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GEOMETRICAL INTERPRETATIONS OF THE π – FUNCTION VALUES OF INTUTIONISTIC FUZZY SETS

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Abstract. This article provides a brief overview of known geometrical interpretations of Intuitionistic Fuzzy Sets[1]. A new classification of IF sets is proposed based on the relations between the ranges of the membership and non-membership functions defined for the elements of an IFS.

1. Introduction

Intuitionistic Fuzzy Sets (IFSs, see [1]) are an extension of fuzzy sets allowing levels of membership and non-membership to be independently defined for each element of the set. Let *E* be an arbitrary universe. IFS *A* in *E* is defined as an object of the following form

$$A = \{<x, \mu_A(x), \nu_A(x)>|x \in E\},$$

where the functions:

$$\mu_A : E \rightarrow [0,1]$$

and

$$\nu_A: E \rightarrow [0,1]$$

define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Therefore, we can define a function

$$\pi_A: E \rightarrow [0,1]$$

such that

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)),$$

which corresponds to the degree of uncertainty about the membership of an element $x \in E$. When

$$\pi_A(x) = 0$$

for each $x \in E$ the set is equivalent to an ordinary fuzzy set.

In [1] five geometric interpretations of IFSs are proposed. Fig.1. illustrates the so called "Interpretation Triangle" (IT) which is denoted by L^*.

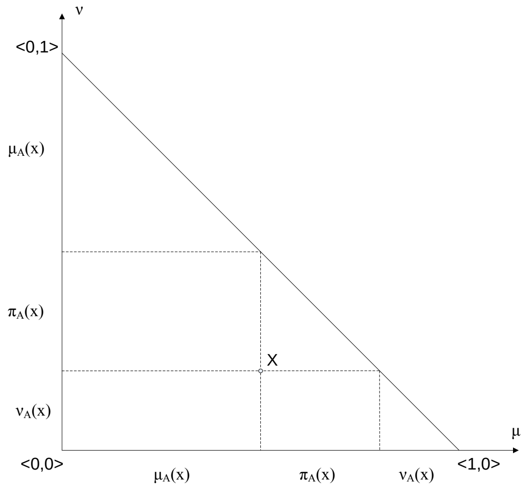

Fig.1. The Interpretation Triangle *IT*

Two transformations that map a square with sides of unit length to the IT are defined in [2]. Another geometric interpretation of the IFS and some of its properties and applications are discussed in [3]. Some details of the IT interpretation - two-dimensional interpretation with a parameter, as well as a three-dimensional interpretation are discussed in [4], while some geometrical properties of function π are discussed in [5]. Topological operators C and I are defined over IFSs [1] by

$$C = \{\langle x, \mu_{max}, \nu_{min}\rangle \mid x \in E\}$$

$$I = \{\langle x, \mu_{min}, \nu_{max}\rangle \mid x \in E\},$$

where:

$$\mu_{max} = \sup \mu_A(y),\ y \in E,\ \mu_{min} = \inf \mu_A(y),\ y \in E,$$

$$\nu_{min} = \inf \nu_A(y),\ y \in E\ \nu_{max} = \sup \nu_A(y),\ y \in E$$

determine the "boundaries" of an IFS – see Fig. 2.

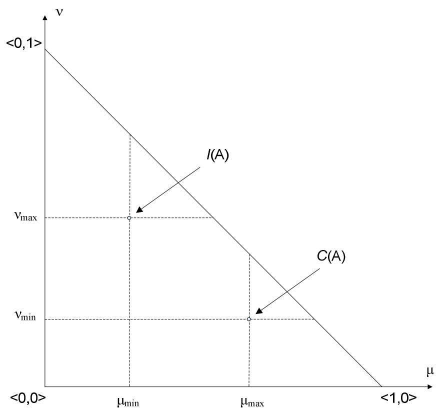

Fig.2. Boundaries of an IFS

2. IFS Shapes

Let

$$\pi_{max} = \sup \pi_A(y),\ y \in E$$

$$\pi_{min} = \inf \pi_A(y), \; y \in E$$

determine the range of π_A. Using the triangle interpretation, we may illustrate the values of π_{min} and π_{max} as distances to a line parallel to the triangle's hypotenuses. Fig. 3 illustrates an example configuration of π_{min} and π_{max}.

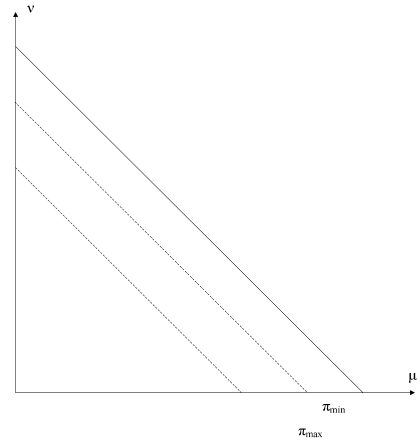

Fig.3. Values of π

The following section proposes a classification of IF sets based on the relations between the values of μ_{max}, μ_{min}, ν_{max}, ν_{min}, π_{max} and π_{min}.

2.1. Relations between the ranges of μ, ν and π

Following some basic properties of IF sets we may introduce a number of relations between the ranges of μ, ν and π.

$$\mu_{min} + \pi_{min} \le \mu + \pi \le 1 - \nu \Rightarrow \nu \le 1 - \mu_{min} - \pi_{min}$$

$$\mu_{max} + \pi_{max} \ge \mu + \pi \ge 1 - \nu \Rightarrow \nu \ge 1 - \mu_{max} - \pi_{max}$$

$$\mu_{min} + \nu_{min} \le \mu + \nu \le 1 - \pi \Rightarrow \nu_{min} \le 1 - \mu_{min} - \pi \Rightarrow \nu_{min} \le 1 - \mu_{min} - \pi_{max}$$

$$\mu_{max} + \nu_{max} \ge \mu + \nu \ge 1 - \pi \Rightarrow \nu_{max} \ge 1 - \mu_{max} - \pi_{min}$$

$$\mu_{min} + \pi_{max} \le 1 \Rightarrow \mu_{min} \le 1 - \pi_{max}$$

$$\mu_{max} + \pi_{min} \le 1 \Rightarrow \mu_{max} \le 1 - \pi_{min}$$

2.2. Geometrical shapes of Intuitionistic Fuzzy Sets

The purpose of this section is to propose a classification of the relations between the ranges of μ, ν and $\boldsymbol{\pi}$ using the triangle geometrical interpretation of IFS. Let us consider the elements of an arbitrary IFS $\boldsymbol{X}$ as points in the plane

$$x(\mu(x), \nu(x)), \text{ where } 1 - \mu(x) - \nu(x) = \pi(x) \ge 0$$

We are interested in constructing a closed convex polygon lying inside the IFS triangle which contains all the points defined by ***X.*** Let us call such figure the set's "*shape*". Such polygon would be uniquely defined by the seven-tuple:

$$S = \langle \mu_{\min}, \mu_{\max}, \nu_{\min}, \nu_{\max}, \pi_{\min}, \pi_{\max} \rangle$$

which we will call the "*shape limit*" of the IFS ***X***. More precisely, we will consider the number of vertices of this figure. Then we will define a number of IFS classes each of them containing all the sets that have shapes with the same number of vertices.

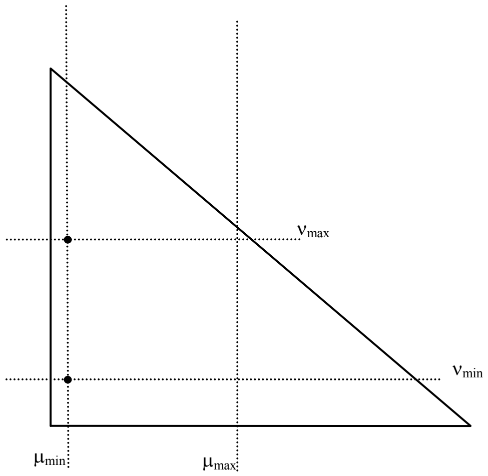

Fig.4. Basic disposition

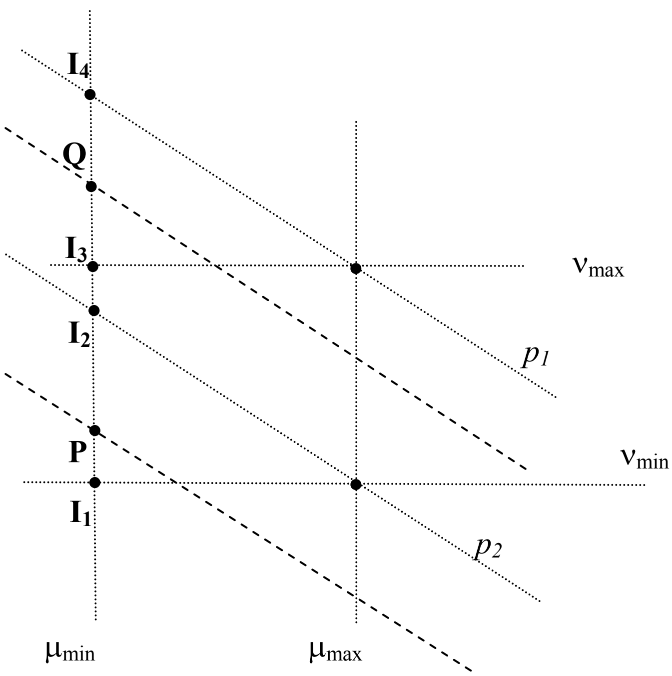

Fig.5. Intersection points and the case of distance 2

We will be interested only in the area closed between $\mu_{\min}$, $\mu_{\max}$, $\nu_{\min}$ and $\nu_{\max}$. Let's draw a line p_1 that goes through the intersection point of $\mu_{\max}$ and $\nu_{\max}$, and is parallel to the hypotenuses of the triangle; a line p_2 that goes through the intersection point of $\mu_{\max}$ and $\nu_{\min}$, and is also parallel to the hypotenuses of the triangle. Now let's sort the intersection points of $(p_1, \mu_{\min}), (p_2, \mu_{\min}), (\mu_{\min}, \nu_{\min})$ and $(\mu_{\min}, \nu_{\max})$ by their ordinates and name them ***I₁***, ***I₂***, ***I₃*** and ***I₄*** respectively. Finally, let the intersection points of $(\pi_{\min}, \mu_{\min})$ and $(\pi_{\max}, \mu_{\min})$ be ***P*** and ***Q*** respectively (see Fig. 5).

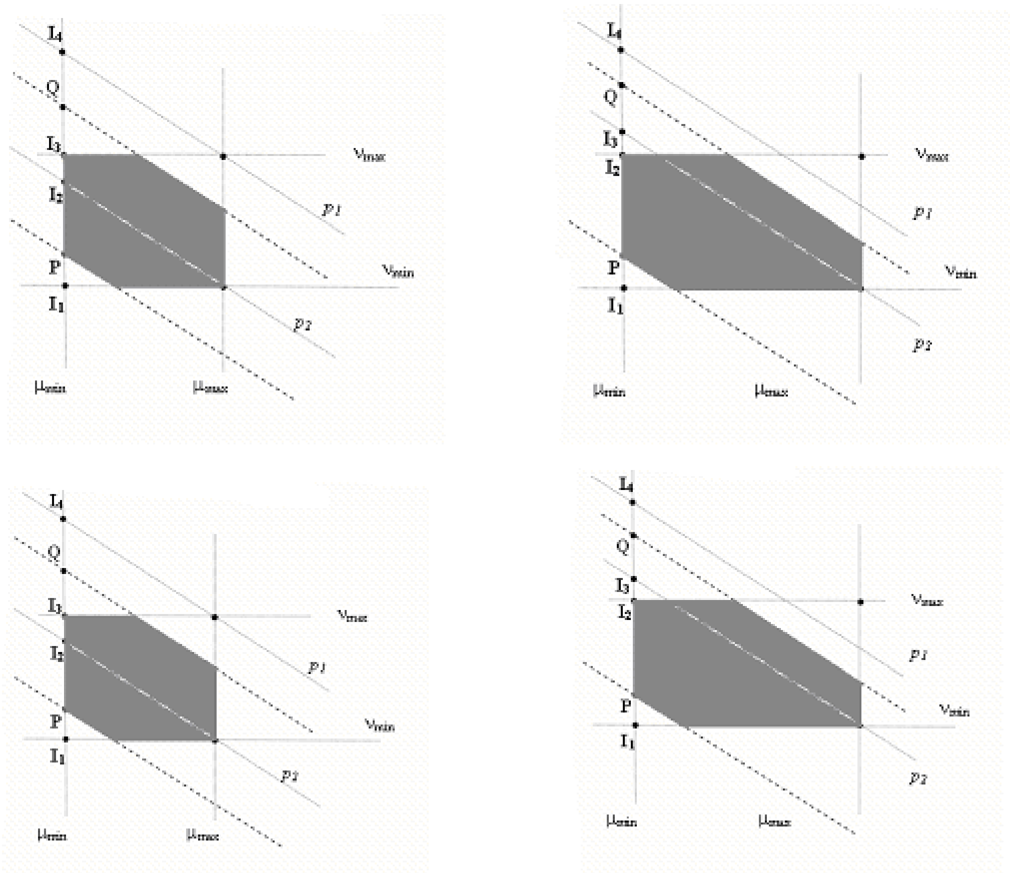

Fig.5.1. Several examples

Let y_k be the ordinate of I_k for $k = 1,..,4$, y_0 be 0, y_5 be 1, p be the ordinate of ***P***, and q be the ordinate of ***Q***. Let $p \in [y_k, y_{k+1})$ and $q \in (y_m, y_{m+1}]$. The following classification of the mutual disposition of ***P, Q,*** I_1, I_2, I_3 and I_4 for any set ***X*** we will call the set's ***distance***:

$$\Delta_X = |m-k| - \begin{cases} 1 & p = y_k \\ 0 & otherwise \end{cases} - \begin{cases} 2 & (k<1) \\ 0 & otherwise \end{cases} - \begin{cases} 2 & (m+1>4) \\ 0 & otherwise \end{cases}.$$

The possible values for this measure are the integers between -1 and 2. It enumerates the classes of mutual dispositions between ***P, Q,*** I_1, I_2, I_3 and I_4 which form equivalent shapes.

Having calculated the distance, the number of vertices of the shape of the set X is:

$$V_X = (\Delta_X + 4),$$

if there are no coinciding values for the maximum and minimum of a degree and:

$$V_X = \left\lceil \frac{(\Delta_X + 4)}{\min(2c, 4)} \right\rceil,$$

where c is the number of coinciding pairs of lines.

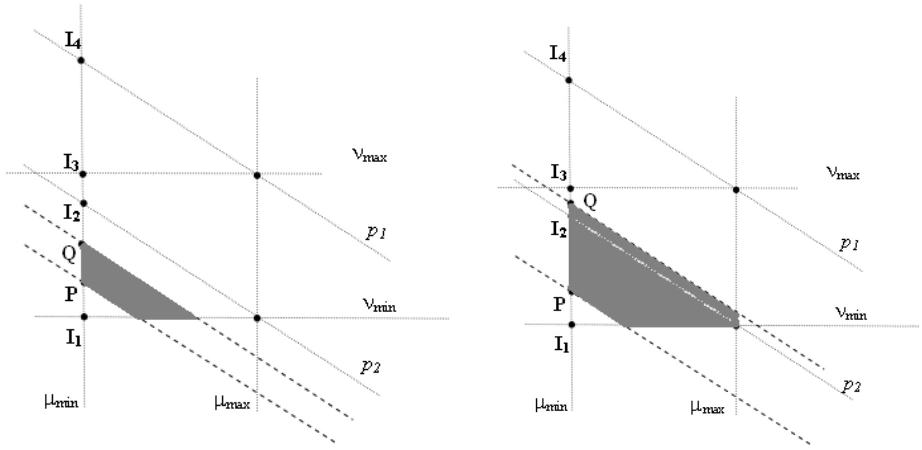

Fig 6. Distances of 0 and 1

Finally, we obtain six classes ***C₀***,..., ***C₅*** of IFSets, where $C_i = \{X \mid i = \Delta_X + 1\}$. The logical properties of these classes are an interesting subject of future investigation.

3. Examples

This section provides a number of examples of different elements of the defined classes.

3.1. *C₀* – point

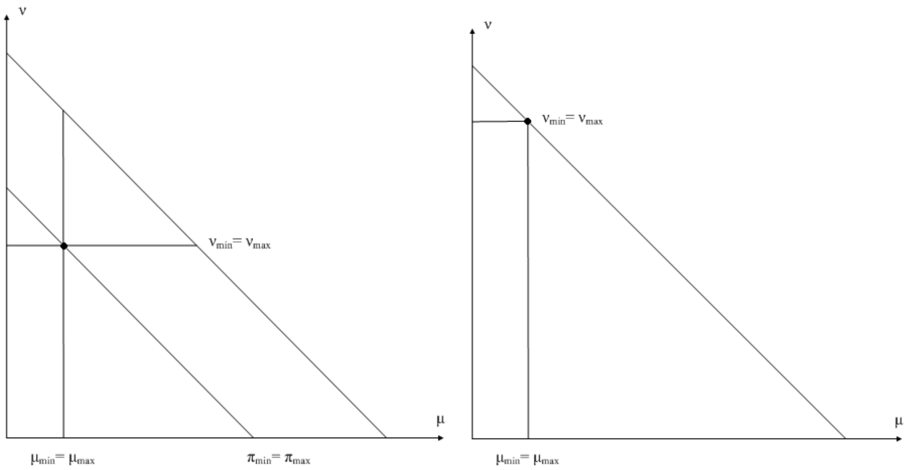

Fig.7a and 7b

All elements of this class satisfy:

$$\mu_{\min} = \mu_{\max}, \nu_{\min} = \nu_{\max}, \pi_{\min} = \pi_{\max}$$

and

$$V_X = 1 .$$

Fig.7a and 7b illustrate two examples of IFS in the case

$$\pi_{\min} = \pi_{\max} = 0 .$$

3.2. C_1 – segment

Elements of this class has two equal degrees. For example:

$$\mu_{\min} = \mu_{\max} \text{ (see Fig. 8a) and } \pi_{\min} = \pi_{\max} \text{ (see Fig. 8b).}$$

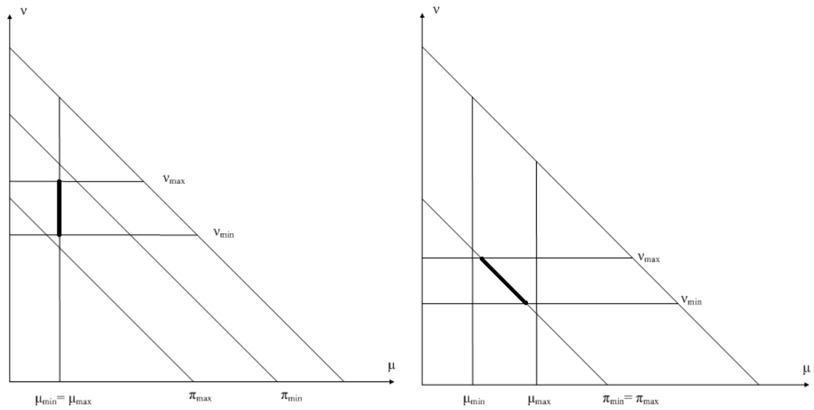

Fig. 8a and 8b.

3.3. C_2 – triangle

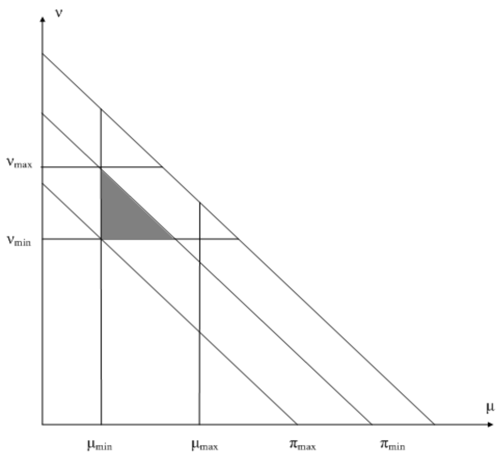

Fig.10. C_2 Triangle

3.4. C_3 – quadrangle

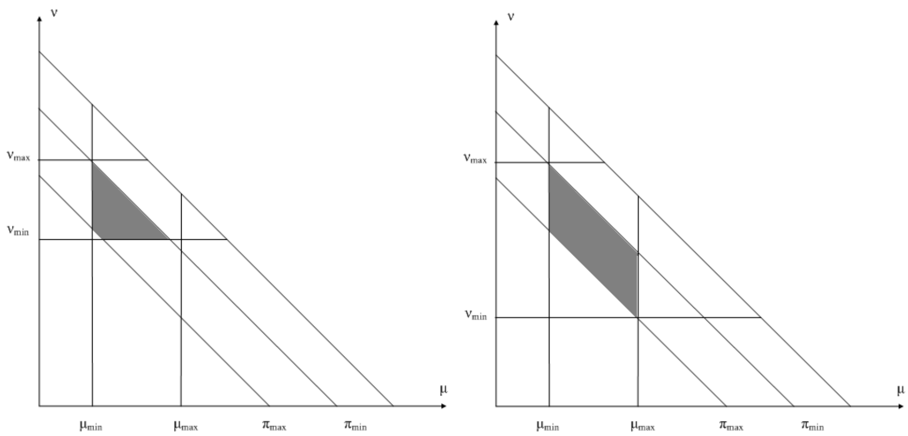

Fig.11. C_3 Quadrangle

3.5. C_4 – pentagon

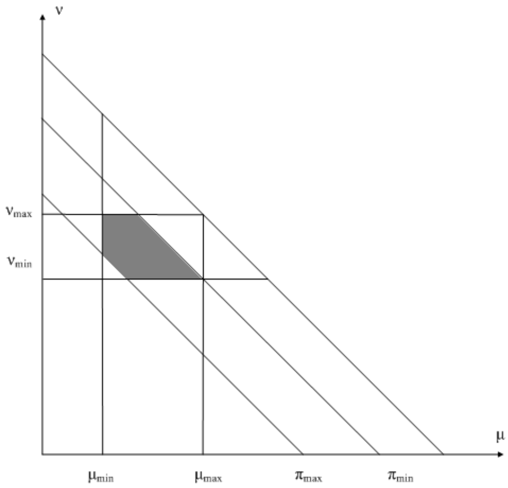

Fig.12. C_4 Pentagon

3.6. C_5 – hexagon

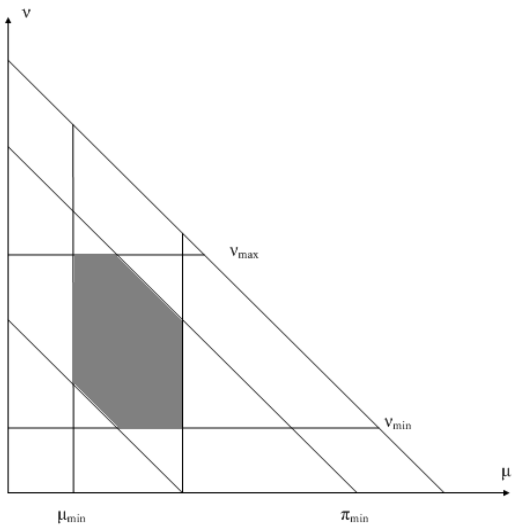

Fig.13. C_5 Hexagon

4. Conclusions

The work considers the elements of an arbitrary IFS as points in the plane and applies the IT geometrical interpretation. A new classification of IF sets is proposed based on the closed convex polygon inside the IT which contains all elements of the set. There are six classes defined - $C_0,...,C_5$ each of them containing sets with the same "*shape*". An interesting approach for future work is to explore the logical properties of the defined classes. For example, an important question would be which are the operations over IFS preserving the "shape" of IF sets.

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