Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy level operators related to the degree of uncertainty

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Abstract: Two new intuitionistic fuzzy operators of level type are introduced and some of their properties are studied. They are related to the degree of uncertainty of a given intuitionistic fuzzy set. Geometrical interpretations onto the intuitionistic fuzzy interpretation triangle are provided.
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1 Introduction

In Intuitionistic Fuzzy Sets (IFSs) theory (see, e.g., [1]) there are some level operators, defined over the IFS

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}.$$

Here, we will assume that each IFS with which we will work is defined over the fixed universe E (all used notations are from [1]). The most useful Intuitionistic Fuzzy Level Operators (IFLOs) are:

$$P_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x))\rangle | x \in E\},$$

$$Q_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x))\rangle | x \in E\}.$$

Their properties and geometrical interpretations were studied by Todorova and Vassilev in [6].

Another level operator, N_γ, was recently defined and investigated by Atanassova, [3], later extended to the level operator $N_{\gamma_1}^{\gamma_2}$ for the case of interval-valued fuzzy sets, [4].

All IFLOs change (to some extent) the values of the membership and non-membership functions of the IFSs.

In the present paper, for the first time, we will discuss IFLOs that change the degree of uncertainty, that is defined (see [1]) as:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

As by definition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, then $\pi_A(x) \in [0, 1]$ for each $x \in E$.

Various aspects of uncertainty (also called *hesitation*) have been studied by Vassilev alone or with co-authors. Noteworthy are the research related to isohesitant intuitionistic fuzzy sets, i.e., ones that maintain the same hesitancy distribution, [8], and the research related to intuitionistic fuzzy subsets with diminishing hesitancy values, [7].

2 Intuitionistic fuzzy level operator R_α

Let (see [1])

$$U^* = \{\langle x, 0, 0\rangle \mid x \in E\}.$$

For it, we see that $\pi_{U^*}(x) = 1$ for each $x \in E$.

Now, using the two Intuitionistic Fuzzy Topological Operators (IFTO)

$$\begin{aligned} \mathcal{C}(A) &= \{\langle x, \sup_{y\in E} \mu_A(y), \inf_{y\in E} \nu_A(y)\rangle \mid x \in E\}; \\ \mathcal{I}(A) &= \{\langle x, \inf_{y\in E} \mu_A(y), \sup_{y\in E} \nu_A(y)\rangle \mid x \in E\}, \end{aligned}$$

we can see that for each $x \in E$ in the first case: $\pi_A(x) \in [0, \max(0, 1 - \sup_{y\in E} \mu_A(y)) - \inf_{y\in E} \nu_A(y)]$, in the second case: $\pi_A(x) \in [0, \max(0, 1 - \inf_{y\in E} \mu_A(y)) - \sup_{y\in E} \nu_A(y)]$, and in general case: $\pi_A(x) \in [0, \max(0, 1 - \sup_{y\in E} \mu_A(y)) - \sup_{y\in E} \nu_A(y)]$.

In general, in each one of these three cases, $\pi_A(x)$ can be a member of a larger interval, that included the above mentioned intervals.

Let the IFSs used below be different than U^*. For each IFS A and for each $\alpha \in [0, 1]$ we define:

$$R_\alpha(A) = \{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x)\rangle | x \in E\}.$$

Theorem 1. *The set R_α is an IFS.*

Proof. First, we see that for each $\alpha \in [0, 1]$, because $\pi_A(x) > 0$:

$$\frac{1-\alpha}{1-\pi_A(x)}\mu_A(x) \leq \frac{1-\alpha}{1-\pi_A(x)}(\mu_A(x) + \nu_A(x)) = \frac{1-\alpha}{1-\pi_A(x)}(1 - \pi_A(x)) = 1 - \alpha \leq 1.$$

The check of the non-membership degree is similar.

Second, for operator R_α we see that

$$0 \leq \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x) + \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x) = \frac{1-\alpha}{1-\pi_A(x)}(\mu_A(x)+\nu_A(x)) \leq 1-\alpha \leq 1.$$

Therefore, the definition of the new operator is correct and it is an IFS. This completes the proof. $\square$

Let $R_\alpha(A)(x)$ denote the result of applying of R_α over element $\langle x, \mu_A(x), \nu_A(x)\rangle$.

Obviously, when $\alpha = \pi_A(x)$:

$$R_\alpha(A)(x) = \langle x, \mu_A(x), \nu_A(x)\rangle,$$

i.e., in a result of applying R_α over element $x \in E$ does not change its degrees.

The geometrical interpretations of the new operator are shown on Figures 1 and 2.

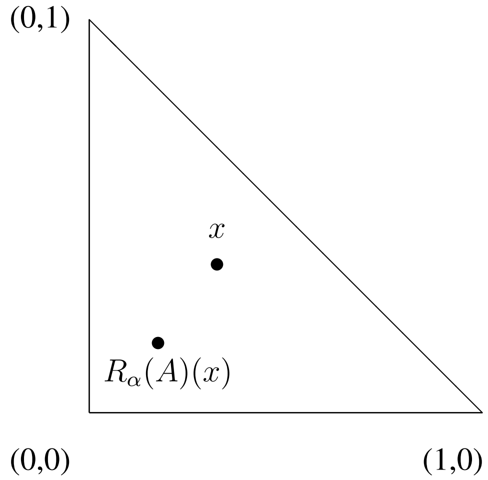

Figure 1. A geometrical interpretation of the operator $R_\alpha(A)$ when $\alpha > \pi_A(x)$.

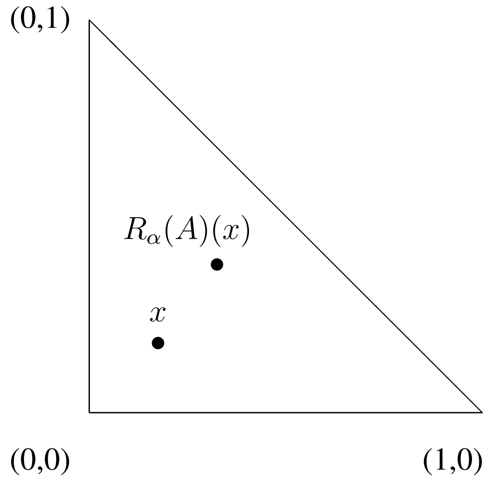

Figure 2. A geometrical interpretation of the operator $R_\alpha(A)$ when $\alpha < \pi_A(x)$.

Having in mind that for evary two IFSs A and B (see [1]):

$$\begin{array}{lll} A \subseteq B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \;\&\; \nu_A(x) \geq \nu_B(x)); \\ A \supseteq B & \text{iff} & B \subseteq A; \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \;\&\; \nu_A(x) = \nu_B(x)); \\ \neg A & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}; \end{array}$$

we prove the following assertions.

Theorem 2. *For each IFS $A \neq U^*$ and for each $\alpha \in [0,1]$:*

$$\neg R_\alpha(\neg A) = R_\alpha(A).$$

Proof. Let $\alpha \in [0,1]$ and the IFS A be given. The check of the equality is the following:

$$\begin{aligned} \neg R_\alpha(\neg A) &= \neg R_\alpha(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}) \\ &= \neg\{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x)\rangle | x \in E\} \\ &= \{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x)\rangle | x \in E\} \\ &= R_\alpha(A). \end{aligned}$$

This completes the proof. □

Theorem 3. *For each IFS $A \neq U^*$ and for every $\alpha, \beta \in [0,1]$:*

$$R_\alpha(R_\beta(A)) = R_\alpha(A).$$

Proof. Let $\alpha, \beta \in [0,1]$ and the IFS A be given. The check of the equality is the following:

$$\begin{aligned} R_\alpha(R_\beta(A)) &= R_\alpha\left(\left\{\left\langle x, \frac{1-\beta}{1-\pi_A(x)}\mu_A(x), \frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\right\rangle | x \in E\right\}\right) \\ &= \left\{\left\langle x, \frac{1-\alpha}{1-\pi_{R_\beta(A)}}\frac{1-\beta}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_{R_\beta(A)}}\frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\right\rangle | x \in E\right\} \\ &= \left\{\left\langle x, \frac{1-\alpha}{1-\left(1-\frac{1-\beta}{1-\pi_A(x)}\mu_A(x)+\frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\right)}\frac{1-\beta}{1-\pi_A(x)}\mu_A(x),\right.\right. \\ &\qquad \left.\left.\frac{1-\alpha}{1-\left(1-\frac{1-\beta}{1-\pi_A(x)}\mu_A(x)+\frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\right)}\frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\right\rangle | x \in E\right\} \end{aligned}$$

$$= \left\{ \left\langle x, \frac{1-\alpha}{1-\left(1-\dfrac{(1-\beta)(\mu_A(x)+\nu_A(x))}{1-\pi_A(x)}\right)} \frac{1-\beta}{1-\pi_A(x)} \mu_A(x), \right.\right.$$

$$\left.\left. \frac{1-\alpha}{1-\left(1-\dfrac{(1-\beta)(\mu_A(x)+\nu_A(x))}{1-\pi_A(x)}\right)} \frac{1-\beta}{1-\pi_A(x)} \nu_A(x) \right\rangle |x \in E \right\}$$

$$= \left\{ \left\langle x, \frac{1-\alpha}{\dfrac{(1-\beta)(\mu_A(x)+\nu_A(x))}{1-\pi_A(x)}} \frac{1-\beta}{1-\pi_A(x)} \mu_A(x), \right.\right.$$

$$\left.\left. \frac{1-\alpha}{\dfrac{(1-\beta)(\mu_A(x)+\nu_A(x))}{1-\pi_A(x)}} \frac{1-\beta}{1-\pi_A(x)} \nu_A(x) \right\rangle |x \in E \right\}$$

$$= \left\{ \left\langle x, \frac{1-\alpha}{1-\beta} \frac{1-\beta}{1-\pi_A(x)} \mu_A(x), \frac{1-\alpha}{1-\beta} \frac{1-\beta}{1-\pi_A(x)} \nu_A(x) \right\rangle |x \in E \right\}$$

$$= \left\{ \left\langle x, \frac{1-\alpha}{1-\pi_A(x)} \mu_A(x), \frac{1-\alpha}{1-\pi_A(x)} \nu_A(x) \right\rangle |x \in E \right\}$$

$$= R_\alpha(A).$$

This completes the proof. $\square$

The simplest IF modal operators are (see [1]):

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle \mid x \in E\},$$
$$\Diamond A = \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle \mid x \in E\}.$$

For them, the following assertions are valid.

Theorem 4. *For each IFS $A \neq U^*$ and for each $\alpha \in [0,1]$:*

(a) $R_\alpha(\Box A) \subseteq \Box R_\alpha(A)$,

(b) $\Diamond R_\alpha(A) \subseteq R_\alpha(\Diamond A)$.

Proof. First, we will prove that for every $a, b, c \in [0,1]$ and $c < 1$:

$$(1-a)(1-b) - 1 + \frac{1-a}{1-c}\, b \geq 0. \tag{1}$$

Let

$$X \equiv (1-a)(1-b) - 1 + \frac{1-a}{1-c}\, b.$$

Then

$$\begin{aligned} X &= 1 - a - b + ab - 1 + \frac{1-a}{1-c}\, b \\ &= -a - b + ab + \frac{1-a}{1-c}\, b \\ &= \frac{1}{1-c}(ab - a - b - abc + ac + bc + b - ab) \\ &= \frac{1}{1-c}(ac - a - abc + bc). \end{aligned}$$

Therefore, $X \geq 0$ if and only if

$$ac - a - abc + bc \geq 0,$$

i.e., when

$$c \geq \frac{a}{a+b-ab} \geq a.$$

Let $\alpha \in [0,1]$. Therefore, for each $x \in E$:

$$\alpha \leq \pi_A(x).$$

Now, using (1) we see that:

$$\begin{aligned} R_\alpha(\Box A) &= R_\alpha(\{\langle x, \mu_A(x), 1-\mu_A(x)\rangle | x \in E\}) \\ &= \{\langle x, \frac{1-\alpha}{1-(1-\mu_A(x)-(1-\mu_A(x)))}\mu_A(x), \\ &\qquad \frac{1-\alpha}{1-(1-\mu_A(x)-(1-\mu_A(x)))}(1-\mu_A(x))\rangle | x \in E\} \\ &= \{\langle x, (1-\alpha)\mu_A(x), (1-\alpha)(1-\mu_A(x)\rangle | x \in E\} \\ &\subseteq \{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), 1 - \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x)\rangle | x \in E\}, \\ &= \Box\{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x)\rangle | x \in E\} \\ &= \Box R_\alpha(A). \end{aligned}$$

This completes the proof. □

Theorem 5. *For each IFS $A \neq U^*$ and for each $\alpha \in [0,1]$:*

(a) $\mathcal{C}(R_\alpha(A)) \subseteq R_\alpha(\mathcal{C}(A))$,

(b) $\mathcal{I}(R_\alpha(A)) \supseteq R_\alpha(\mathcal{I}(A))$.

Proof. For (a), we calculate:

$$\begin{aligned} \mathcal{C}(R_\alpha(A)) &= \mathcal{C}\left(\left\{\left\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x)\right\rangle | x \in E\right\}\right) \\ &= \left\{\left\langle x, \sup_{y\in E}\left(\frac{1-\alpha}{1-\pi_A(y)}\mu_A(y)\right), \inf_{y \in E}\left(\frac{1-\alpha}{1-\pi_A(y)}\nu_A(y)\right)\right\rangle | x \in E\right\} \\ &= \left\{\left\langle x, (1-\alpha)\sup_{y\in E}\frac{\mu_A(y)}{\mu_A(y)+\nu_A(y)}, (1-\alpha)\inf_{y\in E}\frac{\nu_A(y)}{\mu_A(y)+\nu_A(y)}\right\rangle | x \in E\right\}, \end{aligned}$$

$$
\begin{aligned}
R_\alpha(\mathcal{C}(A)) &= R_\alpha\left(\left\{\left\langle x, \sup_{y\in E}\mu_A(y), \inf_{y\in E}\nu_A(y)\right\rangle | x\in E\right\}\right)\\
&= \left\{\left\langle x, \frac{1-\alpha}{1-\pi_{\mathcal{C}(A)}}\sup_{y\in E}\mu_A(y), \frac{1-\alpha}{1-\pi_{\mathcal{C}(A)}}\inf_{y\in E}\nu_A(y)\right\rangle | x\in E\right\}\\
&= \left\{\left\langle x, \frac{1-\alpha}{\sup\limits_{y\in E}\mu_A(y)+\inf\limits_{y\in E}\nu_A(y)}\sup_{y\in E}\mu_A(y),\right.\right.\\
&\qquad \left.\left.\frac{1-\alpha}{\sup\limits_{y\in E}\mu_A(y)+\inf\limits_{y\in E}\nu_A(y)}\inf_{y\in E}\nu_A(y)\right\rangle | x\in E\right\}.
\end{aligned}
$$

Now, we must prove that the membership degree of $\mathcal{C}(R_\alpha(A))$ is higher than this of $R_\alpha(\mathcal{C}A)$ and that the non-membership degree of $R_\alpha(\mathcal{C}A)$ is higher than this of $\mathcal{C}(R_\alpha(A))$.

First, we must mention that for every three real numbers $a, b, c > 0$, from $a \geq b$ it follows that

$$\frac{a}{a+c} \geq \frac{b}{b+c}, \tag{2}$$

because $ab + ac \geq ab + bc$.

Let

$$m = \sup_{y\in E}\mu_A(x),$$
$$n = \inf_{y\in E}\mu_A(x),$$

and let

$$\sup_{y\in E}\frac{\mu_A(y)}{\mu_A(y)+\nu_A(y)} = \frac{p}{p+q}.$$

Let

$$X \equiv \frac{\sup\limits_{y\in E}\mu_A(y)}{\sup\limits_{y\in E}\mu_A(y)+\inf\limits_{y\in E}\nu_A(y)} - \sup_{y\in E}\frac{\mu_A(y)}{\mu_A(y)+\nu_A(y)}.$$

Then, having in mind that $n \leq q$ and (2), we obtain

$$
\begin{aligned}
X &= \frac{m}{m+n} - \frac{p}{p+q}\\
&\geq \frac{m}{m+n} - \frac{p}{p+n} \geq 0.
\end{aligned}
$$

Let

$$Y \equiv \inf_{y\in E}\frac{\nu_A(y)}{\mu_A(y)+\nu_A(y)} - \frac{\inf\limits_{y\in E}\nu_A(y)}{\sup\limits_{y\in E}\mu_A(y)+\inf\limits_{y\in E}\nu_A(y)}$$

and let

$$\inf_{y\in E}\frac{\nu_A(y)}{\mu_A(y)+\nu_A(y)} = \frac{r}{r+s}.$$

Then, having in mind that $m \geq s$ and (2), we obtain

$$
\begin{aligned}
Y &= \frac{r}{r+s} - \frac{n}{m+n}\\
&\geq \frac{r}{r+s} - \frac{n}{s+n} \geq 0.
\end{aligned}
$$

Therefore, (a) is valid. The check of the validity of (b) is similar. This completes the proof. $\square$

3 Intuitionistic fuzzy level operator $R_{\alpha,\beta}$

In this section, we will extend operator R_α adding to it a second argument:

$$R_{\alpha,\beta}(A) = \{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\beta}{1-\pi_A(x)}\nu_A(x)\rangle | x \in E\}.$$

Obviously,

$$R_\alpha(A) = R_{\alpha,\alpha}(A)$$

for each IFS A.

Unfortunately, the new operator loses some of the properties of operator R_α. Yet its geometrical interpretations are similar to these of operator R_α (see Figures 1 and 2).

Theorem 6. *The set $R_{\alpha,\beta}$ is an IFS for every $\alpha, \beta \in [0, 1]$.*

Proof. The check that $\frac{1-\alpha}{1-\pi_A(x)}\mu_A(x), \frac{1-\beta}{1-\pi_A(x)}\nu_A(x) \in [0, 1]$ is as in the proof of Theorem 1. Therefore

$$0 \leq \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x) + \frac{1-\beta}{1-\pi_A(x)}\nu_A(x).$$

Let

$$X \equiv \frac{1-\alpha}{1-\pi_A(x)}\mu_A(x) + \frac{1-\beta}{1-\pi_A(x)}\nu_A(x).$$

If $\alpha \leq \beta$, then

$$X \leq \frac{1-\alpha}{1-\pi_A(x)}(\mu_A(x) + \nu_A(x)) \leq 1 - \alpha \leq 1.$$

If $\alpha > \beta$, then

$$X \leq \frac{1-\beta}{1-\pi_A(x)}(\mu_A(x) + \nu_A(x)) \leq 1 - \beta \leq 1.$$

Therefore, the definition of the operator $R_{\alpha,\beta}$ is correct and it is an IFS. □

Now, Theorem 2 has the following similar, but different form.

Theorem 7. *For each IFS $A \neq U^*$ and for every $\alpha, \beta \in [0, 1]$:*

$$\neg R_{\alpha,\beta}(\neg A) = R_{\beta,\alpha}(A).$$

Proof. Let $\alpha, \beta \in [0, 1]$ and the IFS A be given. The check of the equality is the following:

$$\begin{aligned}
\neg R_{\alpha,\beta}(\neg A) &= \neg R_{\alpha,\beta}(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}) \\
&= \neg\{\langle x, \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x), \frac{1-\beta}{1-\pi_A(x)}\mu_A(x)\rangle | x \in E\} \\
&= \{\langle x, \frac{1-\beta}{1-\pi_A(x)}\mu_A(x), \frac{1-\alpha}{1-\pi_A(x)}\nu_A(x)\rangle | x \in E\} \\
&= R_{\beta,\alpha}(A).
\end{aligned}$$

Theorem 3 is not valid for operators $R_{\alpha,\beta}$ and $R_{\gamma,\delta}$ for $\gamma \neq \delta$. It is valid only in the following form that is proved by analogy with the proof of Theorem 3. □

Theorem 8. *For each IFS $A \neq U^*$ and for every $\alpha, \beta, \gamma \in [0, 1]$:*

$$R_{\alpha,\beta}(R_\gamma(A)) = R_{\alpha,\beta}(A).$$

Theorems 4 and 5 are modified to the following forms.

Theorem 9. *For each IFS $A \neq U^*$ and for every $\alpha, \beta \in [0, 1]$ so that $\alpha + \beta \leq 1$:*

(a) $R_{\alpha,\beta}(\Box A) \subseteq \Box R_{\alpha,\beta}(A)$,

(b) $\Diamond R_{\alpha,\beta}(A) \subseteq R_{\alpha,\beta}(\Diamond A)$.

Theorem 10. *For each IFS $A \neq U^*$ and for every $\alpha, \beta \in [0, 1]$:*

(a) $\mathcal{C}(R_{\alpha,\beta}(A)) \subseteq R_{\alpha,\beta}(\mathcal{C}(A))$,

(b) $\mathcal{I}(R_{\alpha,\beta}(A)) \supseteq R_{\alpha,\beta}(\mathcal{I}(A))$.

4 Conclusion

The two operators, discussed in the present paper, modify the degree of uncertainty of the elements of a given IFS. So, in the IFS theory already there are tools for modifying of each one of the degrees of the IFS elements.

The new operators can be used, e.g., in procedures of decision making in which it is necessary to change degree of uncertainty of some expert's evaluations, when they are wrong. It is important, because in [1] there are procedures for changing only of the two standard degrees (of membership and of non-membership).

Another application of the new operators will be in intercriteria analysis (see, e.g., [2,5], when the degree of uncertainty must be corrected because existing of lacks of evaluations of objects by given criteria. This application will be discussed in near future.

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