First Int. Conf. on IFS, Sofia, 18-19 Oct. 1997
NIFS 3 (1997) 4, 44-49

INTUITIONISTIC FUZZY FEATURE–VALUE OPERATIONS

Nikolai Nikolov
Centre of Biomedical Engineering — Bulgarian Academy of Sciences,
Bl. 105, Acad. G. Bonchev str., 1113 Sofia, BULGARIA
e-mail: ngn@bgcict.acad.bg

Feature–value structures

Feature–value structures (FS) are sort of finite functions, namely, ones whose range intersects with the set of themselves.

Examples:

$$f = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 5 & 25 \end{bmatrix}$$

This was a simple FS ranging over the set of natural numbers.

$$f_1 : \begin{bmatrix} 1 & 1 \\ 2 & f_2 : \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \\ 5 & 25 \end{bmatrix}$$

A FS, f_1, defined as $f_1(1) = 1, f_1(2) = f_2, f_1(5) = 25$, where f_2 is in turn a FS defined as shown in the example. What was important in this example was that a FS, in this case f_1, may have values that are themselves FS's.

A more formal definition follows.

Let a finite set D and a set M be given. Their elements are regarded as elementary.

Definition. Either of the following is a feature–value structure:

1. A function f satisfying the following conditions:

   (a) $\mathrm{dom} f \subseteq D$

(b) $\forall d \in \mathrm{dom} f : f(d) \in M$.

2. A function f satisfying the conditions:

   (a) $\mathrm{dom} f \subseteq D$

   (b) $\forall d \in \mathrm{dom} f : f(d) = \varphi_i$, where $\{\varphi_i\}_{i \in \mathrm{dom} f}$ are such that φ_i is a feature structure or $\varphi_i \in M$, $i \in \mathrm{dom} f$.

The set of all feature–value structures (FS) for fixed D, M will be denoted by $FS_{D,M}$. When D and M are unique in the particular context, we will write FS instead of $FS_{D,M}$.

Complexity ν of a given FS f is defined as:

$$\nu(f) = \begin{cases} 0, \text{ if } \mathrm{dom} f = \emptyset \\ 1 + \max_{d \in D, f(d) \in FS} \nu(f(d)) \end{cases}$$

The set of all FS of complexity not greater than k will be denoted by FS^k.

Relations between feature-value structures

Partial order

Relation of partial ordering can be introduced in the following way:

Let f, g be FS and $\mathrm{dom} f \subset \mathrm{dom} g$ so that $\forall d \in \mathrm{dom} f : f(d) = g(d)$. Then $f \preceq g$.

The so defined relation is reflexive, anti-symmetric and transitive.

Empty FS ($\emptyset$) has the property of being minimal: $\forall f : \emptyset \preceq f$.

The greater (by this order) FS is obviously more informative, as it bears all information contained in smaller FS, plus extra information.

However a comparison of two FS is not always possible.

Unification of feature–value structures

Informally, the idea of unification of FS is the following: from two structures bearing some information to obtain a newer structure to bear the combined information from the two. The resulting FS must not be less informative than any of the two original FS.

Therefore, if two FS f and g are given, their unification (to be denoted by $f \sqcup g$) will give the minimal (by the partial order) FS h such that $f \preceq h$ and $g \preceq h$.

Now we will define the unification operation.

Let $f \in FS_{D_f, M_f}, g \in FS_{D_g, M_g}, \mathrm{dom} f = D_f, \mathrm{dom} g = D_g$ and

$$\forall x \in D_f \cap D_g :$$
$$(f(x) \in M \vee g(x) \in M) \Rightarrow (f(x) \in M) \& (g(x) \in M) \& (f(x) = g(x)).$$

Then $h = f \sqcup g \in FS_{D_f \cup D_g, M_f \cup M_g}, \mathrm{dom} h = D_f \cup D_g$ and $\forall x \in \mathrm{dom} h$:

$$h(x) = \begin{cases} f(x), & \text{if } x \in D_f - D_g, \\ g(x), & \text{if } x \in D_g - D_f, \\ f(x), & \text{if } x \in D_f \cap D_g \text{ and } f(x) \in M_f, \\ f(x) \sqcup g(x), & \text{if } x \in D_f \cap D_g, f(x) \in FS \text{ and } g(x) \in FS \end{cases}$$

In the case when $x \in D_f \cap D_g$ and $f(x) \in M_f, g(x) = f(x)$, so we choose $h(x) = f(x)$.

Intuitionistic Fuzzy FS

Definition. An intuitionistic fuzzy value (ifv) will be called any $\delta \in [0,1] \times [0,1]$ such that $\sum_{i=1}^{2} \mathrm{pr}_i \delta \leq 1$. The set of all ifv's will be denoted by IFV.

Following [1], we will say of an ifv $\langle \mu, \nu \rangle$ that its *truth degree* is μ and that its *falsity degree* is ν.

Definition. An intuitionistic fuzzy plausibility resolvent (ifpr) of two ifv α, β is an ifv $\langle \mu, \nu \rangle$ defined through

$$\mu = \min\{pr_1\alpha, pr_1\beta\}$$
$$\nu = \max\{pr_2\alpha, pr_2\beta\}$$

Definition. Let a finite set D and a set M be given. Either of the following is an intuitionistic fuzzy feature–value structure (IFFS):

1. A function f satisfying the following conditions:
   (a) $\mathrm{dom} f \subseteq D$
   (b) $\forall d \in \mathrm{dom} f : f(d) \in M \times IFV$.

2. A function f satisfying the conditions:
   (a) $\mathrm{dom} f \subseteq D$
   (b) $\forall d \in \mathrm{dom} f : f(d) = \varphi_i$, where $\{\varphi_i\}_{i \in \mathrm{dom} f}$ are such that φ_i is a feature structure or $\varphi_i \in M \times IFV$, $i \in \mathrm{dom} f$.

Notations $IFFS_{D,M}$ and $IFFS$ are introduced in the obvious way.

Intuitionistic fuzzy feature–structure unification

Similarly, the unification will give the minimal IFFS to bear the combined information from the two given IFFS.

Definition. Let $f \in IFFS_{D_f, M_f}, g \in IFFS_{D_g, M_g}$, $\mathrm{dom} f = D_f$, and $\mathrm{dom} g = D_g$.

Then $h = f \sqcup g \in FS_{D_f \cup D_g, M_f \cup M_g}$, $\mathrm{dom}h = D_f \cup D_g$ and $\forall x \in \mathrm{dom}h$:

$$h(x) = \begin{cases} f(x), & \text{if } x \in D_f - D_g, \\ g(x), & \text{if } x \in D_g - D_f, \\ \langle pr_1 f(x), \mathrm{ifpr}(pr_2 f(x), pr_2 g(x))\rangle & \text{if } x \in D_f \cap D_g \text{ and } pr_1 f(x) = pr_1 g(x) \in M \\ \langle pr_1 f(x), \mathrm{ifpr}(pr_2 f(x), pr_2 g(x))\rangle & \text{if } x \in D_f \cap D_g \\ & \text{and } (pr_1 f(x) \in M \text{ or } pr_1 g(x) \in M) \\ & \text{and } pr_1 f(x) \neq pr_1 g(x) \\ & \text{and } \mathrm{ifpr}(pr_2 f(x), pr_2 g(x)) = pr_2 f(x) \\ \langle pr_1 g(x), \mathrm{ifpr}(pr_2 f(x), pr_2 g(x))\rangle & \text{if } x \in D_f \cap D_g \\ & \text{and } (pr_1 f(x) \in M \text{ or } pr_1 g(x) \in M) \\ & \text{and } pr_1 f(x) \neq pr_1 g(x) \\ & \text{and } \mathrm{ifpr}(pr_2 f(x), pr_2 g(x)) = pr_2 g(x) \\ \text{undefined}, & \text{if } x \in D_f \cap D_g \\ & \text{and } (pr_1 f(x) \in M \text{ or } pr_1 g(x) \in M) \\ & \text{and } pr_1 f(x) \neq pr_1 g(x) \\ & \text{and } \mathrm{ifpr}(pr_2 f(x), pr_2 g(x)) \neq pr_2 f(x) \\ & \text{and } \mathrm{ifpr}(pr_2 f(x), pr_2 g(x)) \neq pr_2 g(x) \\ f(x) \sqcup g(x), & \text{if } x \in D_f \cap D_g, f(x) \in FS \text{ and } g(x) \in FS \end{cases}$$

In the case when $x \in D_f \cap D_g$ and $f(x) \in M_f, g(x) = f(x)$, so we choose $h(x) = f(x)$.

Feature–value structures containing variables

Let the following be given: a finite set D, a set M_c of constant symbols and a set M_v of variable symbols. Let $M = M_c \cup M_v$.

A feature–value structure containing variables (VFS) will be called a function f satisfying the conditions:

1. $\mathrm{dom} f \subseteq D$
2. $\forall d \in \mathrm{dom} f$: either $(f(d) \in M)$ or $f(d)$ is a VFS).

The set of all VFS for fixed D, M_c, M_v will be denoted by VFS_{D,M_c,M_v}.

The unification operation defined above for FS's (the abbreviation FS will keep its meaning for feature–value structures without variables) will be used below in the same sense and only applies over FS's. For VFS, a new operation will be defined, called *matching*, which will resemble the well-known unification of terms. For this purpose, first we will introduce the notion of a substitution.

- *A substitution* for a given set VFS_{D,M_c,M_v} will be called any element of the set $FS^1_{M_v, M_c \cup FS_{D,M_c}}$
- We will define the following operation: *application of a substitution σ to a given VFS $F \in VFS_{D,M_c,M_v}$*: $\sigma \sqcap F$.

  The result of this operations and the pre-requisites for its existence are as follows:

$\sigma \sqcap F = h \in FS_{D,M_c}$:

$$\forall d_i \in D : h(d_i) = \begin{cases} f(d_i), \text{ if } f(d_i) \in M_c; \\ \sigma(f(d_i)), \text{ if } f(d_i) \in M_v \text{ and } \sigma(f(d_i)) \text{ is defined}; \\ \sigma \sqcap f(d_i), \text{ if } f(d_i) \in VFS \end{cases}$$

- Let a FS F and a VFS G be given. *Matching* $F \sqcup^+ G$ of F and G is the operation giving the least substitution σ (if one exists) such that $\sigma \sqcap G = F$.

  The matching operation can be introduced by the following definition: (it is easy to demonstrate that the result of the unification will have the above property):

  Let $F \in FS_{D,M_c}, G \in VFS_{D,M_c,M_v}$ and dom$G \subseteq$ dom$F$.

  1. If dom$G = \phi, F \sqcup^+ G = \phi$.
  2. If dom$G \neq \phi$ and for every $d \in$ dom$G$, the expression

  $$\sigma_d = \begin{cases} [G(d)F(d)], \text{ if } G(d) \in M_v; \\ \varphi, \text{ if } G(d) \in M_c, F(d) \in M_c \text{ and } F(d) = G(d); \\ G(d) \sqcup^+ F(d), \text{ if } G(d) \in VFS \text{ and } F(d) \in VFS \end{cases}$$

  is defined and $\bigsqcup_{d \in \text{dom}G} \sigma_d$ exists, then

  $$F \sqcup^+ G = \bigsqcup_{d \in \text{dom}G} \sigma_d$$

Examples:

a) $$\begin{bmatrix} 1 & 1 \\ 2 & \begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix} \\ 5 & 50 \end{bmatrix} \sqcup^+ \begin{bmatrix} 1 & x \\ 2 & \begin{bmatrix} 10 & 20 \\ 20 & y \end{bmatrix} \\ 5 & 50 \end{bmatrix} \text{ is the substitution } \begin{bmatrix} x & 1 \\ y & 40 \end{bmatrix}$$

b) $$\begin{bmatrix} x & 100 \\ y & 300 \end{bmatrix} \sqcap \begin{bmatrix} 10 & x \\ 20 & 200 \\ 30 & y \end{bmatrix} = \begin{bmatrix} 10 & 100 \\ 20 & 200 \\ 30 & 300 \end{bmatrix}$$

Intuitionistic fuzzy feature–value structures containing variables

A slight revision is needed only in the definition of a VFS to obtain a working mechanism.

An intuitionistic fuzzy feature–value structure containing variables (IFVFS) will be called a function f satisfying the conditions:

1. dom$f \subseteq D$
2. $\forall d \in$ dom$f$: either $(f(d) \in M_v)$ or $(f(d) \in M_c \times IFV)$ or $f(d)$ is an IFVFS).

The set of all VFS for fixed D, M_c, M_v will be denoted by VFS_{D,M_c,M_v}.
Again, IF-unification remains only for IFFS's.

- *An intuitionistic fuzzy (IF-) substitution* for a given $IFVFS_{D,M_c,M_v}$ will be called any element of the set $IFFS^1_{M_v,M_c \cup IFFS_{D,M_c}}$.
- We will define the following operation: *application of an IF-substitution σ to a given IFVFS $F \in VFS_{D,M_c,M_v}$*: $\sigma \sqcap F$.

  This operation is defined exactly like the crisp substitution application.

  Moreover, the definition of matching is also in one-to-one correspondence to the above one.

  The rewritten definition of VFS, now IFVFS, and unification (which participates in the matching definition) makes all the difference.

Example:

$$[\ 2 \quad \langle 4, \langle \tfrac{1}{2}, \tfrac{1}{2} \rangle\rangle\ ] \sqcup^+ [\ 2 \quad x\ ] = [\ x \quad \langle 4, \langle \tfrac{1}{2}, \tfrac{1}{2} \rangle\rangle\ ]$$

The so constructed intuitionistic fuzzy extension of feature structures is correctly defined, in the sense that the definitions coincide with the crisp case ones when one takes all associated ifv's to be $\langle 1, 0 \rangle$.

This is, of course, not the only possible approach to building an IF externsion of feature structure theory, though it seems in a way the most natural one. Other approaches, some of them even leading to non-functionality of the resulting FS's, are currently studied and will be the object of subsequent papers by the author.

References

[1] Atanassov, K., *Intuitionistic fuzzy sets*, Fuzzy sets and Systems, 20 (1986) 87-96.

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[3] Nikolov, N., *Situation-semantical Analysis of Natural Language Fragment*, M. Sc. Thesis, Dept. of Information Technologies, Faculty of Math. & Comp. Sci., Sofia University, November 1995.