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A SHORT REMARK ON AN EXTENDED INTUITIONISTIC FUZZY ABSTRACT SYSTEM

Krassimir T. Atanassov
CLBME - Bulg. Academy of Sci., P.O.Box 12, Sofia-1113, Bulgaria
e-mail: krat@bgcict.acad.bg

Following [1-3] the author defined the concept of the *Intuitionistic Fuzzy System (IFSy)* in [4-6]. Now, we will define another type of systems, which will be an extension of the IFSy.

Using the notations from [4-6], we will define the object *Extended IFS (EIFSy).*

Firstly, we shall note that an IFSy

$$S \subset \prod_{i \in I} (V_i \times [0,1]^2) \times [0,1]^2$$

is defined as a proper relation on the sets $\{V_i \mid i \in I\}$, where I is an index set, $\{V_i \mid i \in I\}$ represent the objects which are the constituent parts of the system and each of the sets V_i represents a collection of alternative ways in which the corresponding object appears in the relation which defined the system (as in the case of ordinary systems). Unlike in the standard definition, here there are pairs of real numbers in $[0, 1]$ for which is valid that numbers μ_i and ν_i are associated to V_i $(i \in I), \langle \mu_i, \nu_i \rangle \in [0,1]^2$ and $\mu_i + \nu_i \leq 1$. By analogy with the Intuitionistic Fuzzy Set (IFS) theory (see [5]), the numbers μ_i and ν_i can be interpreted as degrees of validity and non-validity (correctness and incorrectness, etc.). Moreover, two other numbers μ_S and ν_S are associated to S.

They must be interpreted in the same way as the above ones but are related to the system S in general. Therefore, the IFSy S can be described in the form

$$S \subset \prod_{i \in I} \{\langle V_i, \mu_i, \nu_i \rangle\} \times \langle \mu_S, \nu_S \rangle.$$

When $\mu_i = 1, \nu_i = 0$ for every $i \in I$ and $\mu_S = 1, \nu_S = 0$, we obtain an ordinary system.

Now, we shall define an EIFSy

$$\langle S, P\rangle \subset \underset{i\in I}{\Pi} (V_i \times [0,1]^2) \times (P_i \times [0,1]^2) \times [0,1]^2$$

as a proper relation on the sets $\{V_i \mid i \in I\}$ and $\{P_i \mid i \in I\}$, where I is an index set,

$$S \subset \underset{i\in I}{\Pi} (V_i \times [0,1]^2) \times [0,1]^2$$

is an ordinary IFSy and the sets $\{P_i \mid i \in I\}$ represent properties of the objects of the system.

Here there are pairs of real numbers $\mu_{V,i}, \nu_{V,i}, \mu_{P,i}, \nu_{P,i}$ in $[0,1]$ for which they are valid that numbers $\mu_{V,i}$ and $\nu_{V,i}$ are associated to V_i $(i \in I)$ as above, $\mu_{P,i}$ and $\nu_{P,i}$ are associated to P_i $(i \in I)$, $\langle \mu_{P,i}, \nu_{P,i}\rangle \in [0,1]^2$ and $\mu_{P,i} + \nu_{P,i} \leq 1$. By analogy with the IFSy, the numbers $\mu_{P,i}$ and $\nu_{P,i}$ can be interpreted as degrees of validity and non-validity (correctness and incorrectness, etc.) of the object properties.

The two other numbers μ_S and ν_S, which are associated to S, are the same as above.

Obviously, the new type of systems is an extension as of the classical one as well as the IFSy. The use of this extension is to facilitate evaluation of the results of a system's functioning and of the properties of its components (its degrees of validity and non-validity, correctness and incorrectness, etc.) with respect to the different ways of interpretation of the different system's μ- and ν- parameters. For example, we can define the global (final) system's validity (correctness) degrees as:

$$\langle \mu_S, \nu_S\rangle = \langle max(\underset{i\in I}{max}\, \mu_{V,i}, \underset{i\in I}{max}\, \mu_{P,i}), min(\underset{i\in I}{min}\, \nu_{V,i}, \underset{i\in I}{min}\, \nu_{P,i})\rangle,$$

or

$$\langle \mu_S, \nu_S\rangle = \langle min(\underset{i\in I}{min}\, \mu_{V,i}, \underset{i\in I}{min}\, \mu_{P,i}), max(\underset{i\in I}{max}\, \nu_{V,i}, \underset{i\in I}{max}\, \nu_{P,i})\rangle,$$

or

$$\langle \mu_S, \nu_S\rangle = \langle \frac{1}{2}. \sum_{i\in I} \mu_{V,i} + \mu_{P,i}, \frac{1}{2}. \underset{i\in I}{\Pi}\, \nu_{V,i} + \nu_{P,i}\rangle,$$

etc, where for the natural number $n \geq 1$ and for real numbers $a_1, a_2, \ldots \in [0,1]$:

$$\sum_{i\in I} a_i = a_1 + a_2 + \ldots + a_n - a_1.a_2 - a_1.a_3 - \ldots - a_{n-1}.a_n + \ldots + (-1)^{n-1}.a_1. \ldots .a_n$$

and

$$\prod_{i \in I} a_i = a_1.a_2.\dots.a_n$$

(standard operation "production"), if $I = \{1, 2, \dots, n\}$.

Moreover, see [5], we can change the values of the global S-parameters μ_S and ν_S and the local S-parameters $\mu_{V,i}$, $\mu_{P,i}$ and $\nu_{V,i}$, $\nu_{P,i}$ for $i \in I$ by the IF-operators $F_{\alpha,\beta}$, $G_{\alpha,\beta}$, etc. The most interesting case is when these operators (in the general case — the operator $X_{a,b,c,d,e,f}$) and the operators $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ are time-functions.

In this case, we can describe the results of changing the S-parameters values with the time.

For example, we can define

$$\langle \mu_S, \nu_S \rangle = \langle \max_{i \in I} \mu_{V,i}(t) + \mu_{P,i}(t).(1 - \mu_{V,i}(t) - \nu_{V,i}(t)),$$

$$\min_{i \in I} \nu_{V,i}(t) + \nu_{P,i}(t).(1 - \mu_{V,i}(t) - \nu_{V,i}(t))\rangle,$$

where $t \in T$ - a fixed time scale, $\mu_{V,i}, \nu_{V,i}, \mu_{P,i}, \nu_{P,i} : [0,1]^2 \times T \to [0,1]$ are temporal degrees (for temporal IFS see [5]).

Another description of the IFSy S is the following:

$$S \subset \{\langle \Pi\{\langle V_i, \mu_i, \nu_i\rangle | i \in I\}, \mu_S, \nu_S\rangle\}.$$

Now, this representation obtains the form:

$$S \subset \{\langle \Pi\{\langle\langle V_i, P_i\rangle, \varphi(\mu_{V,i}, \mu_{P,i}), \psi(\nu_{V,i}, \nu_{P,i})\rangle | i \in I\}, \mu_S, \nu_S\rangle\},$$

where $\varphi, \psi : [0,1]^2 \to [0,1]$ are some functions, for which

$$\varphi(a,b) + \psi(a,b) \leq 1$$

for every $a, b \in [0,1]$.

This definition (similarly to the definition of the IFSy) enables a uniform representation of a set of systems S_j ($j \in J$; J is an index set) in the form:

$$\{\langle \Pi\{\langle\langle V_i, P_i\rangle, \varphi(\mu_{V,i}, \mu_{P,i}), \psi(\nu_{V,i}, \nu_{P,i})\rangle | i \in I^j\}, \mu_S^j, \nu_S^j\rangle | j \in J\},$$

i.e., in an IFS-form.

The researh in [7] is an example of the idea for the EIFSy.

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