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Concavoconvex intuitionistic fuzzy sets

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Abstract: We introduce the concept of concavoconvexity for intuitionistic fuzzy sets and we give some results in connection with this concept.

Keywords: Triangular norm, convex and concave intuitionistic fuzzy set, temporal intuitionistic fuzzy set.

1 Introduction

In [5] and [6] are studied the convexity and concavity for intuitionistic fuzzy sets and temporal intuitionistic fuzzy sets with help of notions and results from [1], [2], [3], [4] and [7]. This paper is a continuation of these articles. The launch of the idea of concavoconvexity (for fuzzy sets) is maked in [8] but only for the particular case of fuzzy sets in $\mathbf{R}^n$ and operations induced by triangular norm *min* and triangular conorm *max*.

2 Preliminaries and basic definitions

In this paper, all the definitions, the symbols and the results in connection with triangular norms and triangular conorms (t-norms and t-conorms, for short), convexity and concavity of fuzzy sets, intuitionistic fuzzy sets and temporal intuitionistic fuzzy sets follows [5] and [6] (see also [1]-[4], [7]). We will recall some of them.

In further we assume that X is a real vector space and $I = [0, 1]$.

Definition 1 *([7], [5]) Let $\mu \in FS(X)$ and f be a t-norm or t-conorm. μ is called to be*

(1) *f-convex in X if and only if $\mu\left(kx + (1-k)y\right) \geq f\left(\mu(x),\ \mu(y)\right)$ $\forall\ x, y \in X, \forall\ k \in I$.*
(2) *f-concave in X if and only if $\mu\left(kx + (1-k)y\right) \leq f\left(\mu(x),\ \mu(y)\right)$ $\forall\ x, y \in X, \forall\ k \in I$.*

Definition 2 *([5]) Let $A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\} \in IFS(X)$ and f be a t-norm or t-conorm. A is called to be*

(1) *f-convex in X if and only if μ_A is f-convex in X and ν_A is f^c-concave in X*
(2) *f-concave in X if and only if μ_A is f-concave in X and ν_A is f^c-convex in X.*

Definition 3 *([6]) Let $A(T) \in TIFS(X)$ and f be a t-norm or t-conorm. $A(T)$ is called to be f-convex (f-concave) in X if and only if the intuitionistic fuzzy sets $A_t = \{\langle x,\ \mu_A(x,t),\ \nu_A(x,t)\rangle \mid x \in X\}$ are f-convex (f-concave) in X for each $t \in T$.*

Theorem 4 *([5]) Let $A \in IFS(X)$ and f be a t-norm or t-conorm.*

(1) *A is f-convex if and only if A^c is f^c-concave.*
(2) *If A is f-convex (f-concave) then $\Box A$ and $\Diamond A$ are f-convex (f-concave).*

Theorem 5 *([5]) If f is a t-norm (t-conorm), then CA and IA are f-convex (f-concave).*

Theorem 6 *([5]) Let f, g and h be three t-norms (or three t-conorms) and $A, B \in IFS(X)$.*

(1) *If $f \leq g$, $f \ll h$, A is f-convex in X and B is g-convex in X, then $A\tilde{h}B$ is f-convex in X.*
(2) *If $g \leq f$, $h \ll f$, A is f-concave in X and B is g-concave in X, then $A\tilde{h}B$ is f-concave in X.*

We introduce the concept of (f, g)-concavoconvexity of a fuzzy set as follows.

Definition 7 *Let f, g be two t-norms or t-conorms. The fuzzy set μ is called (f, g)-concavoconvex in X if and only if μ is f-convex in X and μ is g-concave in X, that is*

$$f(\mu(x), \mu(y)) \leq \mu(kx + (1-k)y) \leq g(\mu(x), \mu(y)) \ \forall x, y \in X, \forall k \in I.$$

We denote by ${}_f^gFS(X)$ the set of all (f, g)-concavoconvex fuzzy sets in X.

Remark 1 *If $X = \mathbf{R}^n$, $f(u, v) = \min(u, v)$ $\forall u, v \in I$ and $g(u, v) = \max(u, v)$ $\forall u, v \in I$ we obtain the concavoconvexity of a fuzzy set in space introduced in [8].*

Inspired by Definition 7 we can give the next

Definition 8 *Let f, g be two t-norms or t-conorms. The intuitionistic fuzzy set A on X is called (f, g)-concavoconvex in X if and only if A is f-convex in X and A is g-concave in X.*

We denote by ${}_f^gIFS(X)$ the set of all (f, g)-concavoconvex intuitionistic fuzzy sets in X.

Remark 2 *Because $f \leq g$ implies $g^c \leq f^c$ for every t-norms or t-conorms f and g, if $f' \leq f$ and $g \leq g'$ then ${}_f^gIFS(X) \subseteq_{f'}^{g'} IFS(X)$. If $g \leq f$ (for example, g is a t-norm and f is a t-conorm) then ${}_f^gIFS(X) = \emptyset$.*

3 Properties of concavoconvex intuitionistic fuzzy sets

The following theorem is a characterization of concavoconvexity of intuitionistic fuzzy sets with help of concavoconvexity of fuzzy sets.

Theorem 9 *Let f, g be two t-norms or t-conorms.*
$A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\} \in_f^g IFS(X)$ if and only if $\mu_A \in_f^g FS(X)$ and $\nu_A \in_{g^c}^{f^c} FS(X)$.

Proof. $A \in_f^g IFS(X)$ if and only if A is f-convex and g-concave in X if and only if μ_A is f-convex, ν_A is f^c-concave in X and μ_A is g-concave, ν_A is g^c-convex in X if and only if μ_A is (f,g)-concavoconvex and ν_A is (g^c, f^c)-concavoconvex in X ■

In sequel we give some properties of (f,g)-concavoconvex intuitionistic fuzzy sets.

Theorem 10 *Let $A \in IFS(X)$ and f, g be t-norms or t-conorms.*

(1) $A \in_f^g IFS(X)$ *if and only if* $A^c \in_{f^c}^{g^c} IFS(X)$
(2) $A \in_f^g IFS(X)$ *implies* $\Box A \in_f^g IFS(X)$ *and* $\Diamond A \in_f^g IFS(X)$
(3) *If f is a t-norm and g is a t-conorm then* $CA \in_f^g IFS(X)$ *and* $IA \in_f^g IFS(X)$

Proof. $(1), (2)$ Immediately by Theorem 4, (1) and (2).
(3) Immediately by Theorem 5 ■

Theorem 11 *Let f, g and h be three t-norms or three t-conorms such that $f \ll h \ll g$. If $A, B \in_f^g IFS(X)$ then $A\tilde{h}B \in_f^g IFS(X)$.*

Proof. Because A and B are f-convex in X, taking $f = g$ in Theorem 6, (1) we obtain that $A\tilde{h}B$ is f-convex in X. Similar, because A and B are g-concave in X, taking $f = g$ in Theorem 6, (2) we obtain that $A\tilde{h}B$ is g-concave in X.

Therefore $A\tilde{h}B$ is (f,g)-concavoconvex in X ■

Remark 3 *Thanks to Definition* 3 *we can define the (f,g)-concavoconvex temporal intuitionistic fuzzy sets. Also, using Theorem* 3 *and Theorem* 4 *by [6] we can prove properties as above for temporal intuitionistic fuzzy sets.*

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