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Reduced Generalized Nets with Characteristics of the Arcs

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Abstract: In most publications on the applications of Generalized Nets (GNs) in the modelling of real-life processes reduced GNs are used rather than ordinary GNs. The use of reduced GNs allows for a more simple general description of the model as only the minimal number of transition components, i.e. set of input places, set of output places and the Index Matrix (IM) of the transition's condition are described. The reduced GNs have also huge theoretical significance because many theorems can be proved easily with the use of reduced GNs. In recent years, the existence of two classes of minimal reduced GNs has been proven – one in which only the tokens obtain characteristics and one in which only the places obtain characteristics during the functioning of the net. In the present paper, a third class of minimal reduced GNs is proposed in which only the arcs obtain characteristics. Theorems for representation of the GNs from this class with minimal reduced GNs in which only the tokens receive characteristics are proved.

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1 Introduction

GNs may or may not have some of the components in their definition. GNs which do not have some of the components form special classes called reduced

GNs. For more details about the concept of reduced GNs see [6, 7]. Let

$$\Omega = \{A, \pi_A, \pi_L, c, f, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^0, t^*, X, \Phi, b\} \cup \{A_i | 1 \leq i \leq 7\} ,$$

where $A_i = pr_i A$ is the i-th projection of the set of transitions A of an ordinary GN, i.e. $A_i \in \{L', L'', t_1, t_2, r, M, \Box\}$. If Σ, as usual, is the class of all GNs and $Y \in \Omega$, then by Σ^Y we denote the class of all GNs that do not have component Y. We call them Y-reduced GNs. Many assertions for the various classes of reduced GNs are proved in [6].

Let Σ_1 and Σ_2 be two subclasses of Σ. We shall use the following definitions:

- $\Sigma_1 \vdash \Sigma_2$ iff the functioning and the results of the work of every element of Σ_2 can be described by an element of Σ_1
- $\Sigma_1 \equiv \Sigma_2$ iff $\Sigma_1 \vdash \Sigma_2 \,\&\, \Sigma_2 \vdash \Sigma_1$.

The class $\Sigma^* = \Sigma^{A_3, A_4, A_6, A_7, \pi_A, \pi_L, c, \theta_1, \theta_2, \pi_K, \theta_K, T, t^0, t^*, b}$ is the class of minimal reduced GNs (*-GNs). The minimal reduced GNs have the form

$$E' = \langle \langle A', *, *, *, *, *, *, * \rangle, \langle K, *, * \rangle, *, \langle X, \Phi, * \rangle \rangle ,$$

where A' is the set of transitions of the net each of which has the following form

$$Z' = \langle L', L'', *, *, r, *, * \rangle \} ,$$

i.e. every transition of the minimal reduced GNs consists of set of input places, set of output places and Index Matrix (IM) of the transition's condition. For the minimal reduced GNs the following notation is also used

$$E' = \langle A', K^*, X, \Phi \rangle .$$

The minimal elements of Σ^* are denoted by

$$E^* = \langle A^*, K^*, X^*, \Phi^* \rangle ,$$

where A^* is a set of transitions of the form $Z^* = \langle L', L'', r' \rangle$. The following theorem for equivalence in terms of the functioning and results of work of ordinary GNs and minimal reduced GNs is proven in [6].

Theorem 1. $\Sigma \equiv \Sigma^*$.

Many extensions of the ordinary GNs have been defined and all of them have been proven to be conservative extensions of the class Σ. The concept of Generalized Net with Characteristics of the Places (GNCP) has been defined in [3] and studied in [1, 4, 5]. A GNCP is the ordered four-tuple

$$\langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K, \rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle\,,$$

where all components except the characteristic functions Y and Ψ have the same meaning as the ordinary GNs. The characteristic function Y assigns initial characteristics to some of the places of the net. The characteristic function Ψ assign characteristics to some of the output places of a given transition when tokens enter them. These characteristics can be the number of tokens in the place, the time moments when tokens have entered or left the place or other data which is related to the place. Algorithm for transition functioning of GNCP is proposed in [5].

In [1] the concept of reduced GNCP is introduced. We shall mention the basic notation which is used in the present paper.

Let

$$\Omega_{CP} = \{A, \pi_A, \pi_L, c, f, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^0, t^*, X, Y, \Phi, \Psi, b\}$$
$$\cup\{A_i | 1 \leq i \leq 7\}\,,$$

where again $A_i = pr_i A$ is the i-th projection of the set of the transitions of the net A. By Σ^Y_{CP} we denote the class of those GNCP which do not have Y component. They are called Y-reduced GNCP. Obviously, we have

$$\Sigma^A_{CP} = \Sigma^{A_1}_{CP} = \Sigma^{A_2}_{CP} = \Sigma^K_{CP} = \emptyset\,.$$

More generally, if $Y_1, Y_2, ..., Y_k \in \Omega_{CP}$ where $k \geq 1$ is a natural number, then $\Sigma^{Y_1,Y_2,...,Y_k}_{CP}$ is called $(Y_1, Y_2, ..., Y_k)$-reduced class of GNCP.

There are two classes of minimal reduced GNCP. In one of them only the tokens obtain characteristics while in the other characteristics are assigned only to the places. The first class which is denoted by

$$\Sigma^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,Y,\Psi,b}_{CP}$$

coincides with the class Σ^*. All nets from this class have the form $E' = \langle A', K, X, \Phi\rangle$. By Σ^*_{CP} we denote the class of minimal reduced GNCP in which only the places obtain characteristics, i.e.,

$$\Sigma^*_{CP} = \Sigma_{CP}^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,X,\Phi,b} .$$

Using the theorem proved in [3] which states that $\Sigma_{CP} \equiv \Sigma$ in [1] the following theorem is proved:

Theorem 2. $\Sigma_{CP} \equiv \Sigma^*$.

Further in the same paper, a constructive proof is presented that shows the validity of:

Theorem 3. $\Sigma^* \equiv \Sigma^*_{CP}$.

From Theorem 2 and Theorem 3, using the transitive property of the relation $\equiv$ we obtain:

Theorem 4. $\Sigma_{CP} \equiv \Sigma^*_{CP}$.

2 Reduced Generalized Nets with Characteristics of the Arcs

The concept of Generalized Nets with Characteristics of the Arcs (GNCA) is defined in [2]. Formally, a GNCA is the ordered four tuple

$$\langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K, \rangle, \langle T, t^0, t^*\rangle, \langle X, \Omega, \Phi, \Xi, b\rangle\rangle ,$$

where all components with the exception of the characteristic functions Ω and Ξ have the same meaning as in the ordinary GNs. The characteristic function Ω assigns initial characteristics to some of the arcs of the net. The characteristic function Ξ assigns characteristics to the arcs of a given transition if tokens have passed through them. Here, by arc we mean an ordered couple of input and output places for a given transition.

With Σ_{CA} we denote the class of all GNCA. In [2] a construcitve proof of the following theorem is given.

Theorem 5. $\Sigma_{CA} \equiv \Sigma$.

In other words, the theorem states that GNCA are conservative extensions of the ordinary GNs.

Since $\Sigma \equiv \Sigma_{CP}$, using the transitive property of the relation $\equiv$, we obtain

Theorem 6. $\Sigma_{CA} \equiv \Sigma_{CP}$.

Here, for the first time, we propose the concept of *Reduced GNCA*. Let

$$\Omega_{CA} = \{A, \pi_A, \pi_L, c, f, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^0, t^*, X, \Omega, \Phi, \Xi, b\} \cup \{A_i | 1 \leq i \leq 7\} ,$$

where A_i denotes the i-th projection of the set of the transitions A. If $Y \in \Omega_{CA}$, we denote by Σ_{CA}^{Y} the class of all GNCA which do not have Y component. Obviously, if $Y \in \{A, A_1, A_2, K\}$, such reduced GNCA cannot exist, i.e.

$$\Sigma_{CA}^{A} = \Sigma_{CA}^{A_1} = \Sigma_{CA}^{A_2} = \Sigma_{CA}^{K} = \emptyset .$$

If $Y_1, Y_2, ..., Y_k \in \Omega_{CA}$, where $k \geq 1$, then by $\Sigma_{CA}^{Y_1,Y_2,...,Y_k}$ we denote the class of all $Y_1, Y_2, ..., Y_k$-reduced GNCA.

Again, as in the case of reduced GNCP, we have two classes of minimal reduced GNCA. In one of them only the tokens obtain characteristics while in the other characteristics are assigned only to the arcs. The class $\Sigma^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,\Omega,\Xi,b}$ coincides with the class Σ^* of the minimal reduced GNs which have the form $E' = \langle A', K, X, \Phi \rangle$. The other class of minimal reduced GNCA in which only the arcs obtain characteristics we denote by Σ_{CA}^{*}, i.e.

$$\Sigma_{CA}^{*} = \Sigma_{CA}^{A_3,A_4,A_6,A_7,\pi_A,\pi_L,c,\theta_1,\theta_2,\pi_K,\theta_K,T,t^0,t^*,X,\Phi,b} .$$

The functioning and the results of work of every GNCA can be described by a minimal reduced GNs in which only the tokens obtain characteristics, i.e. the following theorem is valid.

Theorem 7. $\Sigma^* \vdash \Sigma_{CA}$.

Proof. In [2] it is shown that $\Sigma \vdash \Sigma_{CA}$. We know from [6] that $\Sigma^* \vdash \Sigma$. Using the transitive property of the relation $\vdash$ we obtain $\Sigma^* \vdash \Sigma_{CA}$. □

The functioning and the results of work of every reduced GN in which only the tokens obtain characteristics can be described by a GNCA, i.e. the following theorem is valid.

Theorem 8. $\Sigma_{CA} \vdash \Sigma^*$.

Proof. Since every ordinary GN can be considered a GNCA in which no arcs obtain characteristics, we have $\Sigma_{CA} \vdash \Sigma$. From [6] we know that $\Sigma \vdash \Sigma^*$. Therefore, using the transitive property of the relation $\vdash$ we obtain $\Sigma_{CA} \vdash \Sigma^*$. □

3 Connection between the Classes Σ^* and Σ^*_{CA}

We shall prove theorems for representation of minimal reduced GNCA through minimal reduced GN in which only the tokens obtain characteristics, and vice versa. The existance of such representations is important not only from theoretical point of view but also from practical because in most models reduced GNs are used. Depending on the model, it can be preferable to use one of the three minimal reduced GNs instead of the others.

Theorem 9. $\Sigma^*_{CA} \vdash \Sigma^*$.

Proof. Let E be an arbitrary reduced GN from the class Σ^*, i.e.

$$E = \langle A', K, X, \Phi \rangle \,.$$

Let $Z = \langle L', L'', r' \rangle$ be arbitrary transition of E. For simplicity, first we consider that characteristics of the tokens are not used in the predicates of the transitions and in the characteristic function Φ. We construct a transition Z_{CA} which has the same graphic representation as Z, i.e. it has the same number of input and output places and the same index matrix of the transitions condition. Let A_{CA} be the set of transitions obtained through repetition of this procedure for all transitions of E. We shall prove that the minimal reduced GNCA

$$G = \langle \langle A_{CA}, *, *, *, *, *, * \rangle, \langle K_{CA}, *, * \rangle, *, \langle *, \Omega, *, \Xi, * \rangle$$

represents the functioning and the results of work of E. The set of tokens K_{CA} of G consists of the same number and types of tokens as the set K of E, i.e. for every token $\alpha' \in K_{CA}$ there is a corresponding token $\alpha \in K$. The function Ω assigns the initial characteristics of the tokens in E to the arcs through which their corresponding tokens in G are transferred for the first time, i.e. if (l', l'') denotes an arc through which a token α which enters E in the input place l' is transferred to the output place l'' and (l'_{CA}, l''_{CA}) is its corresponding arc in G, then $X_{l'} = \Omega_{l'_{CA}, l''_{CA}}$. We will use the following representation of the characteristic function Φ:

$$\Phi = \bigcup_{l \in L - Q^I} \Phi_l = \bigcup_{Z \in A} (\bigcup_{l \in pr_2 Z} \Phi_l) \,,$$

and similarly

$$\Xi = \bigcup_{Z_{CA} \in A_{CA}} (\bigcup_{l'_{CA} \in pr_1 Z_{CA}} (\bigcup_{l''_{CA} \in pr_2 Z_{CA}} \Xi_{l'_{CA}, l''_{CA}})) \,.$$

The function $\Xi_{l'_{CA}, l''_{CA}}$ assigns characteristic to the arc (l'_{CA}, l''_{CA}) if a token is transferred through it. If the token transferred through the arc (l'_{CA}, l''_{CA}) at the current step is α_{CA}, then the characteristic of the arc has the form

$$\text{``}\langle \alpha_{CA}, \Phi_{l''}(\alpha) \rangle\text{''} \,.$$

To prove that the so constructed reduced GNCA G represents the funtioning and the results of work of E we take two correspondig transitions $Z \in pr1pr1E$ and $Z_{CA} \in pr1pr1G$. Let $\alpha \in K$ and $\alpha_{CA} \in K_{CA}$ be two corresponding tokens that are respectively in places l' and l'_{CA}. If the token α is tranferred to output place l'', then α_{CA} is tranferred to the corresponding output place l''_{CA}. The characteristic obtained by α in l'' is assigned by the function Ξ to place l''_{CA}. If the token α cannot be transferred to any output place of Z, then α_{CA} also will not be tranferred to any output place. The case in which splitting of tokens is allowed can be verified in a similar way. From the theorem for the completeness of the GN transition it follows that $\Sigma^*_{CA} \vdash \Sigma^*$.

In the beginning of the proof we assumed that characteristics of tokens are not used in the predicates of the transition Z and in the characteristic function Φ. If such characteristics are used, then they must be substituted with the corresponding characteristics of the places in Z_{CA}. □

The functioning and the results of work of every minimal reduced GNCA in which only the arcs obtain characteristics can be described by a minimal reduced GN in which only the tokens obtain characteristics, i.e. the following theorem is valid.

Theorem 10. $\Sigma^* \vdash \Sigma^*_{CA}$.

Proof. Let G be arbitrary minimal reduced GNCA.

$$G = \langle A_{CA}, *, *, *, *, *, * \rangle, \langle K_{CA}, *, * \rangle, *, \langle *, \Omega, *, \Xi, * \rangle \,.$$

We will contruct a minimal reduced GN which represents the functioning and the results of the work of G. For every transition $Z_{CA} \in pr_1 pr_1 G$ (see Fig. 1) we construct a corresponding transition Z (see Fig. 2).

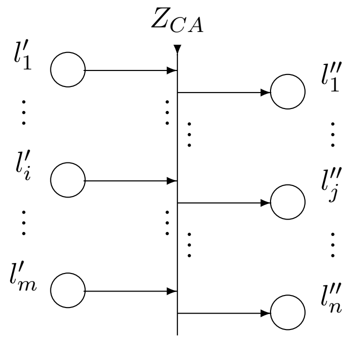

Figure 1. Graphical representation of an arbitrary transition of a minimal reduced GNCA.

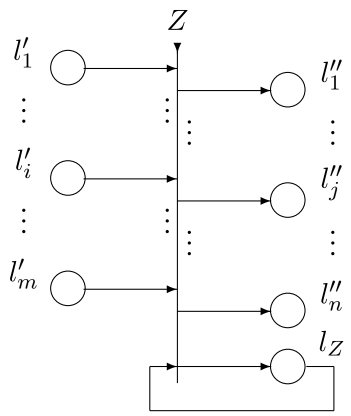

Figure 2. Graphical representation of the transition of the ordinary GN corresponding to the transition in Fig. 1.

Again, we shall consider that characteristics of arcs are not used in the predicates of Z_{CA} and in the characteristic function Ξ. If $Z_{CA} = \langle L'_{CA}, L''_{CA}, r^{CA} \rangle$, then $Z = \langle L', L'', r \rangle$ where

$$L' = L'_{CA} \cup \{l_Z\}\,,$$

$$L'' = L''_{CA} \cup \{l_Z\}\,.$$

To every transition we add additional place which is input and output for the transition where a token α_Z loops and keeps as characteristics the characteristics assigned to the arcs of Z_{CA}. The initial characteristic of the token α_Z is a list of the initial characteristics of the arcs of the transition Z_{CA}. Place l_Z has the lowest priority among the places of the transition.

If

$$r^{CA} = pr_5 Z_{CA} = [L'_{CA}, L''_{CA}, \{r^{CA}_{l_i,l_j}\}]$$

has the form of an IM, then

$$r = pr_5 Z = [L'_{CA} \cup \{l_Z\}, L''_{CA} \cup \{l_Z\}, \{r_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L'_{CA})(\forall l_j \in L''_{CA})(r_{l_i,l_j} = r^{CA}_{l_i,l_j})$$

$$(\forall l_i \in L')(\forall l_j \in L'')(r_{l_i,l_Z} = r_{l_Z,l_j} = \text{“}false\text{”}),$$

$$r_{l_Z,l_Z} = \text{“}true\text{”}\ .$$

Let A be the set of transitions obtained after repeating the above procedure for all transitions of G. We will prove that the minimal reduced GN

$$E = \langle A, K, X, \Phi \rangle$$

represents the functioning and the results of work of G, where

$$K = K_{CA} \bigcup \{\alpha_Z | Z \in A_{CA}\}\,.$$

The characteristic function X assigns initial characteristic $x_0^{\alpha_Z}$ only to the α_Z tokens and it is a list of all arcs of the transition Z and their initial characteristics in G in the form:

$$\text{“}\langle(l'_1, l''_1), \Omega_{l'_1,l''_1}\rangle, \langle(l'_1, l''_2), \Omega(l'_1, l''_2)\rangle, ..., \langle(l'_m, l''_n), \Omega_{l'_m,l''_n}\rangle\text{”}\,.$$

The characteristic function Φ assigns to the α_Z-tokens a list with the arcs of the transition and the characteristics obtained by them in G in the form

$$\Phi_{\{l_Z|Z\in A\}}(\alpha_Z) = \text{“}\{\langle(l'_i, l''_j), \Xi_{l'_i,l''_j}\rangle | l'_i \in L' \& l''_j \in L''\}\text{”}.$$

The proof that the so constructed minimal reduced GN represents the functioning and the results of the work of G is similar to the proof that $\Sigma^*_{CA} \vdash \Sigma^*$. Now all characteristics of the arcs of G are kept as characteristics of the α_Z tokens. Thus, we obtain $\Sigma^* \vdash \Sigma^*_{CA}$.

In the beginning of the proof, we assumed that characteristics of the places are not used in the predicates of the transition Z_{CA} and in the characteristic function Ξ. If such characteristics are used, then they must be substituted with the corresponding characteristics of the tokens in Z. □

From Theorem 9 and Theorem 10 we obtain the following theorem:

Theorem 11. $\Sigma^* \equiv \Sigma^*_{CA}$.

In [1] it is proved that $\Sigma^* \vdash \Sigma^*_{CP}$. Using this and Theorem 9 we obtain:

Theorem 12. $\Sigma^*_{CA} \vdash \Sigma^*_{CP}$.

Again, in [1] it is shown that $\Sigma^*_{CP} \vdash \Sigma^*$. Using this and Theorem 10 we obtain:

Theorem 13. $\Sigma^*_{CP} \vdash \Sigma^*_{CA}$.

From Theorem 12 and Theorem 13 we obtain:

Theorem 14. $\Sigma^*_{CP} \equiv \Sigma^*_{CA}$.

4 Conclusions

The concept of reduced GNCA, defined here, allows for a third class of minimal reduced GNs to be defined. Apart from the minimal reduced GNs in which only the tokens obtain characteristics (the class Σ^*) and the minimal reduced GNCP (the class Σ^*_{CP}) in which only the places obtain characteristics, now we have minimal reduced GNCA in which only the arcs obtain characteristics (the class Σ^*_{CA}).

Here, we have studied the connection between the classes Σ^* and Σ^*_{CA}. The theorems for representation – Theorem 9 and Theorem 10 – have not only theoretical significance but, also, they can be used in the models of real processes. The proofs show which components of the given reduced GN have to be changed and in what way so that we can construct a GN from the other class of minimal reduced GNs.

Theorems 12 - 14 specify the connection between the minimal reduced GNCP and the minimal reduced GNCA. Their validity is obtained as a corollary from previously proven theorems and the theorems for representation proved here. However, constructive proofs of these theorems would be very useful and would show how we can obtain a minimal reduced GN from one of the classes, given a net from the other. Obtaining such proofs is a part of our future research.

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