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Isomorphism theorems for intuitionistic fuzzy submodules of *G*-modules

P. K. Sharma

P. G. Department of Mathematics, D. A. V. College
Jalandhar City, Punjab, India
e-mail: pksharma@davjalandhar.com

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Abstract: The concept of intuitionistic fuzzy *G*-modules and their properties are defined and discussed by the author et al. in [11]. In this paper, we give three fundamental theorems of isomorphism for intuitionistic fuzzy submodules of *G*-modules.
Keywords: Intuitionistic fuzzy *G*-submodule, Quotient *G*-modules, Intuitionistic fuzzy isomorphism theorem.
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1 Introduction

The concept of intuitionistic fuzzy sets was introduced by K. T. Atanossov [1, 2] as a generalization to the notion of fuzzy sets by L.A. Zedah [16]. R. Biswas was the first to introduce the intuitionistic fuzzification of algebraic structure and developed the concept of intuitionistic fuzzy subgroup of a group in [4]. Later on many mathematicians worked on it and introduced the notion of intuitionistic fuzzy subring, intuitionistic fuzzy submodule etc. (see [6–10]). The notion of intuitionistic fuzzy *G*-modules was introduced by the author et al. in [11]. Many properties like representation, reducibility, complete reducibility and injectivity of intuitionistic fuzzy *G*-modules have been discussed in [11–13]. The idea, for proving these results came from Meena and Thomas [8], Sinha and Dewangan [15], which was originally proved for intuitionistic fuzzy L-rings and fuzzy submodules of *G*-modules respectively.

Throughout this article, concepts related with *G*-modules are mainly taken from [5] and concepts related with intuitionistic fuzzy set theory are taken from [1–3, 7–11].

2 Preliminaries

For proving the isomorphism theorems for intuitionistic fuzzy submodules of G-modules, we use the following definitions and results.

Definition (2.1)[5] Let G be a group and M be a vector space over a field K. Then M is called a G-***module*** if for every $g\in G$ and $m\in M$, there exists a product (called the action of G on M), $gm\in M$ satisfies the following axioms

- i) $1_G\cdot m = m, \forall m\in M$ (1_G being the identity of G)
- ii) $(g\cdot h)\cdot m = g\cdot(h\cdot m), \ \forall m\in M, g, h\in G$
- iii) $g\cdot(k_1m_1 + k_2m_2) = k_1(g\cdot m_1) + k_2(g\cdot m_2), \ \forall k_1, k_2\in K; m_1, m_2\in M$ and $g\in G$.

Since G acts on M on the left hand side, M may be called a **left *G*-module**. In a similar way, we can define a **right *G*-module**. But here we shall consider only leftmodules. *A* parallel study is possible using right G-modules also.

Definition (2.2) [5] Let M be a G-module. *A* vector subspace N of M is a ***G-submodule*** if N is also a G-module under the same action of G.

Proposition (2.3) [5] If M is a G-module and N is a G-submodule of M, then M/N is a G-module which is called ***Quotient G-modules.***

Definition (2.4) [5] Let M and M^* be G-modules. *A* mapping $f : M \to M^*$ is a***G-module homomorphism*** if

- (i) $f(k_1m_1 + k_2m_2) = k_1f(m_1) + k_2f(m_2)$
- (ii) $f(gm) = gf(m), \forall k_1, k_2 \in K; m, m_1, m_2 \in M$ and $g \in G$.

Definition (2.5) [5] Let $f : M \to M^*$ is a G-module homomorphism. Then $\ker f = \{m\in M : f(m) = 0^*\}$is a G-submodule of M and $\operatorname{img} f = \{ f(m) : m\in M\}$ is a G-submodule of M^*.

Definition (2.6) [2] Let X be a non-empty set. An ***intuitionistic fuzzy set*** (IFS) A of X is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where $\mu_A: X\to [0,1]$ and $\nu_A : X\to [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$, respectively, and for any $x \in X$, we have $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition (2.7) [2] Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$ be any two IFSs of X, then

- (i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x\in X$;
- (ii) $A = B$ if and only if$\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x\in X$;
- (iii) $A^c= \{\langle x, (\mu_A{}^c)(x) , (\nu_A{}^c)(x)\rangle: x\in X\}$, where$(\mu_A{}^c)(x) = \nu_A(x)$ and $(\nu_A{}^c)(x) = \mu_A(x)$ for all $x\in X$;
- (iv) $A\cap B = \{\langle x, (\mu_{A\cap B})(x) , (\nu_{A\cap B})(x)\rangle : x\in X\}$, where $(\mu_{A\cap B})(x) = \mu_A(x) \wedge \mu_B(x)$ and $(\nu_{A\cap B})(x) = \nu_A(x) \vee \nu_B(x)$

(v) $A \cup B = \{\langle x, (\mu_{A\cup B})(x), (\nu_{A\cup B})(x)\rangle : x \in X\}$, where $(\mu_{A\cup B})(x) = \mu_A(x) \vee \mu_B(x)$ and $(\nu_{A\cup B})(x) = \nu_A(x) \wedge \nu_B(x)$.

Definition (2.8) [2, 3, 7, 9] Let X and Y be two non-empty sets and f: X → Y be a mapping. Let A and B be IFSs of X and Y respectively. Then the ***image of A*** under the map $f$is denoted by $f(A)$ and is defined as$f(A)(y) = (\mu_{f(A)}(y), \nu_{f(A)}(y))$, where

$$\mu_{f(A)}(y) = \begin{cases} \vee\{ \mu_A(x) : x \in f^{-1}(y)\}; & if\ f^{-1}(y) \neq \emptyset \\ 0 & ; \quad \text{otherwise} \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \wedge\{ \nu_A(x) : x \in f^{-1}(y)\}; & if\ f^{-1}(y) \neq \emptyset \\ 1 & ; \quad \text{otherwise} \end{cases}, \ \forall\, y \in Y.$$

Also the ***pre-image of B*** under f is denoted by $f^{-1}(B)$ and is defined as

$$f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)), \text{ where}$$
$$\mu_{f^{-1}(B)}(x) = \mu_B(f(x)) \quad \text{and} \quad \nu_{f^{-1}(B)}(x) = \nu_B(f(x)); \ \forall\, x \in X.$$

Remark (2.9) In general, $\mu_{f(A)}(f(x)) \geq \mu_A(x)$ and $\nu_{f(A)}(f(x)) \leq \nu_A(x)$ and equality holds iff is one-one.

Definition (2.10) [10] Let $(X, .)$ be a groupoid and A, B be two IFSs of X. Then the ***intuitionistic fuzzy sum*** of A and B is denoted by $A + B$ and is defined as:$(A + B)(x) = (\mu_{A+B}(x), \nu_{A+B}(x))$, where

$$\mu_{A+B}(x) = \begin{cases} \vee\{\mu_A(a) \wedge \mu_B(b)\} & ; if\ x = a + b \\ 0 & ; \text{ otherwise} \end{cases}$$

and

$$\nu_{A+B}(x) = \begin{cases} \wedge\{\nu_A(a) \vee \nu_B(b)\} & ; if\ x = a + b \\ 1 & ; \text{ otherwise} \end{cases}; \forall\, x \in X.$$

Definition (2.11) [3, 9, 11] For any IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ of set X. We denote the ***support*** of the IFS set A by A^* and is defined as $A^* = \{ x \in X : \mu_A(x) > 0 \text{ and } \nu_A(x) < 1 \}$.

Proposition (2.12) [9] Let f : X → Y be a mapping and A, B are IFSs of X and Y respectively. Then the following result holds

(*i*) $f(A^*) \subseteq (f(A))^*$ and equality hold when the map f is bijective

(*ii*) $f^{-1}(B^*) = (f^{-1}(B))^*$.

Definition (2.13) [11] Let $A\in G^M$ (where G^M denotes the intuitionistic fuzzy power set of G-module M). Then A is called an ***intuitionistic fuzzy submodule of G-module M***, if it satisfies the following:

(i) $\mu_A(0) = 1$ and $\nu_A(0) = 0$;

(ii) $\mu_A(gm) \geq \mu_A(m)$ and $\nu_A(gm) \leq \nu_A(m)$, $\forall g\in G$, $m\in M$;

(iii) $\mu_A(m_1+m_2) \geq \mu_A(m_1)\wedge\mu_A(m_2)$ and $\nu_A(m_1+m_2) \leq \nu_A(m_1)\vee\nu_A(m_2)$, $\forall m_1, m_2\in M$.

We denote the set of all intuitionistic fuzzy submodules of G-module M by $G(M)$.

Theorem (2.14) [11] Let $A\in G(M)$. Then A^* is a G-submodule of M.

Theorem (2.15) [3, 11] For any A, $B\in G(M)$, we have

$$(A + B)^* = A^* + B^* \text{ and } (A\cap B)^* = A^*\cap B^* .$$

Definition (2.16) [8] Let A, $B\in G(M)$. Then A is said to be ***normal*** in B if $A \subseteq B$ and

$$\mu_A(yxy^{-1}) \geq \mu_A(x) \wedge \mu_B(y) \text{ and } \nu_A(yxy^{-1}) \leq \nu_A(x) \vee \nu_B(y) \text{ for all } x, y\in M.$$

Theorem (2.17) [11] Let $A\in G(M)$ and let N be a G-submodule of M. Define $A|_N\in G^N$ (where G^N is the intuitionistic fuzzy power set of G-module N) as follows:
$\mu_{A|_N}(x) = \mu_A(x)$ and $\nu_{A|_N}(x) = \nu_A(x)$, $\forall x\in N$. Then, $A|_N\in G(N)$.

Theorem (2.18) [11] Let $A\in G(M)$ and let N be a G-submodule of M. Define $A_N \in G^{(M/N)}$ as follows:

$$\mu_{A_N}(x + N) = \vee\{\mu_A(x + n) : n\in N\}$$

and

$$\nu_{A_N}(x + N) = \wedge\{\nu_A(x + n) : n\in N\}, \forall x\in M,$$

where M/N denote the quotient module of M with respect to N. Then $A_N\in G(M/N)$.

Let A, $B\in G(M)$ be such that $A \subseteq B$. It is known that both $A^*$and $B^*$are G-submodules of M. Clearly, $A^*\subseteq B^*$. Thus $A^*$is a G-submodule of B^*. Moreover it is clear that $B|_{B^*} \in G(B^*)$.

Therefore, it follows from Theorem (2.18) that if we define $C\in G^{(B^*/A^*)}$as follows: $C(x+A^*) = (\vee\{\mu_B(x+y) : y\in A^*\}, \wedge\{\nu_B(x+y) : y\in A^*\})$, $\forall x\in B^*$. Then $C \in G(B^*/A^*)$ is called the quotient of B w.r.t. A and is written as B/A, i.e., $C = B/A$.

Definition (2.19) Let A, $B\in G(M)$ be such that $A \subseteq B$. Then $B/A \in G(B^*/A^*)$ is called the ***quotient of B with respect to A*** and is defined as $B/A(x+A^*) = (\mu_{B/A}(x+A^*), \nu_{B/A}(x+A^*))$, where $\mu_{B/A}(x+A^*) = \vee\{\mu_B(x+y) : y\in A^*\}$ and $\nu_{B/A}(x+A^*) = \wedge\{\nu_B(x+y) : y\in A^*\}$, where $x\in B^*$.

Theorem (2.20) Let A, $B\in G(M)$ be such that $A \subseteq B$. Then $(B|_{B^*})_{A^*} = B/A$.

Proof. Since A and B are intuitionistic fuzzy submodules of G-module M such that $A \subseteq B$. Therefore A^* and B^* are G-submodules of M such that $A^* \subseteq B^*$. Clearly, both $\left(B|_{B^*}\right)_{A^*}$ and B/A are intuitionistic fuzzy submodules of G-module B^*/A^*.

Let $x + A^* \in B^*/A^*$ be any element, where $x \in B^*$. Then we have

$$\mu_{\left(B|_{B^*}\right)_{A^*}}\left(x + A^*\right) = \vee\left\{\mu_{\left(B|_{B^*}\right)}(x+y): \ y \in A^*\right\} = \vee\left\{\mu_B(x+y): \ y \in A^*\right\} = \mu_{B/A}\left(x + A^*\right).$$

Similarly, we can show that $\nu_{\left(B|_{B^*}\right)_{A^*}}\left(x + A^*\right) = \nu_{B/A}\left(x + A^*\right)$. Hence, $\left(B|_{B^*}\right)_{A^*} = B/A$. □

3 Isomorphism theorems for intuitionistic fuzzy submodules of *G*-modules

In this section, we give three fundamental theorems of isomorphism for the intuitionistic fuzzy submodules of G-modules.

Definition (3.1)[11]Let M and M^* be G-modules and let A,B be two intuitionistic fuzzy G-submodules on M and M^* respectively. Let $f: M \to M^*$ be a G-module homomorphism. Then f is called a ***weak intuitionistic fuzzy G-homomorphism*** of A into B if $f(A) \subseteq B$. The homomorphism f is an intuitionistic fuzzy G-homomorphism of A onto B if $f(A) = B$. We say that A is an intuitionistic fuzzy G-homomorphic onto B and we write it as $A \approx B$.

Let $f: M \to M^*$ be a G-module isomorphism. Then f is called a ***weak intuitionistic fuzzy G-isomorphism*** if $f(A) \subseteq B$ and f is an ***intuitionistic fuzzy G-isomorphism*** if $f(A) = B$ and we write it as $A \cong B$.

Theorem (3.2)(First isomorphism theorem) Let $A \in G(M)$ and $B \in G(N)$ such that $A \approx B$. Then there exists $C \in G(M)$ such that $C \subseteq A$ and $A/C \cong B$.

Proof. Since $A \approx$ B there exists a G-epimorphism $f: M \to N$ such that $f(A) = B$.

Define $C \in G^M$ as follows:

$$\mu_C(x) = \begin{cases} \mu_A(x) & if \ x \in \ker f \\ 0 & if \ x \notin \ker f \end{cases} \quad \text{and} \quad \nu_C(x) = \begin{cases} \nu_A(x) & if \ x \in \ker f \\ 1 & if \ x \notin \ker f \end{cases}.$$

Then, it is easy to see that $C \in G(M)$ and $C \subseteq A$. If $x \in \ker f$, then $yxy^{-1} \in \ker f$, $\forall y \in M$.

$$\mu_C(yxy^{-1}) = \mu_A(yxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y) = \mu_C(x) \wedge \mu_A(y),$$

i.e.,

$$\mu_C(yxy^{-1}) \geq \mu_C(x) \wedge \mu_A(y).$$

Similarly, we can show that $\nu_C(yxy^{-1}) \leq \nu_C(x) \vee \nu_A(y)$.If $x \notin \ker f$, then $\mu_C(x) = 0$ and $\nu_C(x) = 1$ and so $\mu_C(yxy^{-1}) \geq \mu_C(x) \wedge \mu_A(y)$ and $\nu_C(yxy^{-1}) \leq \nu_C(x) \vee \nu_A(y)$ is obviously true. So, C is a normal G-submodule of A. Also, $A \overset{f}{\approx} B$ this implies that $f(A) = B$ and hence $(f(A))^* = B^*$.

It implies that $f(A^*)=B^*$.Let $G = f|_{A^*}$, then g: $A^* \to B^*$ is a G-homomorphism onto with ker $G = C^*$.Then there exists a G-isomorphism h: $A^*/C^* \to B^*$ such that $h(x+C^*) = g(x) = f(x)$, $\forall x \in A^*$.

For such an h, we have $h(A/C)(z) = \left(\mu_{h(A/C)}(z),\ \nu_{h(A/C)}(z)\right)$, where

$\mu_{h(A/C)}(z) = \vee\{\mu_{A/C}(x+C^*) : x \in A^*, h(x+C^*) = z\}$ and $\nu_{h(A/C)}(z) = \wedge\{\nu_{A/C}(x+C^*) : x \in A^*, h(x+C^*) = z\}$.

Now,
$$
\begin{aligned}
\mu_{h(A/C)}(z) &= \vee\{\mu_{A/C}(x+C^*)\ :\ x \in A^*\ ,\ h(x+C^*) = z\} \\
&= \vee\{\vee\{\mu_A(y) : y \in x+C^*\} : x \in A^*,\ g(y) = z\} \\
&= \vee\{\mu_A(y) : y \in A^*,\ g(y) = z\} \\
&= \vee\{\ \mu_A(y) : y \in M, f(y) = z\} \\
&= \mu_{f(A)}(z) \\
&= \mu_B(z),\ \forall\ z \in B^*
\end{aligned}
$$
and
$$
\begin{aligned}
\nu_{h(A/C)}(z) &= \wedge\{\nu_{A/C}(x+C^*)\ :\ x \in A^*,\ h(x+C^*) = z\} \\
&= \wedge\{\wedge\{\nu_A(y) : y \in x+C^*\} : x \in A^*,\ g(y) = z\} \\
&= \wedge\{\nu_A(y) : y \in A^*,\ g(y) = z\} \\
&= \wedge\{\ \nu_A(y) : y \in M, f(y) = z\} \\
&= \nu_{f(A)}(z) \\
&= \nu_B(z),\ \forall\ z \in B^*.
\end{aligned}
$$
Thus, $h(A/C) = B$. Hence, $A/C \overset{h}{\cong} B$. □

Theorem (3.3) (Second isomorphism theorem) Let A, B be intuitionistic fuzzy submodules of G-module M, then $B/(A \cap B) \cong (A+B)/A$.

Proof. We know that A^* and B^* are G-submodules of M. By the second isomorphism theorem for modules, we have $B^*/\left(A^* \cap B^*\right) \cong \left(A^*+B^*\right)/A^*$ i.e., $B^*/\left(A^* \cap B^*\right) \cong \left(A+B\right)^*/A^*$.Let $B^*/(A \cap B)^* \overset{f}{\cong} (A+B)^*/A^*$, where f is given by $f\left(x+(A \cap B)^*\right) = x + A^*$, $\forall\ x \in B^*$. Now,

$$
\begin{aligned}
\mu_{f\left(\frac{B}{A\cap B}\right)}\left(x+A^*\right) &= \mu_{\left(\frac{B}{A\cap B}\right)}\left(x+\left(A\cap B\right)^*\right) \quad \text{[Since } f \text{ is one-one]} \\
&= \vee\left\{\mu_B(z) :\ z \in x+\left(A\cap B\right)^*\right\} \\
&= \vee\left\{\mu_{A+B}(z) :\ z \in x+\left(A\cap B\right)^*\right\} \\
&\le \vee\left\{\mu_{A+B}(z) :\ z \in x+A^*\right\} \\
&= \mu_{\left(\frac{A+B}{A}\right)}\left(x+A^*\right),\ \forall\ x \in B^*
\end{aligned}
$$

and

$$\nu_{f\left(\frac{B}{A\cap B}\right)}\left(x+A^*\right)=\nu_{\left(\frac{B}{A\cap B}\right)}\left(x+\left(A\cap B\right)^*\right) \quad \text{[Since } f \text{ is one-one]}$$

$$=\wedge\left\{\nu_B(z):\ z\in x+\left(A\cap B\right)^*\right\}$$

$$=\wedge\left\{\nu_{A+B}(z):\ z\in x+\left(A\cap B\right)^*\right\}$$

$$\geq\wedge\left\{\nu_{A+B}(z):\ z\in x+A^*\right\}$$

$$=\nu_{\left(\frac{A+B}{A}\right)}\left(x+A^*\right),\ \forall\ x\in B^*.$$

Thus, $f\left(\dfrac{B}{A\cap B}\right)\subseteq\left(\dfrac{A+B}{A}\right)$.

Hence, $\left(\dfrac{B}{A\cap B}\right)\cong\left(\dfrac{A+B}{A}\right)$ (weak intuitionistic fuzzy *G*-isomorphism). □

Theorem (3.4)(Third isomorphism theorem) Let *A, B, C* be intuitionistic fuzzy *G*-submodules of *M* with $A\subseteq B\subseteq C$, then $\dfrac{(C/A)}{(B/A)}\cong\dfrac{C}{B}$.

Proof. Since $A\subseteq B\subseteq C$, then A^* is a *G*-submodule of B^* and both A^* and B^* are *G*-submodules of C^*. Then by third isomorphism theorem for modules $\dfrac{(C^*/A^*)}{(B^*/A^*)}\overset{f}{\cong}\dfrac{C^*}{B^*}$, where *f* is defined as $f(x+A^*+(B^*/A^*))=x+B^*,\ \forall x\in C^*$.

Now, $\mu_{f\left(\frac{C/A}{B/A}\right)}(x+B^*)=\mu_{\left(\frac{C/A}{B/A}\right)}\left(x+A^*+(B^*/A^*)\right)$ [Since *f* is one-one]

$$=\vee\left\{\mu_{\left(\frac{C}{A}\right)}(y+A^*):\ y\in C^*,\ y+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\vee\left\{\vee\{\mu_C(z):\ z\in y+A^*\}:\ y\in C^*,\ y+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\vee\left\{\mu_C(z):\ z\in C^*, z+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\vee\left\{\mu_C(z):\ z\in x+A^*+(B^*/A^*)\right\}$$

$$=\vee\left\{\mu_C(z):\ z\in C^*, f(z)\in x+B^*\right\}$$

$$=\mu_{\left(\frac{C}{B}\right)}(x+B^*),\ \forall\ x\in C^*$$

and $\nu_{f\left(\frac{C/A}{B/A}\right)}(x+B^*)=\nu_{\left(\frac{C/A}{B/A}\right)}\left(x+A^*+(B^*/A^*)\right)$ [Since *f* is one-one]

$$=\wedge\left\{\nu_{\left(\frac{C}{A}\right)}(y+A^*):\ y\in C^*,\ y+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\wedge\left\{\wedge\{\nu_C(z):\ z\in y+A^*\}:\ y\in C^*,\ y+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\wedge\left\{\nu_C(z):\ z\in C^*, z+A^*\in x+A^*+(B^*/A^*)\right\}$$

$$=\wedge\left\{\nu_C(z):\ z\in x+A^*+(B^*/A^*)\right\}$$

$$=\wedge\left\{\nu_C(z):\ z\in C^*, f(z)\in x+B^*\right\}$$

$$=\nu_{\left(\frac{C}{B}\right)}(x+B^*),\ \forall\ x\in C^*$$

Thus, $f\left(\frac{C/A}{B/A}\right)=\left(\frac{C}{B}\right)$ Hence, $\left(\frac{C/A}{B/A}\right)\cong\left(\frac{C}{B}\right)$. □

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