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On Łukasiewicz's intuitionistic fuzzy subtraction

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Abstract: On the basis of Łukasiewicz's intuitionistic fuzzy implication, the new operation named Łukasiewicz's intuitionistic fuzzy subtraction has been defined, and its basic properties have been investigated.
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In the present remark, we will introduce a new operation subtraction as a continuation of our research [2, 6]. In the beginning, the necessary concepts from intuitionistic fuzzy set theory will be given.

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

In [3], intuitionistic fuzzy logic interpretation of Łukasiewicz's fuzzy implication is given on the basis of Klir and Yuan's book [5]. In [4] it is re-writen for IFS-form as follows

$$A \to_L B = \{\langle x, \min(1, \nu_A(x) + \mu_B(x)), \max(0, \mu_A(x) + \nu_B(x) - 1)\rangle | x \in E\}.$$

This implication generates the classical negation of IFS A, because

$$\overline{A} = A \to_L O^* = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}.$$

Now, using the well-known equality

$$A - B = \overline{A \to B},$$

we will introduce the new operation "Łukasiewicz's intuitionistic fuzzy subtraction". It is obtained from

$$A -_L B = \overline{A \to_L B} = \{\langle x, \min(1, \max(0, \mu_A(x) + \nu_B(x) - 1), \nu_A(x) + \mu_B(x))\rangle | x \in E\}.$$

First, we have to check that in a result of the operation we obtain an IFS. Let the IFSs A and B be given. Let for an arbitrary $x \in E$

$$X \equiv \min(1, \nu_A(x) + \mu_B(x)) + \max(0, \mu_A(x) + \nu_B(x) - 1).$$

If $\nu_A(x) + \mu_B(x) \geq 1$, then $\mu_A(x) + \nu_B(x) \leq 2 - \nu_A(x) + \mu_B(x) \leq 1$ and therefore

$$X \leq 1 + \max(0, 1 - 1) = 1.$$

If $\nu_A(x) + \mu_B(x) < 1$, then

$$X = \nu_A(x) + \mu_B(x) + \max(0, \mu_A(x) + \nu_B(x) - 1).$$

If $\mu_A(x + \nu_B(x) - 1 \geq 0$, then

$$X = \nu_A(x) + \mu_B(x) + \mu_A(x) + \nu_B(x) - 1 \leq 2 - 1 = 1.$$

If $\mu_A(x + \nu_B(x) - 1 < 0$, then

$$X = \nu_A(x) + \mu_B(x) + 0 < 1.$$

Therefore, $A -_L B$ is an IFS.

By analogy, we can prove the following assertions.

Theorem: For every two IFSs A and B:

(a) $A -_L E^* = O^*$,

(b) $A -_L O^* = A$,

(c) $E^* -_L A = \overline{A}$,

(d) $O^* -_L A = O^*$,

(e) $(A \cap B) -_L C = (A -_L C) \cap (B -_L C)$,

(f) $(A \cup B) -_L C = (A -_L C) \cup (B -_L C)$,

(g) $(A -_L B) -_L C = (A -_L C) -_L B$.

Obviously,

$$O^* -_L U^* = O^*,\ O^* -_L E^* = O^*,\ O^* -_L O^* = O^*,$$
$$U^* -_L U^* = U^*,\ U^* -_L E^* = O^*,\ U^* -_L O^* = U^*,$$
$$E^* -_L U^* = U^*,\ E^* -_L E^* = O^*,\ E^* -_L O^* = E^*.$$

References

[1] Atanassov, K. Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] Atanassov, K., B. Riecan. On two operations over intuitionistic fuzzy sets. Journal of Applied Mathematics, Statistics and Informatics, Vol. 2, 2006, No. 2, 145–148.

[3] Atanassov, K. On some intuitionistic fuzzy implication. Comptes Rendus de l'Academie bulgare des Sciences, Tome 59, 2006, No. 1, 21–26.

[4] Atanassov, K., D. Dimitrov, Intuitionistic fuzzy implications and axioms for implications. Notes on Intuitionistic Fuzzy Sets, Vol. 16, 2010, No. 1, 10–20.
`http://ifigenia.org/wiki/issue:nifs/16/1/10-20`

[5] Klir, G. and Bo Yuan, *Fuzzy Sets and Fuzzy Logic.* Prentice Hall, New Jersey, 1995.

[6] Riecan, B. and K. Atanassov. A set-theoretical operation over intuitionistic fuzzy sets. Notes on Intuitionistic Fuzzy Sets, Vol. 12, 2006, No. 2, 24–25.
`http://ifigenia.org/wiki/issue:nifs/12/2/24-25`