Notes on Intuitionistic Fuzzy Sets
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Bondage and non-bondage sets in regular intuitionistic fuzzy graphs

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Abstract: The concept of strong edges in domination set and its properties are discussed. The increasing or reducing domination numbers using cardinality are also studied. Bondage $(\alpha(G))$ and non-bondage $(\alpha_K(G))$ sets are defined in regular intuitionistic fuzzy graph. The properties of bondage and non-bondage number of intuitionistic fuzzy graph analyzed. A minimum 2-bondage set X of an intuitionistic fuzzy graph (IFG) G is a bondage set of regular intuitionistic fuzzy graph in G.
Keywords: Intuitionistic fuzzy graph (IFG), Regular IFG, Strong edge, Bondage $(\alpha(G))$, Non-bondage $(\alpha_K(G))$, Domination numbers, Cardinality of domination sets.
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1 Introduction

Euler first introduced the concept of graph theory in the year 1736. Cockayne and Hedetniemi [5] introduced the domination number and the independent domination number of graphs but Ore and Berge [4] introduced the concept of dominating sets in graphs. Fink, Jacobson, Kinch and Roberts [6] introduced the concept of the bondage number in graphs in 1990. Krzywkowski [9] discussed the concept of 2-bondage number in graph theory in 2012.

Zadeh [11] introduced the concept of fuzzy relation in his classical paper in 1965. Rosenfeld [10] introduced the notion of fuzzy graph and several fuzzy examples of graph theoretic concepts such as paths, cycles and connectedness. Atanassov [1,3] introduced intuitionistic fuzzy sets.

Karunambigai and Parvathi [7] introduced intuitionistic fuzzy graph as a special case of Shannon and Atanassov IFG [2]. These concepts have been applied to find the shortest path in networks using dynamic programming problem approach of strong edge. Intuitionistic fuzzy graph is an extension of traditional graphs where each vertex and edge have associated membership and non-membership in G. Constant intuitionistic fuzzy graph introduced by Karunambigai, Parvathi and Buvaneswari [8]. In Fuzzy Graph Theory, a dominating set of a fuzzy graph $G : (v, \mu, \nu)$ is a set D of edges of G such that every edge in $V - D$ has at least one strong neighbor in D. If an edge (v_i, v_j) is a strong edge, then v_i dominates v_j. The vertex domination number of G is the minimum number of vertex in D also defined.

Using the weight of strong edges in bondage and non-bondage set, in this study the vertex dominance number of an intuitionistic fuzzy graph is determined.

2 Preliminaries

In this section, some basic definitions and theorems which used in constructing the properties relating to intuitionistic fuzzy graph are given.

Definition 2.1. *Minmax intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_i : V \to [0, 1]$ and $\nu_i : V \to [0, 1]$ denotes the degrees of membership and degrees of non-membership of the vertex $v_i \in V$, respectively, and $0 \leq \mu_i + \nu_i \leq 1$, for every $v_i \in V (i = 1, 2, \ldots, n)$. $E \subset V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $\nu_{ij} : V \times V \to [0, 1]$ the degrees of membership and degree of non-membership of an edge such that*

$$\mu_{ij} \leq \min\{\mu_i, \mu_j\}$$

$$\nu_{ij} \leq \max\{\nu_i, \nu_j\}$$

$$0 \leq \mu_{ij} + \nu_{ij} \leq 1, \text{ for all } (v_i, v_j) \in E.$$

Note 1. Here the triple (v_i, μ_i, ν_i) denotes the degree of membership and degree of non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{ij}, \nu_{ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

Definition 2.2. *An IFG $G = (V, E)$ is said to be a semi-μ strong intuitionistic fuzzy graph if $\mu_{ij} = \min(\mu_i, \mu_j)$ for every i and j.*

Definition 2.3. *An IFG $G = (V, E)$ is said to be a semi-ν strong intuitionistic fuzzy graph if $\nu_{ij} = \max(\nu_i, \nu_j)$ for every i and j.*

Definition 2.4. *An IFG $G = (V, E)$ is said to be a strong intuitionistic fuzzy graph if $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\nu_{ij} = \max(\nu_i, \nu_j)$ for all i and j.*

Definition 2.5. *The μ-strength of a path $P = \{v_1, v_2, \ldots, v_n\}$ is defined as $\min\{\mu_{ij}\}$ for all $\{i, j = 1, 2, \ldots, n\}$ and it is denoted by S_μ.*

Definition 2.6. *The ν-strength of a path $P = \{v_1, v_2, \ldots, v_n\}$ is defined as $\max\{\nu_{ij}\}$ for all $\{i, j = 1, 2, \ldots, n\}$ and it is denoted by S_ν.*

Definition 2.7. *In an intuitionistic fuzzy graph, the μ-strength of connectedness between two vertices v_i and v_j is $\text{CON}_{\mu(G)}(v_i, v_j) = \max\{S_\mu\}$ and the ν-strength of connectedness between two vertices v_i and v_j is $\text{CON}_{\nu(G)}(v_i, v_j) = \min\{S_\nu\}$, for all v_i and v_j.*

Definition 2.8. *An edge (v_i, v_j) is said to be a strong edge if $\mu_{ij} \geq \text{CON}_{\mu(G)}(v_i, v_j)$ and $\nu_{ij} \leq \text{CON}_{\nu(G)}(v_i, v_j)$ for every $v_i, v_j \in V$.*

Definition 2.9. *An edge (v_i, v_j) is said to be a weak edge if $\mu_{ij} < \text{CON}_{\mu(G)}(v_i, v_j)$ and $\nu_{ij} > \text{CON}_{\nu(G)}(v_i, v_j)$ for every $v_i, v_j \in V$.*

Definition 2.10. *Let G be an IFG. If there exists a set $H \subseteq S$ such that $\eta(G)$, then H is called bondage set of G, where S is the set of all strong edges in G. The bondage set of G is denoted by $\alpha(G)$.*

Note 2. The $\eta(G)$ of an IFG G is the minimum cardinality among all bondage sets of G.

Definition 2.11. *The set of strong edges in $H \subseteq S$ is called a non-bondage set if $\eta(G - H) \leq \eta(G)$, where S is the set of all strong edges in G. Non-Bondage set of G is denoted by $\alpha_K(G)$.*

Definition 2.12. *Let $G = (V, E)$ be an IFG. If $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all v_i, v_j, then IFG G is called as (k_i, k_j)-regular intuitionistic fuzzy graph.*

Definition 2.13. *Let $G = (V, E)$ be an IFG. The degree of vertex is defined as*

$$d(v_i) = \left[\sum_{v_i, v_j \in E} \mu_{ij}, \sum_{v_i, v_j \in E} \nu_{ij} \right]$$

and $\mu_{ij} = \nu_{ij} = 0$ for $(v_i, v_j) \notin E$.

Definition 2.14. *Let $G = (V, E)$ be an IFG. Then the cardinality of G is defined as*

$$|\eta(G)| = \left| \sum_{v_i \in V} \frac{1 + \mu_i - \nu_i}{2} \right| \text{ where } \nu_i, i = 1, 2, \ldots, n$$

3 Properties of the bondage and the non-bondage set in intuitionistic fuzzy graphs

Theorem 3.1. *If an IFG G has an isolated edge, then $\alpha(G) = 1$.*

Proof. Let G be an IFG with an isolated edge.

Suppose that u and v are the terminating vertices of the isolated edge (v_i, v_j). Obviously, (v_i, v_j) is a strong edge and either v_i or v_j belongs to the minimum dominating set of G, but not both. Thus, removing e_{ij} results in v_i and v_j as isolated vertices. Therefore, both v_i and v_j are considered to belong to each dominating set of $(G - e_{ij})$. Subsequently, $\eta(G - e_{ij}) > \eta(G)$ and e_{ij} is a Bondage Set of G. Hence $\alpha(G) = 1$. □

Definition 3.1. *A set $X \subseteq S$ is said to be a 2-bondage set of the IFG $\eta_2(G - X) > \eta_2(G)$, where S is the set of all strong edges in G.*

Theorem 3.2. *A minimum 2-bondage set X of an intuitionistic fuzzy graph G is a bondage set of G if $\eta_2(G - e_{ij}) = \eta(G - e_{ij})$.*

Proof. By Definition 3.1, Let G be an IFG and X be the minimum 2-bondage set of G with

$$\eta_2(G - e_{ij}) = \eta(G - e_{ij}).$$
$$\eta_2(G) \geq \eta(G)$$
$$\eta_2(G - e_{ij}) = \eta(G - e_{ij}).$$

Since X is a minimum 2-bondage set of G, then we have $\eta_2(G - e_{ij}) > \eta(G)$.

Thus

$$\eta_2(G - e_{ij}) > \eta_2(G) \geq \eta(G).$$
$$\eta_2(G - e_{ij}) > \eta(G).$$

Therefore X is a bondage set of G. Hence the proof. □

4 Bondage and non-bondage sets in regular intuitionistic fuzzy graphs

In this section, a procedure to identify the intuitionistic fuzzy bondage and non-bondage sets in regular IFG $G = (V, E)$ is given. The proposed algorithm identifies the IF bondage and non-bondage sets in IFG. The steps involved in the algorithm are as follows:

- Step (i): Let $G = (V, E)$ be a regular intuitionistic fuzzy graph.
- Step (ii): Finding dominating set in G.

  By the definition, a dominating set of a fuzzy graph $G : (v, \mu, \nu)$ is a set D of edges of G such that every edge in $V - D$ has at least one strong neighbor in D.

- Step (iii): Calculation of strength of connectedness for all the edges in G.

  By the definition, the μ_{ij}-strength of the connectedness and the μ_{ij}-strength of the connected are $\text{CON}_{\mu(G)}(v_i, v_j) = \max\{\mu_{ij}\}$, $\text{CON}_{\nu(G)}(v_i, v_j) = \min\{\nu_{ij}\}$, respectively including all paths of v_i, v_j.

- Step (iv): Determine set of strong edges satisfying the following conditions.

$$\mu_{ij}(G) \geq \text{CON}_{\mu}(G)(v_i, v_j) \text{ and } \nu_{ij}(G) \leq \text{CON}_{\nu}(G)(v_i, v_j).$$

- Step (v): Calculating of bondage and non-bondage by satisfying condition for $\eta(G - H) > \eta(G)$. Let $G = (V, E)$ be an IFG. Then the cardinality of G is defined be

$$|\eta(G)| = \left| \sum_{v_i \in V} \frac{1 + \mu_i - \nu_i}{2} \right|, \quad \text{where } i = 1, 2, \ldots, n$$

- Step (vi): Repeat the steps to calculate bondage and non-bondage sets by the formula from Step (iv).

Numerical Example 4.1. Consider a regular intuitionistic fuzzy graph G as the one shown in Figure 4.1 to find the bondage set and non-bondage set.

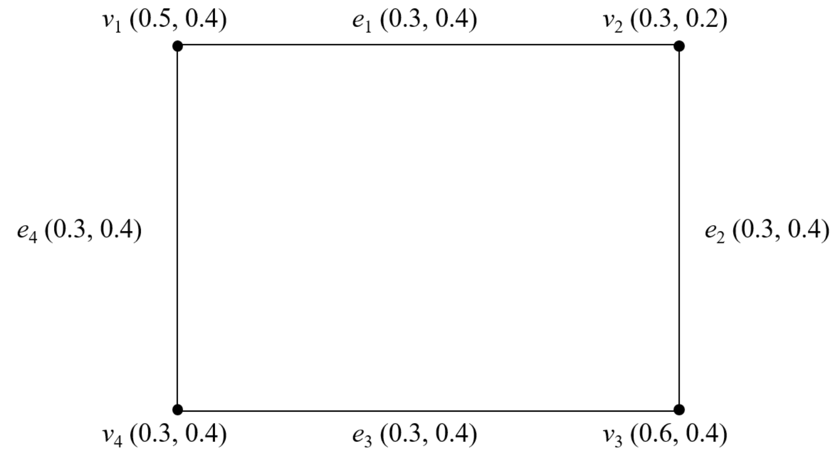

Figure 1. Regular intuitionistic fuzzy graph G

- Step(i): Let $G = (V, E)$ be a regular intuitionistic fuzzy graph as in Figure 4.1.
- Step (ii): By definition of dominating edges all $\{e_i\}$ are dominating edges.
- Step (iii): Calculating of strength of connectedness for all the edges $\{e_i\}$ in IFG.

$$\text{CON}_{\nu}(G)(\nu_1, \nu_2) = \text{CON}_{\nu}(G)(e_1) = (0.3 \vee\ 0.3, 0.4 \wedge 0.4) = (0.3, 0.4),$$
$$\text{CON}_{\nu}(G)(\nu_2, \nu_3) = \text{CON}_{\nu}(G)(e_2) = (0.3 \vee\ 0.3, 0.4 \wedge 0.4) = (0.3, 0.4),$$
$$\text{CON}_{\nu}(G)(\nu_3, \nu_4) = \text{CON}_{\nu}(G)(e_3) = (0.3 \vee\ 0.3, 0.4 \wedge 0.4) = (0.3, 0.4),$$
$$\text{CON}_{\nu}(G)(\nu_1, \nu_2) = \text{CON}_{\nu}(G)(e_4) = (0.3 \vee\ 0.3, 0.4 \wedge 0.4) = (0.3, 0.4).$$

- Step (iv): In Figure 4.1, based on its strength of connectedness, all the edges are strong by satisfying the conditions

$$\mu_{12} \geq \text{CON}_{\mu}(G)(v_1, v_2) \text{ and } v_{12} \leq \text{CON}_{\nu}(G)(v_1, v_2)$$
$$\mu_{23} \geq \text{CON}_{\mu}(G)(v_2, v_3) \text{ and } v_{23} \leq \text{CON}_{\nu}(G)(v_2, v_3)$$
$$\mu_{34} \geq \text{CON}_{\mu}(G)(v_3, v_4) \text{ and } v_{34} \leq \text{CON}_{\nu}(G)(v_3, v_4)$$
$$\mu_{41} \geq \text{CON}_{\mu}(G)(v_1, v_2) \text{ and } v_{41} \leq \text{CON}_{\nu}(G)(v_4, v_1)$$

- Step (v): Hence all $\{e_i\}$'s are strong edges in Figure 4.1.

  Computing of Bondage and non-bondage by $\eta(G)$ for all $\{e_i\}$.

  The set of strong edges will be $S = \{e_1, e_2, e_3, e_4\}$. The dominating set of G with the lowest cardinality $\{v_3, v_4\}$. Then,

$$\eta(G) = \frac{1 + 0.6 - 0.4}{2} + \frac{1 + 0.3 - 0.4}{2}$$
$$\eta(G) = 1.05.$$

- Step (vi): Continue the same process to calculate the bondage and non-bondage sets.
  Consider the subset $H = \{e_2\}$ of the set of strong edge.
  If the dominating set of $G - \{e_2\}$ with the lowest cardinality is $\{v_1, v_4\}$, then its $\eta(G)$ will be

$$\eta(G) - \{e_2\} = \frac{1 + 0.5 - 0.4}{2} + \frac{1 + 0.3 - 0.4}{2}$$
$$\eta(G) - \{e_2\} = 1.00 < 1.05.$$

  Here, $H = \{e_2\}$ is a non-bondage set.

$$\eta\ (G\{e_3\}) = 1.1 > 1.05.$$

  $H = \{e_3\}$ is a bondage set.

$$\eta(G\{e_4\}) = 1.15 > 1.05.$$

  $H = \{e_4\}$ is a bondage set.
  Hence, G has a bondage and non-bondage set in a regular IFG.

5 Conclusion

The concept of regular IFG is very rich in both theoretical developments and applications. In this article, the strong edge domination number of regular intuitionistic fuzzy graphs is obtained based on domination set. The vertices are classified into Bondage $\alpha(G)$ and Non-Bondage $\alpha_k(G)$ using the strength of connectedness. These concepts can be explored in real life applications using intuitionistic fuzzy environment.

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