9th Int. Workshop on IFSs, Banská Bystrica, 8 October 2013
Notes on Intuitionistic Fuzzy Sets
Vol. 19, 2013, No. 2, 6–9

Modifications of the weight-center operator, defined over intuitionistic fuzzy sets with a countable universe

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Abstract: K. Atanassov and A. Ban in [2] introduce the operator $W(A)$, defined for IFSs over a finite universe E. In [3, 4, 5] three modifications of the weight-center operator were studied. In the paper, the first modification is considered over a countable universe.
Keywords: Intuitionistic fuzzy sets, First weight-center operator.
AMS Classification: 03E72.

1 Introduction

Intuitionistic Fuzzy Sets (IFSs) A, B in E by [1] are objects of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\}$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in E\}$$

where $\mu_A : E \to \langle 0, 1\rangle$, $\nu_A : E \to \langle 0, 1\rangle$ and for every $x \in E$: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

For every two IFSs A and B a lot of operations, relations and operators are defined in [1], but we need only a few of these:

$$A^{|} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in E\}$$

$$I(A) = \{\langle x, k, l\rangle \mid x \in E\}$$

$$C(A) = \{\langle x, K, L\rangle \mid x \in E\}$$

where

$$k = \inf_{y \in E} \mu_A(y)$$

$$l = \sup_{y \in E} \nu_A(y)$$

$$K = \sup_{y \in E} \mu_A(y)$$

$$L = \inf_{y \in E} \nu_A(y)$$

In [2] was introduced the following operator, defined for IFSs over a finite universe E:

$$W(A) = \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y)}{card(E)}, \frac{\sum_{y \in E} \nu_A(y)}{card(E)} \right\rangle \mid x \in E \right\},$$

where $card(E)$ is the number of the elements of a finite universe E.

Let $B \neq U^*$ where $U^* = \{\langle x, 0, 0 \rangle \mid x \in E\}$. The first modification of weight-center operator over IFSs A and B over the finite universe E is introduced by:

$$W_B^1(A) = \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{card(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{card(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\}$$

We see that $W_B^1(A)$ is an IFS because, for every $x \in E$:

$$0 \leq \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{card(E) \cdot \sum_{y \in E} \mu_B(y)} + \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{card(E) \cdot \sum_{y \in E} \nu_B(y)} \leq$$

$$\leq \frac{\sum_{y \in E} \mu_A(y)}{card(E)} + \frac{\sum_{y \in E} \nu_A(y)}{card(E)} = \frac{\sum_{y \in E} \mu_A(y) + \nu_A(y)}{card(E)} \leq 1$$

In this paper, we study a modification of $W_B^1(A)$ in the case that E is countable.

2 First modification of weight-center operator

Let a set E be countable. Suppose that there exists $c \in R$ such that

$$\sum_{y \in E} \mu_A(y) + \sum_{y \in E} \nu_A(y) < c$$

where $c < \infty$. We define

$$W_B^1(A) = \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{c \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{c \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\}$$

Theorem 1. *For every two IFSs A and $B \neq U^*$ over the countable universe E:*

(a)

$$W_B^1(A^|)^| = W_B^1(A)$$

(b)

$$I(W_B^1(A) = W_B^1(I(A))$$

(c)

$$C(W_B^1(A)) = W_B^1(C(A))$$

Proof. (a)

$$W_B^1(A^|)^| = W_B^1\left(\{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in E\}\right)^| =$$

$$= \left\{\left\langle x, \frac{\sum\limits_{y\in E} \nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}, \frac{\sum\limits_{y\in E} \mu_A(y)\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}\right\rangle \mid x \in E\right\}^| =$$

$$= \left\{\left\langle x, \frac{\sum\limits_{y\in E} \mu_A(y)\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}, \frac{\sum\limits_{y\in E} \nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}\right\rangle \mid x \in E\right\} = W_B^1(A)$$

(b)

$$W_B^1(I(A)) = W_B^1\left(\{\langle x, k, l\rangle \mid x \in E\}\right) =$$

$$= W_B^1\left(\left\{\left\langle x, \inf_{y\in E}\mu_A(y), \sup_{y\in E}\nu_A(y)\right\rangle \mid x \in E\right\}\right) =$$

$$= \left\{\left\langle x, \frac{\sum\limits_{y\in E} \inf\limits_{y\in E}\mu_A(y)\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}, \frac{\sum\limits_{y\in E} \sup\limits_{y\in E}\nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}\right\rangle \mid x \in E\right\} =$$

$$= I\left(\left\{\left\langle x, \frac{\sum\limits_{y\in E} \mu_A(y)\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}, \frac{\sum\limits_{y\in E} \nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}\right\rangle \mid x \in E\right\}\right) = I(W_B^1(A))$$

(c)

$$W_B^1(C(A)) = W_B^1\left(\{\langle x, K, L\rangle \mid x \in E\}\right) =$$

$$= W_B^1\left(\left\{\left\langle x, \sup_{y\in E}\mu_A(y), \inf_{y\in E}\nu_A(y)\right\rangle \mid x \in E\right\}\right) =$$

$$= \left\{\left\langle x, \frac{\sum\limits_{y\in E} \sup\limits_{y\in E}\mu_A(y))\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}, \frac{\sum\limits_{y\in E} \inf\limits_{y\in E}\nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}\right\rangle \mid x \in E\right\} =$$

$$= C\left(\left\{\left\langle x, \frac{\sum\limits_{y\in E} \mu_A(y)\cdot\mu_B(x)}{c\cdot\sum\limits_{y\in E}\mu_B(y)}, \frac{\sum\limits_{y\in E} \nu_A(y)\cdot\nu_B(x)}{c\cdot\sum\limits_{y\in E}\nu_B(y)}\right\rangle \mid x \in E\right\}\right) = C(W_B^1(A))$$

□

Corollary 1. *For a finite universe E, where $E = \{y_1, ..., y_n\}$ put the countable universe F, where $F = \{y_1, ..., y_n, y_{n+1}, ...\}$ and $\mu_A(y_i) = \nu_A(y_i) = 0$ for $i \geq n+1$:*

$$\sum_{y \in F} \mu_A(y) + \sum_{y \in F} \nu_A(y) = \sum_{y \in E} \mu_A(y) + \sum_{y \in E} \nu_A(y) = \sum_{y \in E} (\mu_A(y) + \nu_A(y)) \leq card(E) = c.$$

Acknowledgements

This paper was supported by Grant VEGA 1/0621/11.

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