Notes on Intuitionistic Fuzzy Sets
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On the 3D-figure areas based on Pick's formula

Krassimir Atanassov

Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 105, 1113 Sofia, Bulgaria
e-mail: krat@bas.bg

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Abstract: This work is based on previous research focused on iterative procedures for estimation of the area surrounded by a simple closed curve in the 2D space. Pick's formula is employed for calculating the area surrounded by a special type of polygons. Here, we propose 3D-figure areas based on Pick's formula and give intuitionistic fuzzy and interval-valued intuitionistic fuzzy estimations of these areas.
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1 Introduction

In [3–5], we proposed intuitionistic fuzzy and interval-valued intuitionistic fuzzy estimations of the planar figure's area surrounded by a continuous simple closed curve, where by a "simple curve" we mean one that has no self-intersections and if, moreover, it is continuous, this implies that its interior is a simply connected domain (cf. Munkres [6, Ch. 9]).

For example, if we have an elevated landform as the one illustrated on Fig. 1, we can plot its contour lines (see Fig. 2), i.e., the lines connecting points of equal elevation (height) above a given level. If we do not take into account these lines and estimate the area of the figure using the map (when the figure is planar), its estimated area will be less than the real area.

In the paper, we will use the following two concepts of the theory of intuitionistic fuzziness.

The pair $x = \langle a, b \rangle$ is called an Intuituinistic Fuzzy Pair (IFP, see [2]) if $a, b \in [0, 1]$ and $a + b \leq 1$.

By analogy, the pair $x = \langle A, B \rangle$ is called an Interval-Valued IFP (IVIFP, see [1]) if $A, B \subseteq [0, 1]$ and $\sup A + \sup B \leq 1$.

In [5] we introduced a formula of the intuitionistic fuzzy estimation of a simple closed curve surrounded by two polygons (an inner and an outer one) with sides parallel to the coordinate axes.

Figure 1. A hill (with a simpler profile form)

In [3,4] we extended the IFS formula from [5] for the area surrounding the closed curve through the introduction of new types of polygons that can better approximate the curve in the same grid step size. In that second type of polygons, the Pick's formula (see [7]) for the computation of the surrounded by the polygons' area, has been employed in the computation of an estimation of the closed simple curve's interior. *Pick's theorem* provides a simple formula for calculating the area S surrounded by a polygon in terms of the number I of grid-points in the interior of the polygon, i.e. not touching any of the sides, and the number B of grid-points on the boundary, i.e. placed on the polygon's perimeter. Assuming that we have a grid with grid-step equal to one, Pick's formula provides the number of unit squares through the following expression:

$$S = I + \frac{B}{2} - 1 \tag{1}$$

An approach to the estimation of the area surrounded by a closed curve line through estimation of the area of two polygons, one of which is inscribed in the curve, while the other one is circumscribed. The area of the polygon is estimated using the Pick's formula.

This work actually can be considered as a next step of our previous research [3–5]. Here, we will discuss the case, when the area is 3-dimensional (3D), but it is surrounded by a *continuous simple closed* curve, i.e., it is in the real 3D space.

Here, we propose an approximate way to estimate the area of a 3D surface closed by the curve.

2 Main results

Let the distance between two neighbouring isohypses be h and let the map onto which the hill is projected be covered with a square grid of unit length (grid-step) equal to a.

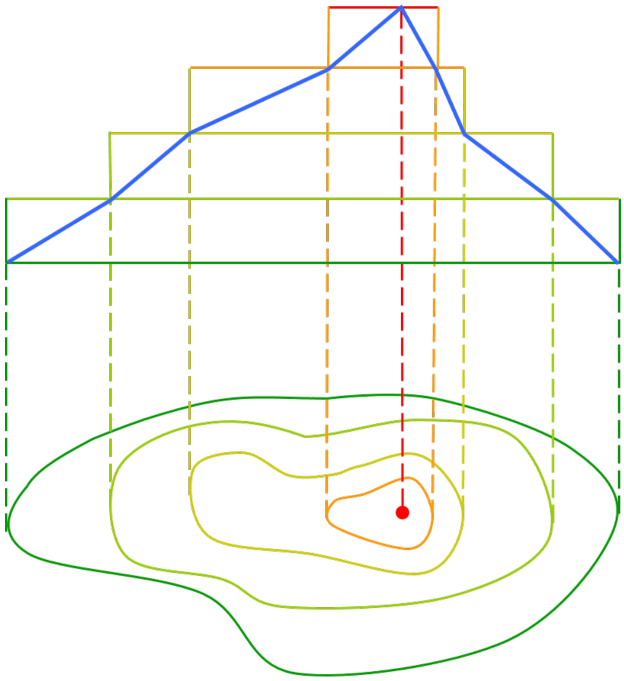

Figure 2. The landform and its contour lines

Therefore, the formula (1) is modified to the form:

$$S(a) = a^2\left(I + \frac{B}{2} - 1\right) \tag{2}$$

for the area surrounded by a given polygon.

We will show how to estimate the area of one of the layers from Fig. 2. We will use as an example the closed simple curve from Fig. 3 that corresponds to one of the intermediate layers.

Let us denote for Fig. 3:

- F_1 – the red polygon that is inner for the curve F (in black),
- F_2 – the green polygon that is inner for the curve F,
- F – the given curve (coloured in black),
- F_3 – the light green polygon that is outer for the curve F,
- F_4 – the light red polygon that is outer for the curve F,
- F_5 – the blue rectangle that is outer for the curve F.

Let the horizontal area of the arbitrary figure G be denoted by $S(G)$. Obviously, for figures F_1 and F_4, their areas $S(F_1)$ and $S(F_2)$ coincide with the numbers of the unit squares in them, multiplied by a^2, while for figures F_2 and F_3, we can use Pick's formula (2).

Let the perimeter of figure G be denoted by $P(G)$.

We must have in mind that in reality, the area of each next layer (going up) is included in the area of its previous layer. So, for the perimeters of the figures X and Y lying in the i-th and j-th

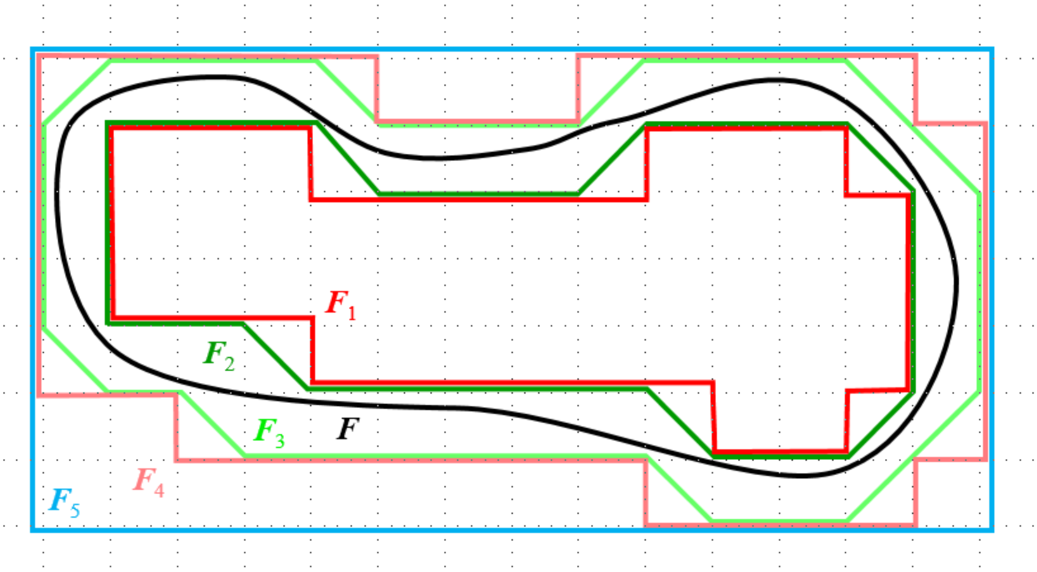

Figure 3. Simple curve F in a grid (coloured in black). We have five polygons approximating that curve: two polygons (in red, F_1, and light red, F_4) whose segments are parallel to the coordinate axes, and two polygons (in green, F_2, and light green, F_3) whose segments are parallel to the coordinate axes or to the unit square diagnoals.

layers, respectively, for $i < j$ it must be valid that $P(X) \geq P(Y)$. Therefore, the opposite case must be omitted, in spite that it can exists.

For the above example from Fig 3., G_i coincides with F_i for $i = 1, 2, 3, 4, 5$.

Let us denote the i-th layer of the arbitrary figure G by G^i. As we mentioned above, in this layer there are four polygons related to G^i that we will denote by G^i_j for $j = 1, 2, 3, 4, 5$.

Let the given 3D-figure G has n layers. Therefore, for its i-th layer ($1 \leq i \leq n$), we can calculate the areas of its inner and outer polygons as follows:

$$A(G^i_j) = S(G^i_j)a^2 + P(G^i_j)ah.$$

Now, we see that for the whole 3D-figure G there are five polygons G_1, G_2, G_3, G_4, G_5 – two inner and three outer polygons, and for their 3D-areas we obtain:

$$A(G_j) = S(G_j)a^2 + ah\sum_{i=1}^{n} P(G^i_j).$$

As we mentioned above, we assume that for every $1 \leq i < j \leq 5$:

$$A(G_i) \leq A(G_j). \tag{3}$$

Hence, for the 3D-area of figure G we can use one of the three following formulas, based on function A:

$$\mathcal{A}_1(G) = \frac{1}{2}(A(G_1) + A(G_4)),$$

$$\mathcal{A}_2(G) = \frac{1}{2}(A(G_2) + A(G_3)),$$

$$\mathcal{A}_3(G) = \frac{1}{4}\sum_{j=1}^{4} A(G_j).$$

On the other hand, the formulas for $A(G_1), \dots, A(G_5)$ can be used for a basis of a given intuitionistic fuzzy evaluations for the area of G.

If we would like to obtain the evaluation of the 3D-area of figure G in the form of an IFP, then we can define it, e.g., in one of the following two ways:

$$\left\langle \frac{A(G_1}{A(G_5)}, \frac{A(G_5) - A_4(G)}{A(G_5)} \right\rangle \text{ more rough IFP evaluation,} \tag{4}$$

$$\left\langle \frac{A(G_2)}{A(G_4)}, \frac{A(G_4) - A(G_3)}{A(G_4)} \right\rangle \text{ more precise IFP evaluation,} \tag{5}$$

where $A(G_5)$ is the area of the minimal parallelepiped that contains figure G.

If we would like to obtain the evaluation of the 3D-area of figure G to have the form of an IVIFP, then we can define it, e.g., in one of the following two ways:

$$\left\langle \left[\frac{A(G_1)}{A(G_5)}, \frac{A(G_2)}{A(G_5)}\right], \left[\frac{A(G_5) - A(G_4)}{A(G_5)}, \frac{A(G_5) - A(G_3)}{A(G_5)}\right] \right\rangle \text{ more rough IVIFP evaluation,} \tag{6}$$

$$\left\langle \left[\frac{A(G_1)}{A(G_4)}, \frac{A(G_2)}{A(G_4)}\right], \left[0, \frac{A(G_4) - A(G_3)}{A(G_4)}\right] \right\rangle \text{ more precise IVIFP evaluation.} \tag{7}$$

Now, we can check that the evaluations (4) and (5) are IFPs, and (6) and (7) are IVIFPs. Really, from the above restriction (3) it follows that

$$A(G_1) \le A(G_2) \le A(G_3) \le A(G_4) \le A(G_5).$$

Hence, $\frac{A(G_1)}{A(G_5)}, \frac{A(G_5) - A(G_4)}{A(G_5)} \in [0, 1]$ and

$$0 \le \frac{A(G_1)}{A(G_5)} + \frac{A(G_5) - A(G_4)}{A(G_5)} = \frac{A(G_5) - A(G_4) + A(G_1)}{A(G_5)} \le 1.$$

Also, $\frac{A(G_2)}{A(G_4)}, \frac{A(G_4) - A(G_3)}{A(G_4)} \in [0, 1]$ and

$$0 \le \frac{A(G_2)}{A(G_4)} + \frac{A(G_4) - A(G_3)}{A(G_4)} = \frac{A(G_4) - A(G_3) + A(G_2)}{A(G_4)} \le \frac{A(G_4)}{A(G_4)} = 1;$$

and $\frac{A(G_1)}{A(G_5)}, \frac{A(G_2)}{A(G_5)}, \frac{A(G_5) - A(G_4)}{A(G_5)}, \frac{A(G_5) - A(G_3)}{A(G_5)} \in [0, 1]$,

$$\frac{A(G_1)}{A(G_5)} \le \frac{A(G_2)}{A(G_5)},$$

$$\frac{A(G_5) - A(G_4)}{A(G_5)} \leq \frac{A(G_5) - A(G_3)}{A(G_5)},$$

and

$$0 \leq \frac{A(G_2)}{A(G_5)} + \frac{A(G_5) - A(G_3)}{A(G_5)} = \frac{A(G_5) - A(G_3) + A(G_2)}{A(G_5)} \leq 1,$$

and $\frac{A(G_1)}{A(G_4)}, \frac{A(G_2)}{A(G_4)}, \frac{A(G_4) - A(G_3)}{A(G_4)} \in [0, 1],$

$$\frac{A(G_1)}{A(G_4)} \leq \frac{A(G_2)}{A(G_5)},$$

and

$$0 \leq \frac{A(G_2)}{A(G_4)} + \frac{A(G_4) - A(G_3)}{A(G_4)} = \frac{A(G_4) - A(G_3) + A(G_2)}{A(G_4)} \leq 1.$$

Example 1 . *For the figure F from Fig. 3 (one layer, i.e., $n = 1$) and for $a = 3e, h = e$, where e is a fixed unit, we obtain the following values:*

$$A(F_1) = 477.00e^2,$$
$$A(F_2) = 493.40e^2,$$
$$A(F_3) = 627.40e^2,$$
$$A(F_4) = 870.00e^2,$$
$$A(F_5) = 1008.00e^2,$$

from where,

$$\mathcal{A}_1(F) = 673.50e^2,$$
$$\mathcal{A}_2(F) = 560.40e^2,$$
$$\mathcal{A}_3(F) = 616.95e^2.$$

This leads to the following evaluations:

- *more rough evaluation in IFP:* $\langle 0.473, 0.137 \rangle$,
- *more precise evaluation in IFP:* $\langle 0.567, 0.279 \rangle$,
- *more rough evaluation in IVIFP:* $\langle [0.473, 0.489], [0.137, 0.378] \rangle$,
- *more precise evaluation in IVIFP:* $\langle [0.548, 0.567], [0.00, 0.279] \rangle$.

We can see that there are relations between formulas (4) and (6), and formulas (5) and (7).

3 Conclusion

The idea for this research came after the author's visited to a upland zone that had been affected by a forest fire. From the local foresters, it became clear that their estimations of the surface affected by the wildfire are made according to a planar topographical map, i.e., the estimated areas are less than the actually affected ones.

For a more abstract purpose, restrictions of the kind $P(X) \geq P(Y)$ and $A(G_2) < A(G_3)$ are not necessary.

It is clear that for the 3D-area much more precise estimations from a mathematical point of view can be made, but the ones proposed here are easier for use by non-specialists.

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