{H}$  is defined to be (l, m)-uniform if all the *k*-partite hyperedges have same cardinality, i.e.,  $|\operatorname{supp}(\mu_{k_ij}, \nu_{k_ij})| = (l, m)$ .

**Example 7.** Consider  $\mathcal{H}$  with  $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$  with  $E = \{E_1, E_2, E_3\}$  where  $\psi = \{\psi_1, \psi_2, \psi_3\}$ . The following figure represents  $\langle 0.3, 0.5 \rangle$ -uniform IFk-PHG.



Fig. 3. Uniform IFk-PHG

**Remark 2.** An uniform IF*k*-PHG is also called balanced IF*k*-PHG because in both cases, the *k*-partite hyperedges have the same number of vertices.

**Definition 3.10.** *The dual of an IFk-PHG,*  $\mathcal{H} = (V, E, \psi)$ *,*  $V = \{v_1, v_2, ..., v_k\}$  *is an IFk-PHG*  $\mathcal{H}^* = \{\psi, V_1, V_2, ..., V_k\}$  *where* 

- $\psi = \{\xi_1, \xi_2, \dots, \xi_k\}$  is the set of vertices corresponding to  $\psi_1, \psi_2, \dots, \psi_k$ , respectively.
- $\{V_1, V_2, \ldots, V_k\}$  is the set of hyperedges corresponding to  $v_1, v_2, \cdots, v_k$ , respectively.

**Example 8.** Consider an IF*k*-PHG,  $\mathcal{H} = (V, E, \psi)$  such that  $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ ,  $E = \{E_1, E_2, E_3\}, \psi = \{\psi_1, \psi_2, \psi_3, \psi_4\}$  where  $\psi_1 = \{v_1 \langle 0.1, 0.7 \rangle, v_6 \langle 0.8, 0.1 \rangle\}, \psi_2 = \{v_2 \langle 0.2, 0.5 \rangle, v_5 \langle 0.6, 0.2 \rangle\}, \psi_3 = \{v_3 \langle 0.4, 0.3 \rangle, v_7 \langle 0.3, 0.4 \rangle\}, \psi_4 = \{v_4 \langle 0.5, 0.1 \rangle, v_8 \langle 0.2, 0.7 \rangle\}.$ 

The corresponding graph is shown below.



Fig. 4. IFk-PHG

The IFk-PHG can be represented by the following incidence matrix:

	$\psi_1$	$\psi_2$	$\psi_3$	$\psi_4$
$v_1$	$\langle 0.1, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1\rangle$
$v_2$	$\langle 0,1 \rangle$	$\langle 0.2, 0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
$v_3$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4, 0.3 \rangle$	$\langle 0,1 \rangle$
$v_4$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.5, 0.1 \rangle$
$v_5$	$\langle 0,1 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
$v_6$	$\langle 0.8, 0.1 \rangle$	$\langle 0,1  angle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
$v_7$	$\langle 0,1 \rangle$	$\langle 0,1  angle$	$\langle 0.3, 0.4 \rangle$	$\langle 0,1 \rangle$
$v_8$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$

The dual IF*k*-PHG  $\mathcal{H}^*$  is shown in Figure 5.



Fig. 5. Dual IFk-PHG  $\mathcal{H}^*$ 

Its incidence matrix is defined below.

	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$	$V_8$
$\xi_1$	$\langle 0.1, 0.7 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1\rangle$
$\xi_2$	$\langle 0,1  angle$	$\langle 0.2, 0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
$\xi_3$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4, 0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.3, 0.4 \rangle$	$\langle 0,1 \rangle$
$\xi_4$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.5, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.7 \rangle$

### 4 Conclusion

The concepts of IF*k*-PHGs are applied in several areas of computer science and engineering. In this paper, we introduce total degree, regular and totally regular IF*k*-PHG. It has been proved that if  $\mathcal{H}$  is both regular and totally regular IF*k*-PHG then it is a constant function. Also, the order and size of IF*k*-PHG satisfies certain properties. Finally, the concept of dual IF*k*-PHG has been defined with example. The authors plan to work on Robotics as an application part of IF*k*-PHG.

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