

**A GENERALIZED NET FOR INTELLECTUAL GAMES LEARNING:
AN INTUITIONISTIC FUZZY APPROACH**

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Game Theory has been an object of active research for already 60 years. Forty years ago, a new branch emerged on the boundary of Game Theory and Artificial Intelligence, namely, the so called "intellectual games" (e.g., [1-4])

Here we shall give a Generalized Net (GN; see [5]) model of the machine learning of an intellectual game, briefly and in principle. The details of each particular intellectual game can be interpreted by appropriate sub-nets of the GN from Fig. 1. Here we shall assume that there are n players, where $n \geq 1$ is determined by the type of the game. Let the i -th player is represented by an α_i -token ($1 \leq i \leq n$). This token can have some data on the corresponding player as initial characteristics. The α_i -token enters the net through place a_1 and passes through places a_2, a_3, \dots, a_9 .

On the other hand, the initial and the current status of the game are represented by the corresponding characteristics of the unique β -token, which enters the net through place b_1 and passes through places b_2, b_3 and b_4 .

The game rules determine who of the players should start first, who will play next etc. This information can be included in the initial characteristics of the α -tokens, or (and this situation is not standard in the traditional games!) as their current characteristic. Similarly to the model investigated in the present paper, in future we can develop models of some intellectual business-modelling games that cannot be described by ordinary mathematical means. In this case the players do not play sequentially, following some fixed order, but simultaneously (i.e., in parallel). Examples for the last case are discussed in [6-8].

The constructed GN-model is capable of modelling a situation in which one or more tokens will come into an active state of "playing" (in place a_2), while the other will be in an inactive state of "waiting" (in place a_3). On the other hand, in chess both players may

be simultaneously thinking over the situation - one about his move to make and the other about the possible moves of his opponent. Therefore, the state of “waiting” can be replaced by the state of “active waiting” and the tokens in place a_2 can obtain next characteristics as well. For this purpose, the tokens (all or some of them) in place a_2 will circulate there for several time-steps, so as to obtain new characteristics.

The tokens from place a_3 would transfer either into place a_5 if the decision is made by an AI algorithm, or into a_6 if the player is human or the decision is made by some other means not involving this model (for the second case, we will use the term indirectly modelled). If the token possesses as a characteristic an algorithm for choosing a move and a training algorithm, then the token enters place a_5 . If it does not have such an algorithm or it is indirectly modelled, the token enters place a_6 . From a_5 the token transfers into a_7 and loops there until the last step from the algorithm for choosing a new move is accomplished. For example, at each step one can use the following intuitionistic fuzzy approach

To each node of the decision tree [1] an intuitionistic fuzzy estimation of the move is assigned.

$$a_{i,j} = \langle \mu_{i,j}, \nu_{i,j} \rangle$$

where $\mu_{i,j}$ is the degree of usefulness of the move and $\nu_{i,j}$ is the degree of harmfulness of the move. and $1 \leq i \leq \text{number of possible moves}$, and $1 \leq j \leq \text{number of forward investigated moves}$

For this estimation, either static estimation can be used function, or it can be calculated on the basis of the estimations of the sub-tree of the current node. It is assumed that better quality of the game is achieved when deeper investigation of the decision tree is made. In that case the process of learning can be treated as finding such a static estimation function of the game situation that gives as close as possible an estimation to the one obtained by investigating the decision tree.

The composite estimation for a sub-tree can be obtained by the min-max method. It assumes that the opponent will play the moves with minimal estimation and the AI player will play those with maximal estimation. For calculating the overall IF estimation for sub - tree we can use the following method.

Firstly we must assign IF estimations to the leaves of the decision tree using static estimation function. Then from those nodes we must choose the node with minimal ν denoted as ν^* if it is the opponents turn, or the node with maximal μ if it is algorithm's turn. Let $A_j = \{\langle \mu, \nu \rangle\}$ is the set of estimations for all possible moves for AI player from j -th layer of the decision tree

$$\phi(A_j) = \langle \mu^*, \nu^* \rangle$$

where

$$\forall i(\langle \mu_i, \nu_i \rangle \in A_j \& \mu_i \leq \mu^*, \nu^* = \nu_i)$$

Similarly, let $B_j = \{\langle \mu, \nu \rangle\}$ is the set of estimations for all possible moves for AI player from j -th layer of the decision tree

$$\phi(B_j) = \langle \mu^*, \nu^* \rangle$$

where

$$\forall i(\langle \mu_i, \nu_i \rangle \in A_j \& \mu^* = \mu_i, \nu^* \leq \nu_i)$$

The estimations of the nodes from preceding layer is obtained by operator $F_{\mu^*, \nu^*}(\mu, \nu)$ by means of which the uncertainty of IF estimation is decreased.

The static estimation function can be obtained as a superposition of the estimations with respect to different criteria for the advantages or the drawbacks to the given move. These estimations are strongly related to the game rules and aims, and that is why they will not be discussed in the present paper. The example of the construction such functions for a Chess game is given in [9].

From a_7 the token transfers to a_8 and the changes relevant to the process of learning take place during this transfer. Then, the token transfers to place a_9 , when the planned move is executed and the procedure repeats. At this moment the current game-situation changes again, and for this reason the β -token transfers parallelly from place b_3 to place b_4 . After a certain number of steps, the game is over and the α -tokens leave the GN through place a_4 , while the β -token leaves the GN through place b_2 .

We can generalize the standard idea for a game, assuming that one or more of the players may participate simultaneously in several games. In this case we shall use simultaneously m in number β -tokens (where m is the number of the parallel games). Also, the number of players can vary during the game. Therefore, the capacities of the a -places can be ∞ , while the capacities of the b -places can be m .

The GN transitions have the following forms:

$$Z_1 = \langle \{a_1, a_2, a_9, b_1, b_4\}, \{a_2, a_3, a_4, b_2, b_3\}, r_1, M_1, \square_1 \rangle,$$

where

	a_2	a_3	a_4	b_2	b_3
$r_1 =$	a_1	$W_1 \& \neg W_3$	$W_2 \& \neg W_3$	W_3	$false$ $false$
	a_2	$W_1 \& \neg W_3$	$W_2 \& \neg W_3$	W_3	$false$ $false$
	a_9	$W_1 \& \neg W_3$	$W_2 \& \neg W_3$	W_3	$false$ $false$
	b_1	$false$	$false$	$false$	W_4 $\neg W_4$
	b_4	$false$	$false$	$false$	W_4 $\neg W_4$

and

W_1 = “the player must wait”,

W_2 = “the player must play”,

W_3 = “the game for the current player is over”,

W_4 = “the game is over”,

	a_2	a_3	a_4	b_2	b_3
$M_1 =$	a_1	∞	∞	∞	0 0
	a_2	∞	∞	∞	0 0
	a_9	∞	∞	∞	0 0
	b_1	0	0	0	1 1
	b_4	0	0	0	1 1

$$\square_1 = \bigwedge (\bigvee (a_1, a_2, a_9), \bigvee (b_1, b_4)).$$

In place a_2 the tokens are assigned active state of playing and in place a_3 they are assigned inactive state of waiting. In place a_4 they are assigned the characteristic of a ”winner” or a ”loser”.

$$Z_2 = \langle \{a_3\}, \{a_5, a_6\}, r_2, M_2, \bigvee(a_3) \rangle,$$

where

	a_5	a_6
$r_2 =$	a_3	W_5 $\neg W_5$

where

W_5 = “the player has a game algorithm”, and

	a_5	a_6
$M_2 =$	a_3	∞ ∞

In a_5 they obtain the characteristic of ”an algorithm for choosing the move and a learning algorithm”. In place a_6 they do not obtain any characteristic.

$$Z_3 = \langle \{a_5, a_7\}, \{a_7, a_8\}, r_3, M_3, \vee(a_5, a_7) \rangle,$$

where

$$r_3 = \begin{array}{c|cc} & a_7 & a_8 \\ \hline a_5 & W_6 & \neg W_6 \\ a_7 & W_6 & \neg W_6 \end{array}$$

where

$W_6 =$ “the game algorithm has not finished”, and

$$M_3 = \begin{array}{c|cc} & a_7 & a_8 \\ \hline a_5 & \infty & \infty \\ a_7 & \infty & \infty \end{array}.$$

In place a_7 in the characteristic the state of the learning algorithm and the algorithm for choosing a new move is renewed. In the place a_8 the tokens obtain no new characteristic.

$$Z_4 = \langle \{a_6, a_8, b_3\}, \{a_9, b_4\}, r_4, M_4, \square_4 \rangle,$$

where

$$r_4 = \begin{array}{c|cc} & a_9 & b_4 \\ \hline a_6 & true & false \\ a_8 & true & false \\ b_3 & false & true \end{array},$$

$$M_4 = \begin{array}{c|cc} & a_9 & b_4 \\ \hline a_6 & \infty & 0 \\ a_8 & \infty & 0 \\ b_3 & 0 & m \end{array},$$

and

$$\square_4 = \wedge(\vee(a_6, a_8), b_3).$$

The tokens do not obtain any new characteristic in place a_9 . In place b_4 obtain the characteristic of the current game-situation.

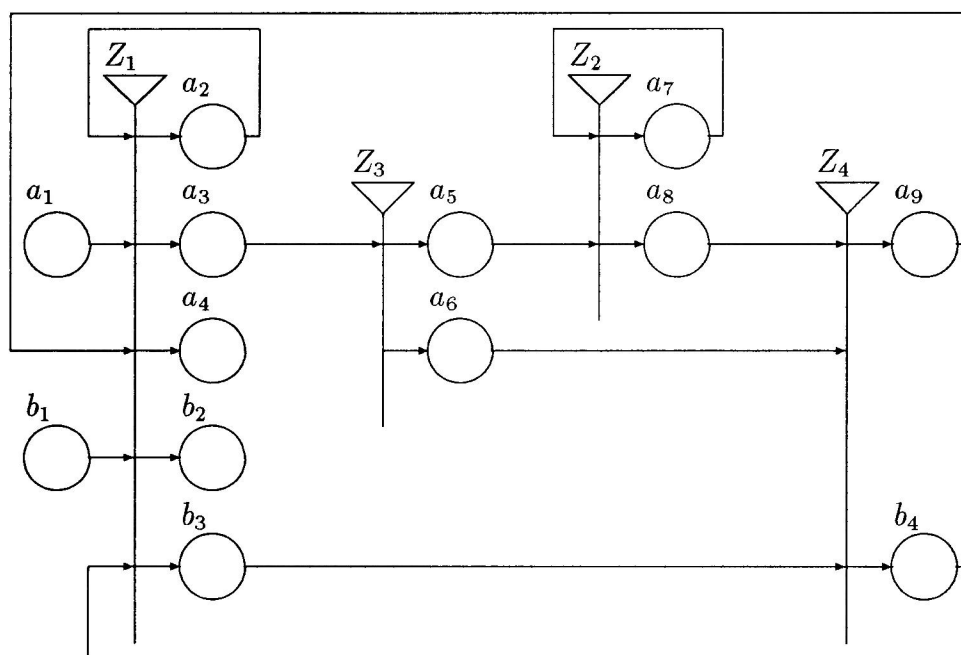


Fig. 1.

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