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The Inclusion-Exclusion principle for IF-events

Karol Samuelčík, Jana Kelemenová
Faculty of Natural Science, Matej Bel University
Department of Mathematics
Tajovského 40
974 01 Banská Bystrica, Slovakia

1 Introduction.

Let $(\Omega, \mathcal{S}, P)$ denote a Kolmogorov probability space, Ω be an abstract set, $\mathcal{S}$ be a $\sigma-$ algebra of subsets of $\Omega, P : \mathcal{S} \to [0,1]$ be a probability measure. If $A_1, ..., A_n$ is a collection of arbitrary members of $\mathcal{S}$ and $n \in N$ then

$$(*)P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - ... + (-1)^{n+1} P(\bigcap_{i=1}^{n} A_i).$$

Atanassov [1] introduced the concept of IF-events. Atanassov theory works with IF-events, i.e. each pairs

$$A = (\mu_A, \nu_A),$$

such that $\mu_A, \nu_A : \Omega \to [0,1]\,, \mu_A + \nu_A \leq 1$, and μ_A, ν_A are measurable. Denote by $\mathcal{F}$ the set of all IF-events. We shall use basic operations on IF-sets.Probability theory on IF-events depends on connective used for the definition of the additivity of corresponding probability measure. We shall use Lukasiewicz connectives;

$$\mu_A \oplus \mu_B = (\mu_A + \mu_B) \wedge 1,$$

$$\nu_A \odot \nu_B = (\nu_A + \nu_B - 1) \vee 0,$$

$$A \oplus B = (\mu_A \oplus \mu_B, \nu_A \odot \nu_B)\,,$$

$$A \odot B = (\mu_A \odot \mu_B, \nu_A \oplus \nu_B)$$

Representation theorem for probabilities on IF-events is given by Ciungu-Riečan in [2] and by Petrovičov-Riečan in [3]. Grzegorzewski [4] generalize the probability version of the inclusion-exclusion principle for IF-events for two versions corresponding to different t-conorms.

In this paper we generalize the probability version of the inclusion-exclusion principle for IF-events with probability measure $\mathcal{P}: \mathcal{F} \to \mathcal{J}$,

$$\mathcal{P}(\mu_A, \nu_A) = \left[\int_\Omega \mu_A dP + \alpha(1 - \int_\Omega (\mu_A + \nu_A) dQ), \int_\Omega \mu_A dR + \beta(1 - \int_\Omega (\mu_A + \nu_A) dS)\right].$$

The Grzegorzewski result can be obtained with special choises $\alpha = 0, \beta = 1, P = Q = R = S$, then

$$\mathcal{P}(A) = \left[\int_\Omega \mu_A dP, 1 - \int_\Omega \nu_A dP\right].$$

2 Probability on family of IF-events.

Definition 1 *Probability is considered as a mapping*

$$\mathcal{P}: \mathcal{F} \to \mathcal{J},$$

where $\mathcal{J} = \{[a, b]\,; a, b \in R, a \leq b\})$ *satisfying the following conditions:*

(i) $\mathcal{P}((1, 0)) = [1, 1]\,, \mathcal{P}((0, 1)) = [0, 0]$,

(ii) if $A \odot B = (0, 1)$, *then* $\mathcal{P}(A \oplus B) = P(A) + P(B)$,

(iii) if $A_n \nearrow A$,*then* $\mathcal{P}(A_n) \nearrow \mathcal{P}(A)$.

$\mathcal{P}(A)$ *is a compact interval on* R, *denote it by*

$$\mathcal{P}(A) = \left[P^\flat(A), P^\sharp(A)\right].$$

It is to see that the main results can be described by the mappings $P^\flat(A), P^\sharp(A): \mathcal{F} \to [0, 1]$.

Definition 2 *A state on* $\mathcal{F}$ *is any function* $m: \mathcal{F} \to [0, 1]$ *satisfying the following properties:*

(i) $m((1, 0)) = 1, m((0, 1)) = 0$,

(ii) if $A \odot B = (0, 1)$, *then* $m(A \oplus B) = m(A) + m(B)$,

(iii) if $A_n \nearrow A$, *then* $m(A_n) \nearrow m(A)$.

Theorem 2.1 Let mapping $\mathcal{P}: \mathcal{F} \to \mathcal{J}$ be given, $P^\flat, P^\sharp: \mathcal{F} \to [0, 1]$ be defined by the formula

$$\mathcal{P}(\mu_A, \nu_A) = \left[P^\flat((\mu_A, \nu_A)), P^\sharp((\mu_A, \nu_A))\right].$$

Then $\mathcal{P}$ is a probability if and only if $P^\flat, P^\sharp$ are states.
Proof. See [3]

In calculations of probabilities of IF-events we apply interval arithmetic;

$$\mathcal{P}(A) + \mathcal{P}(B) = \left[P^\flat(A), P^\sharp(A)\right] + \left[P^\flat(B), P^\sharp(B)\right] =$$

$$= \left[P^\flat(A) + P^\flat(B), P^\sharp(A) + P^\sharp(B)\right].$$

Theorem 2.2 Let $\mathcal{P}: \mathcal{F} \to \mathcal{J}$ be a probability measure. Then there exist the Kolmogorov probability measures $P, Q, R, S: \mathcal{S} \to [0, 1]$ and constants $\alpha, \beta \in [0, 1]$ such that

$$\mathcal{P}(\mu_A, \nu_A) = \left[\int_\Omega \mu_A dP + \alpha(1 - \int_\Omega (\mu_A + \nu_A) dQ), \int_\Omega \mu_A dR + \beta(1 - \int_\Omega (\mu_A + \nu_A) dS)\right].$$

Proof. See [2, 3].

3 The inclusion-exclusion principle for IF-events.

3.1. Gödel connectives In this section we use the operations

$$A \vee B = (max(\mu_A, \mu_B), min(\nu_A, \nu_B))$$

$$A \wedge B = (min(\mu_{A,B}), max(\nu_A, \nu_B))\,.$$

Theorem 3.1.

For any $A_1, ..., A_n \in F, n \in N$ there holds

$$\mathcal{P}(\bigvee_{i=1}^{n} A_i) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \mathcal{P}(\bigwedge_{i=1}^{k} A_{j_i}).$$

Proof. It follows by Theorem 2.2. and equality

$$\bigvee_{i=1}^{n} (\mu_{A_i}, \nu_{A_i}) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \bigwedge_{i=1}^{k} (\mu_{A_{j_i}}, \nu_{A_{j_i}}).$$

3.2. Lukasziewicz connectives In this section we use the operations

$$A \oplus B = (min(\mu_A + \mu_B, 1), max(\nu_A + \nu_B - 1), 0)$$

$$A \odot B = (max(\mu_A + \mu_B - 1, 0), min(\nu_A + \nu_B, 1))\,.$$

Theorem 3.2.

For any $A_1, ..., A_n \in F, n \in N$ there holds

$$\mathcal{P}(\bigoplus_{i=1}^{n} A_i) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \mathcal{P}(\bigodot_{i=1}^{k} A_{j_i}).$$

Proof. It follows by Theorem 2.2. and equality

$$\bigoplus_{i=1}^{n} (\mu_{A_i}, \nu_{A_i}) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \bigodot_{i=1}^{k} (\mu_{A_{j_i}}, \nu_{A_{j_i}}).$$

3.3. Product connectives

In this section we use the operations

$$A \bigtriangledown B = (\mu_A + \mu_B - \mu_A.\mu_B, \nu_A.\nu_B)$$

$$A \bigtriangleup B = (\mu_A.\mu_B, \nu_A + \nu_B - \nu_A.\nu_B)\,.$$

Theorem 3.3.

For any $A_1, ..., A_n \in F, n \in N$ there holds

$$\mathcal{P}(\bigtriangledown_{i=1}^{n} A_i) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \mathcal{P}(\bigtriangleup_{i=1}^{k} A_{j_i}).$$

Proof. It follows by Theorem 2.2. and equality

$$\bigtriangledown_{i=1}^{n} (\mu_{A_i}, \nu_{A_i}) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \le j_1 < ... < j_k \le n} \bigtriangleup_{i=1}^{k} (\mu_{A_{j_i}}, \nu_{A_{j_i}}).$$

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