**Second International Workshop on IFSs**
**Banska Bystrica, Slovakia, 3 Dec. 2006**
**NIFS 12 (2006), 4, 12-21**

# LATTICE OF IFLs ON A SET

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**Abstract**

In this paper we introduce the notion of intuitionistic fuzzy sublattices of a lattice and study their properties .Also the lattice structure of the set of all intuitionistic fuzzy sublattices of a lattice is discussed.

## 1.Introdution

Krassimir T.Atnassov [2] introduced the concept of Intuitionistic Fuzzy seta as a generalization of Fuzzy sets and this notion has been adapted in many areas, for example Dogan Coker [5] initiated the idea of Intuitionistic Fuzzy topological spaces.

In this paper we introduce the notion of Intuitionistic fuzzy sublattices of a lattice and study their properties. Also the lattice structure of the set of all Intuitionistic fuzzy sublattices of a lattice is investigated. Some results on the Intuitionistic fuzzy ideals of a lattice and ideals of intuitionistic fuzzy lattices are introduced in another paper [6]

Subject Classification: 06D72,O3E72
Keywords: Lattice ,Sublattice,Fuzzy sets, Intuitionistic fuzzy sets, Fuzzy sublattice

## 2.Preliminaries

In this section we recall some definition and results that we will be needed in the sequel .For details see [2] ,[3] and [5]

**Definition 2.1**

Let X denote a fixed non empty set .An intuitionistic Fuzzy Set (IFS) A of X is an object of the form $A=\{<x ,\mu_A(x),\nu_A(x) > ,x \varepsilon X \}$ where $\mu_A: X \to [0\ 1]$ and $\nu_A :X \to [0\ 1]$ define the degree of membership and the degree of nonmember ship of the element $x \varepsilon X$ respectively , and for every $x \varepsilon X$ , $0 \le \mu_A(x) +\nu_A (x) \le 1$

**Remark 2.2**

An Intuitionistic fuzzy set $A =\{ <x, \mu_A(x),\nu_A(x)> : x \varepsilon X \}$ can be identified to an ordered pair $(\mu_{A,} \nu_A )$ in $I^X \times I^X$ .For the sake of simplicity we shall use the symbol $A =( \mu_{A,} \nu_A )$ for the IFS $A=\{<x ,\mu_A(x),\nu_A(x) > ,x \varepsilon X \}$ in X. ( The set of all intuitionistic fuzzy subsets of X is denoted by IFS(X) )

**Example 2.1**

Every fuzzy set $\mu_A$ on a nonempty set X is an IFS having the form $A=(\mu_A, 1-\mu_A)$ and every subset A of X is an IFS,Since $A=(\chi_A, 1-\chi_A)$

**Definition 2.2**

Let X be a nonempty set and the IFS's A and B be given by $A=(\mu_A, \nu_A)$ and $B=(\mu_B, \nu_B)$.Then $A \subseteq B$ ,iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ε X

It can be shown that the relation '⊆' is a partial ordering on IFS(X) and under this ordering IFS(X) is a complete lattice with $\cap A_i = (\cap \mu_{Ai}, \cup \nu_{Ai})$ and $\cup A_i = (\cup \mu_{Ai}, \cap \nu_{Ai})$.Its largest element is $I_x=(1,0)$ and smallest element is $0_x = (0,1)$ ,where $I_x(x)=1$ and $0_x(x)=0$ ∀ x ε X

**3.Intuitionistic Fuzzy sublattice of X**

Fuzzy sublattices of a lattice was introduced and some elementary results obtained in [1].In this section we define intuitionistic fuzzy sublattices and investigate about them.

Let $(X, \leq)$ be a lattice. An IFS , $A = (\mu_A, \nu_A)$ of X is called an intuitionistic fuzzy sublattice (IFL) of X if

1. $\mu_A(x \vee y) \geq \mu_A(x) \wedge \mu_A(y)$
2. $\mu_A(x \wedge y) \geq \mu_A(x) \wedge \mu_A(y)$
3. $\nu_A(x \vee y) \leq \nu_A(x) \vee \nu_A(y)$
4. $\nu_A(x \wedge y) \leq \nu_A(x) \vee \nu_A(y)$ for x,y ε X

Eg 3.1 : Consider the standard nonmodular lattice $N_5$ as X . ie , X={a,b,c,d,e} where a > c > e and a > b > d > e , b,c and c,d are noncomparable.

An intuitionistic fuzzy set $(\mu, \nu)$ of X is defined as follows

$\mu(a) = 0.6$ $\nu(a) = 0.3$
$\mu(b) = 0.1$ $\nu(b) = 0.9$
$\mu(c) = 0.5$ $\nu(c) = 0.4$
$\mu(d) = 0.6$ $\nu(d) = 0.3$
$\mu(e) = 0.7$ $\nu(e) = 0.2$

Here $(\mu, \nu)$ is an intuitionistic fuzzy sublattice of X . <u>Convention</u> : Throughout this paper X is an arbitrary nonempty lattice with a partial order ⊆

**Result 3.1**

If Y is any nonempty sublattice of X and if a ε I ,then $A=(\mu_A, \nu_A)$ defined

$$\text{by } \mu_A(x) = \begin{cases} a \ \forall \ x \in Y \\ 0 \text{ if } x \in X-Y \end{cases}$$

$$\text{and } \nu_A(x) = \begin{cases} 1-a \ \forall \ \ x \in Y \\ 1 \ \text{ if } x \in X-Y \end{cases}$$

is an intuitionistic fuzzy sublattice of X.

Proof: Easy and omitted

**Result 3.2**

Let A be a nonempty subset of X .Then A is a sublattice of X iff $(\chi_A, 1-\chi_A)$ is an IFL of X.

**Proof**

Suppose A is a sublattice of X .Then for any x , y ε A , x ∨ y and x ∧ y ε A

So $\chi_A(x) = \chi_A(y) = \chi_A(x \vee y) = \chi_A(x \wedge y) = 1$ and $(1-\chi_A)(x) = (1-\chi_A)(y) = (1-\chi_A)(x \vee y) = (1-\chi_A)(x \wedge y) = 0$

Then $\chi_A(x \vee y) = \chi_A(x) \wedge \chi_A(y)$

$\chi_A(x \wedge y) = \chi_A(x) \wedge \chi_A(y)$

$(1-\chi_A)(x \vee y) = (1-\chi_A)(x) \vee (1-\chi_A)(y)$

$(1-\chi_A)(x \wedge y) = (1-\chi_A) \vee (1-\chi_A)(y)$

Suppose x,y ε X and at least one of them ,say $y \notin A$.Then

$\chi_A(y) = 0$ , $(1-\chi_A)(y) = 1$ , $\chi_A(x) \wedge \chi_A(y) = 0$

$(1-\chi_A)(x) \vee (1-\chi_A)(y) = 1$

$\chi_A(x \vee y) \geq \chi_A(x) \wedge \chi_A(y)$

$\chi_A(x \wedge y) \geq \chi_A(x) \wedge \chi_A(y)$

$(1-\chi_A)(x \vee y) \leq (1-\chi_A)(x) \vee (1-\chi_A)(y)$

$(1-\chi_A)(x \wedge y) \leq (1-\chi_A)(x) \vee (1-\chi_A)(y)$

Thus $(\chi_A, 1-\chi_A)$ satisfies the properties of an IFL.
Conversly , suppose $(\chi_A, 1-\chi_A)$ is an IFL of X.
Let x,y ε A .

$\therefore \chi_A(x) = \chi_A(y) = 1$

$\therefore \chi_A(x) \wedge \chi_A(y) = 1$

But both $\chi_A(x \vee y)$ and $\chi_A(x \wedge y) \geq \chi_A(x) \wedge \chi_A(y)$ by assumption

Thus both $\chi_A(x \vee y)$ and $\chi_A(x \wedge y) = 1$

$\therefore x \vee y$ and $x \wedge y \in A$

∴ A is a sublattice of X '

**Notation**

The set of all intuitionistic fuzzy sublattices of X is denoted by IFL(X)

**Defenition 3.2**

Let A = ( $\mu_A$,$\nu_A$ ) be an IFL of X ,then for any ($t_1$ , $t_2$ ) ε I × I , the set $A_{(t_1, t_2)}$ = { x ε X ; $\mu_A(x) \geq t_1$ and $\nu_A(x) \leq t_2$ } is called a ($t_1$, $t_2$ ) level subset of A. ( $A_{(t_1, t_2)}$ may be empty )

**Theorem (3.1)**

Let A = ($\mu_A$,$\nu_A$ ) ε IFS (X).A ε IFL (X) iff the level subsets $A_{(t_1, t_2)}$ of A

for every $t_1 \leq \sup_{x \varepsilon X} \mu_A(x)$ and $t_2 \geq \inf_{x \varepsilon X} \nu_A(x)$ are sublattices of X and are

called level sublattices of A.

**Proof**

Assume that A ε IFL(X) and ($t_1$, $t_2$) ε I × I such that $t_1 \leq \sup_{x \varepsilon X} \{ \mu_A(x)\}$ and

$t_2 \geq \inf_{x \varepsilon X} \{\nu_A(x)\}$. Then $A_{(t_1, t_2)}$ is nonempty.

Let x,y ε $A_{(t_1, t_2)}$

$\therefore \mu_A(x) \geq t_1 , \mu_A(y) \geq t_1$ and

$\nu_A(x) \leq t_2 , \nu_A(y) \leq t_2$ .

since A ε IFL(X) , We have

$\mu_A(x \vee y) \geq \mu_A(x) \wedge \mu_A(y) \geq t_1 \wedge t_1 = t_1$ , $\nu_A(x \vee y) \leq \nu_A(x) \vee \nu_A(y)$

$\leq t_2 \vee t_2 = t_2$

$\therefore x \vee y$ ε $A_{(t_1, t_2)}$ → (1)

Similarly $\mu_A(x \wedge y) \geq t_1$ and $\nu_A(x \wedge y) \leq t_2$

$\therefore x \wedge y$ ε $A_{(t_1, t_2)}$ → (2)

From (1) and (2) , $A_{(t_1, t_2)}$ is a sublattice of X

<u>Converse</u>

Assume that ∀ ($t_1$, $t_2$) ε I × I , where $t_1 \leq \sup_{x \varepsilon X} \mu_A(x)$ and $t_2 \geq \inf_{x \varepsilon X} \nu_A(x)$ ,

$A_{(t_1, t_2)}$ is a sublattice of X.

Let x,y ε X,and let $t_1 = \mu_A(x) \wedge \mu_A(y)$

$\therefore t_1 \leq \mu_A(x)$ and $t_1 \leq \mu_A(y)$

Let $t_2 = \nu_A(x) \vee \nu_A(y)$

$\therefore t_2 \geq \nu_A(x)$ and $t_2 \geq \nu_A(y)$

Thus $\mu_A(x) \geq t_1 , \nu_A(x) \leq t_2$ and

$\mu_A(y) \geq t_1 , \nu_A(y) \leq t_2$

$\therefore$ x , y ε $A_{(t_1, t_2)}$

$\therefore$ $x \vee y$ and $x \wedge y \varepsilon A_{(t_1, t_2)}$ , since $A_{(t_1, t_2)}$ is a sublattice of X.

$\therefore$ $\mu_A(x \vee y) \geq t_1$ , $\nu_A(x \vee y) \leq t_2$ , $\mu_A(x \wedge y) \geq t_1$ and $\nu_A(x \wedge y) \leq t_2$

ie $\mu_A(x \vee y) \geq \mu_A(x) \wedge \mu_A(y)$

$\nu_A(x \vee y) \leq \nu_A(x) \vee \nu_A(y)$

$\mu_A(x \wedge y) \geq \mu_A(x) \wedge \mu_A(y)$

and $\nu_A(x \wedge y) \leq \nu_A(x) \vee \nu_A(y)$

Thus $A = (\mu_A, \nu_A)$ is an IFL of X

**Theorem 3.2**

If $A=(\mu_A, \nu_A)$ is an intuitionistic fuzzy sublattice of X,then $A^* = (\mu_A^*, \nu_A^*) = \{x \varepsilon X / \mu_A(x) > 0$ and $\nu_A(x) < 1\}$ is a sublattice of X if it is non empty

**Proof**

Easy and omitted.

**Theorem 3.3**

Any intuitionistic fuzzy subset of a chain X is an IFL of X

**Proof**

Let X be a chain and $A = (\mu_A, \nu_A)$ be an IFS of X.Let x,y ε X .Then either $x \geq y$ or $y \geq x$.If $x \geq y$ then $x \vee y = x$ and $x \wedge y = y$.

$\therefore$ $\mu_A(x \vee y) = \mu_A(x) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x \vee y) = \nu_A(x) \leq \nu_A(x) \vee \nu_A(y)$

Similarly $\mu_A(x \wedge y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x \wedge y) \leq \nu_A(x) \vee \nu_A(y)$

Hence A is an IFL of X.

On the other hand if $y \geq x$ , using the same arguments , we can prove that A is IFL of X.

Hence in either case A is an IFL of X.

**Defenition 3.2**

Let X and Y be two nonempty set and $f : X \rightarrow Y$ be a function

1. If $B = (\mu_B, \nu_B)$ is an IFS in Y ,then the preimage of B under f denoted by $f^{-1}(B)$ is the IFS in X defined by

   $$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B)) = (\mu_B \circ f, \nu_B \circ f)$$

2. If $A = (\mu_A, \nu_A)$ is an IFS in X ,then the image of A under f , denoted by f(A) is the IFS in Y defined by $f(A) = (f(\mu_A), 1 - f(1 - \nu_A))$

Note that

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \,\varepsilon\, f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \Phi \\ 0 & \text{otherwise} \end{cases}$$

and

$$(1- f(1-\nu_A))(y) = \begin{cases} \inf\limits_{x \,\varepsilon\, f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \Phi \\ 0 & \text{otherwise} \end{cases}$$

**Theorem 3.4**

Let X and Y be lattices and B = $(\lambda_B,\theta_B)$ be an IFL of Y.If f is a lattice homomorphism of X into Y ,then $f^{-1}(B)$ ε IFL (X)

**Proof**

We have $f^{-1}(B) = (f^{-1}(\lambda_B), f^{-1}(\theta_B))$

Let $x_1, x_2$ ε X

$f^{-1}(\lambda_B)(x_1 \vee x_2) = \lambda_B(f(x_1 \vee x_2))$ ; by the definition of $f^{-1}(\lambda_B)$

$= \lambda_B(f(x_1) \vee f(x_2))$ ;since f is a lattice homomorphism

$\geq \lambda_B(f(x_1) \wedge \lambda_B f(x_2))$ ;since B ε IFL(Y)

$= f^{-1}(\lambda_B(x_1)) \wedge f^{-1}(\lambda_B(x_2))$

Similarly $f^{-1}(\lambda_B)(x_1 \wedge x_2) \geq f^{-1}(\lambda_B(x_1)) \wedge f^{-1}(\lambda_B(x_2))$

Now $f^{-1}(\theta_B)(x_1 \vee x_2) = \theta_B((f(x_1 \vee x_2))$ ; by definition of $f^{-1}(\theta_B)$

$= \theta_B(f(x_1) \vee f(x_2))$ ;since f is a lattice homomorphism

$\leq \theta_B(f(x_1) \vee \theta_B(f(x_2))$ ;since B ε IFL(Y)

$= f^{-1}(\theta_B(x_1)) \vee f^{-1}(\theta_B(x_2))$

Similarly $f^{-1}(\theta_B)(x_1 \wedge x_2) \leq f^{-1}(\theta_B)(x_1) \vee f^{-1}(\theta_B)(x_2)$

$\therefore f^{-1}(B) = (f^{-1}(\lambda_B), f^{-1}(\theta_B))$ ε IFL(X)

**Theorem 3.5**

Let X and Y be sublattices and A = $(\mu_A, \nu_A)$ ε IFL(X).Suppose that f is an epimorphism of X onto Y.Then f(A) ε IFL (Y)

**Proof**

$f(A) = (f(\mu_A), 1- f(1-\nu_A))$ where

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \,\varepsilon\, f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \Phi \\ 0 & \text{otherwise} \end{cases}$$

and

$$(1-f(1-\nu_A))(y) = \begin{cases} \inf\limits_{x\,\varepsilon\, f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \Phi \\ 1 & \text{otherwise} \end{cases}$$

Now let $y_1, y_2 \,\varepsilon\, Y$ .Then by the definition of $f(\mu_A)$ ,

$$f(\mu_A)(y_1 \vee y_2) = \vee \{ \mu_A(z), z \,\varepsilon\, X : f(z) = (y_1 \vee y_2) \}$$
$$\geq \vee \{ \mu_A(x_1 \vee x_2) : x_1, x_2 \,\varepsilon\, X, f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$

( since f is a lattice homomorphism, if $z = x_1 \vee x_2$ , $f(z) = f(x_1) \vee f(x_2) = y_1 \vee y_2$ )

$$\geq \vee \{ \mu_A(x_1) \wedge \mu_A(x_2) : x_1, x_2 \,\varepsilon\, X, f(x_1) = y_1, f(x_2) = y_2 \}$$
since $A \,\varepsilon\, IFL(X)$

$$= (\vee \{ \mu_A(x_1) : x_1 \,\varepsilon\, X, f(x_1) = y_1 \}) \wedge (\vee \{ \mu_A(x_2) : x_2 \,\varepsilon\, X, f(x_2) = y_2 \})$$
$$= f(\mu_A)(y_1) \wedge f(\mu_A)(y_2)$$

Similarly $f(\mu_A)(y_1 \wedge y_2) = \vee \{ \mu_A(z) : z \,\varepsilon\, X : f(z) = (y_1 \wedge y_2) \}$

$$\geq \vee \{ \mu_A(x_1 \wedge x_2) : x_1, x_2 \,\varepsilon\, X, f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$

(since f is a lattice homomorphism, if $z = x_1 \wedge x_2$,
then $f(z) = f(x_1) \wedge f(x_2) = y_1 \wedge y_2$ )

$$\geq \vee \{ \mu_A(x_1) \wedge \mu_A(x_2) : x_1, x_2 \,\varepsilon\, X, f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$ Since $A \,\varepsilon\, IFL(X)$

$$= (\vee \{ \mu_A(x_1) : x_1 \,\varepsilon\, X, f(x_1) = y_1 \}) \wedge (\vee \{ \mu_A(x_2) : x_2 \,\varepsilon\, X, f(x_2) = y_2 \})$$
$$= f(\mu_A)(y_1) \wedge f(\mu_A)(y_2)$$

and

$(1-f(1-\nu_A))(y_1 \vee y_2)$

$$= \wedge \{ \nu_A(z) : z \,\varepsilon\, X, z \,\varepsilon\, f^{-1}(y_1 \vee y_2) \}$$
$$= \wedge \{ \nu_A(z) : z \,\varepsilon\, X : f(z) = y_1 \vee y_2 \}$$
$$\leq \wedge \{ \nu_A(x_1 \vee x_2) : x_1, x_2 \,\varepsilon\, X : f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$
$$\leq \wedge \{ \nu_A(x_1) \vee \nu_A(x_2) : x_1, x_2 \,\varepsilon\, X : f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$
$$= (\wedge \{ \nu_A(x_1) : x_1 \,\varepsilon\, X, f(x_1) = y_1 \}) \vee (\wedge \{ \nu_A(x_2) : x_2 \,\varepsilon\, X \text{ and } f(x_2) = y_2 \})$$
$$= (1-f(1-\nu_A))(y_1) \vee (1-f(1-\nu_A))(y_2)$$

Similarly $(1-f(1-\nu_A))(y_1 \wedge y_2) = \wedge \{ \nu_A(z) : z \,\varepsilon\, X : z \,\varepsilon\, f^{-1}(y_1 \wedge y_2) \}$

$$= \wedge \{ \nu_A(z) : z \,\varepsilon\, X : f(z) = (y_1 \wedge y_2) \}$$
$$\leq \wedge \{ \nu_A(x_1 \wedge x_2) : x_1, x_2 \,\varepsilon\, X : f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$
$$\leq \wedge \{ \nu_A(x_1) \vee \nu_A(x_2) : x_1, x_2 \,\varepsilon\, X : f(x_1) = y_1 \text{ and } f(x_2) = y_2 \}$$

$= (\wedge \{ \nu_A(x_1), x_1 \varepsilon X , f(x_1) = y_1\} ) \vee (\wedge \{ \nu_A(x_2), x_2 \varepsilon X , f(x_2) = y_2\} )$
$= (1- f(1- \nu_A)( y_1) \vee ((1- f(1- \nu_A ))( y_2)$
$\therefore (\mu_A , 1 -f(1- \nu_A )) = f(A) \varepsilon IFL(Y)$

**Theorem 3.6**

Let $\{ A_j | j \varepsilon J \} \subseteq IFL(X)$.Then $\bigcap_{j \varepsilon J} A_j \varepsilon IFL(X)$

**Proof**

Let $A_j = (\mu_{Aj} , \nu_{Aj} )$
By definition we have $\bigcap_{j \varepsilon J} A_j = ( \bigcap_{j \varepsilon J} \mu_{Aj} , \bigcup_{j \varepsilon J} \nu_{Aj} )$

Let $x_1, x_2 \varepsilon X$

$( \bigcap_{j \varepsilon J} \mu_{Aj} )( x_1 \wedge x_2 ) = \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_1 \wedge x_2 )\}$

$\geq \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_1) \wedge \mu_{Aj} (x_2) \}$ Since $A_j \varepsilon IFL(X)$

$= ( \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_1) \} ) \wedge ( \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_2) \} )$

$= ( \bigcap_{j \varepsilon J} \mu_{Aj} ) (x_1) \wedge ( \bigcap_{j \varepsilon J} \mu_{Aj} ) (x_2))$

Similarly $( \bigcap_{j \varepsilon J} \mu_{Aj} )( x_1 \vee x_2) = \bigwedge_{J} (\mu_{Aj} (x_1 \vee x_2) , j \varepsilon J )$

$\geq \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_1) \wedge \mu_{Aj} (x_2) \}$ Since $A_j \varepsilon IFL(X)$

$= (\bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_1)\} ) \wedge ( \bigwedge_{j \varepsilon J} \{ \mu_{Aj} (x_2)\} )$

$= (\bigcap_{j \varepsilon J} \mu_{Aj} )( x_1) \wedge (\bigcap_{j \varepsilon J} \mu_{Aj})( x_2)$

and $( \bigcup_{j \varepsilon J} \nu_{Aj} )( x_1 \vee x_2 ) = \bigvee_{j} \{ \nu_{Aj} ( x_1 \vee x_2 ) , j \varepsilon J \}$

$\leq \bigvee_{j} \{ \nu_{Aj} (x_1) \vee \nu_{Aj} (x_2) , j \varepsilon J \}$

Since $\nu_{Aj} (x_1 \vee x_2 ) \leq \nu_{Aj} (x_1) \vee \nu_{Aj} (x_2)$

$= (\vee_j \{ \nu_{Aj}(x_1) , j \varepsilon J \} ) \vee (\vee_j \{ \nu_{Aj} (x_2) , j \varepsilon J \} )$
$= (\cup_j \nu_{Aj} )( x_1) \vee (\bigcup_{j \varepsilon J} \nu_{Aj} )( x_2)$

Similarly $(\bigcup_{j \varepsilon J} \nu_{Aj} )( x_1 \wedge x_2) = \bigvee_{j} \{ \nu_{Aj} (x_1 \wedge x_2) , j \varepsilon J \}$

$$\leq \bigvee_{j} \{ \nu_{Aj}(x_1) \vee \nu_{Aj}(x_2), j \in J \}$$

Since $\nu_{Aj}(x_1 \wedge x_2) \leq \nu_{Aj}(x_1) \vee \nu_{Aj}(x_2)$

$$= (\vee_j \{ \nu_{Aj}(x_1), j \in J \}) \vee (\vee_j \{ \nu_{Aj}(x_2), j \in J \})$$

$$= (\cup_j \nu_{Aj})(x_1) \vee (\cup_{j \in J} \nu_{Aj})(x_2)$$

Thus $(\cap_{j \in J} \mu_{Aj}, \cup_{j \in J} \nu_{Aj}) \in IFL(X)$

ie $\cap_{j \in J} A_j \in IFL(X)$

But union of two IFL's need not be an IFL

Eg 3.2: Consider the following lattice X = { a,b,c,d } where a > c > d and a > b > d , b , c are non comparable.

Define A and B in IFL(X) as follows
$A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ where

$\mu_A(a) = 0.5$ $\nu_A(a) = 0.4$
$\mu_A(b) = 0.7$ $\nu_A(b) = 0.3$
$\mu_A(c) = 0.3$ $\nu_A(c) = 0.6$
$\mu_A(d) = 0.3$ $\nu_A(d) = 0.7$

and
$\mu_B(a) = 0.4$ $\nu_B(a) = 0.5$
$\mu_B(b) = 0.4$ $\nu_B(b) = 0.6$
$\mu_B(c) = 0.6$ $\nu_B(c) = 0.3$
$\mu_B(d) = 0.6$ $\nu_B(d) = 0.3$

Both A and B are IFLs.But $A \cup B$ is not an IFL.
Since $(\mu_A \cup \mu_B)(b \vee c) \ngeq (\mu_A \cup \mu_B)(b) \wedge (\mu_A \cup \mu_B)(c)$

**<u>Defenition 3.4</u>**

Let $A \in IFL(X)$ .Let $\langle A \rangle = \cap \{B / A \subseteq B , B \in IFL(X) \}$.By theorem 3.6 $\langle A \rangle$ is an IFL of X .$\langle A \rangle$ is called the intuitionistic fuzzy sublattice of X generated by A.

## <u>4. Lattice of intuitionistic fuzzy lattices</u>

In this section we show that for a lattice X , the set of all intuitionistic fuzzy sublattices is a complete lattice under the ordering $\subseteq$.
Recall that the set of all intuitionistic fuzzy sets in a set forms a complete lattice under the ordering $\subseteq$, where the infimum and supremum are intersection and union of intuitionistic fuzzy subsets. To construct the lattice of intuitionistic fuzzy sublattices of a lattice X we define the infimum of a family $\{A_i\}$ of intuitionistic fuzzy sublattices to be the intersection $\cap A_i$. However the supremum is defined as the intuitionistic fuzzy sublattice generated by $\langle \cup A_i \rangle$. Thus we have the following theorem.

**Theorem 4.1**
The set IFL(X) of all intuitionistic fuzzy sublattices of X is a complete lattice under the ordering ⊆ of intuitionistic fuzzy set inclusion.

**Proof**
Let $\{A_i\}$ be a family of intuitionistic fuzzy sublattices of X .Then $\cap A_i$ is an intuitionistic fuzzy sublattice and it is the largest intuitionistic fuzzy sublattice contained in each $A_i$ .Then $\wedge A_i = \cap A_i$ .On the other hand consider the intuitionistic fuzzy sublattice generated by $\cup A_i$ i.e., $< \cup A_i >$ . It is an intuitionistic fuzzy sublattice and it is the smallest IFL containing each $A_i$ .
∴ We see that $\vee A_i = < \cup A_i >$ . Thus IFL(X) is a complete lattice.

**Acknowledgement**
The author wishes to thank Dr. T. Thrivikraman for his guidance during the preparation of this paper.

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