Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy Laplace Adomian decomposition method for solving intuitionistic fuzzy differential equations of higher order

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Abstract: The aim of this work is to introduce the intuitionistic fuzzy Laplace Adomian decomposition method, in which we combine intuitionistic fuzzy Laplace transform with the intuitionistic fuzzy Adomian decomposition method. Furthermore, the methodology of this approach for solving differential equations of higher order in an intuitionistic fuzzy environment has been proposed in detail. Finally, the paper concludes with two numerical examples to demonstrate the application and efficiency of this method, and we have studied their precision.

Keywords: Intuitionistic fuzzy solution, Intuitionistic fuzzy number, Intuitionistic fuzzy Laplace Adomian decomposition.
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1 Introduction

The theory of intuitionistic fuzzy sets was originally proposed by Atanassov [4,5] as an extension of fuzzy sets [24]. This theory has many applications in various disciplines including decision-making problems, medical diagnostics, programming, and others [6, 7, 10, 11]. The concepts of differential and integral calculus for intuitionistic fuzzy-set-valued functions were developed using the Hukuhara difference in intuitionistic fuzzy theory [15]. Papers [20,21] explored differential and partial differential equations in an intuitionistic fuzzy context, respectively. The existence and uniqueness of solutions to intuitionistic fuzzy differential equations were examined using successive approximations in [15], while in [12] demonstrated these properties through the fixed point theorem in complete metric spaces, also providing explicit solution formulas using α-cuts method. The solution to first-order linear intuitionistic fuzzy differential equations using the variation of constants formula was presented in [14]. Intuitionistic fuzzy Laplace transformation was employed to solve differential equations of second-order in intuitionistic fuzzy environments [13].

Adomian introduced the Adomian Decomposition Method (ADM) in 1980 [1]. This powerful technique is utilized to address algebraic, differential, integral, and integro-differential equations with non-linear functions [2, 3]. In [9, 23], researchers applied the ADM to find approximate solutions for intuitionistic fuzzy differential equations.

The Laplace–Adomian decomposition method (LADM), which combines the ADM with the Laplace transform, serves as an effective analytical approach for solving non-linear FDES. Several studies [8, 18, 19] have successfully employed this strategy. LADM's efficiency stems from its fewer parameters compared to other analytical techniques, eliminating the need for discretization and linearization [16]. Inspired by these works, the idea of this paper involves using intuitionistic fuzzy Laplace transformation and decomposing nonlinear terms into Adomian polynomials, resulting in a recursive, iterative technique for solving higher order linear and nonlinear intuitionistic fuzzy differential equations.

To the best of our knowledge, this is the first work to combine the Laplace transform and Adomian decomposition method in the intuitionistic fuzzy environment for solving higher-order IFDEs as a generalization of the classical approaches to manage uncertainty and vagueness more effectively. The results demonstrate the method's efficiency, accuracy, and applicability to real-world problems such as control theory, population dynamics, fluid mechanics, and other fields where uncertainty and higher-order dynamics are critical, making it a valuable contribution to the literature.

The structure of this paper is as follows: in Section 2 we give preliminaries, which we will use throughout this work. In Section 3 we construct a procedure for solving differential equations of higher order in an intuitionistic fuzzy environment. In Section 4, we provide examples to clarify the main results of this study. In the last section, we give a short conclusion.

2 Preliminaries

Through this paper $\mathbb{I} = [b, c]$ is a real interval.

Definition 2.1. [22] We define the space of any intuitionistic fuzzy number as

$$\mathrm{IF}_1 = \mathbb{IF}(\mathbb{R}) = \left\{\mathcal{E}_{\langle\Phi,\Psi\rangle} \,:\, \mathbb{R} \to [0,1]^2 \mid (\forall z \in \mathbb{R})(0 \le \Phi(z) + \Psi(z) \le 1)\right\}.$$

An element $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ of IF_1 is called an intuitionistic fuzzy number if the following assertions hold:

1. $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ is normal, i.e., there exist $z_0, z_1 \in \mathbb{R}$ such that $\Phi(z_0) = 1$ and $\Psi(z_1) = 1$,
2. Φ is fuzzy convex and Ψ is fuzzy concave,
3. Φ is upper semi-continuous and Ψ is lower semi-continuous,
4. $\mathrm{supp}(\mathcal{E}_{\langle\Phi,\Psi\rangle}) = \mathrm{cl}\{z \in \mathbb{R} \mid \Psi(z) < 1\}$ is bounded.

Definition 2.2. [17] The parametric form (PF) of intuitionistic fuzzy number $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ denoted as $\mathcal{E}_{\langle\Phi,\Psi\rangle}(\gamma) = \Big((\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,l}(\gamma), \mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,r}(\gamma)), (\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,l}(\gamma), \mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,r}(\gamma))\Big)$, where the functions $\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,l}(\gamma)$, $\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,r}(\gamma)$, $\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,l}(\gamma)$ and $\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,r}(\gamma)$, verify the following conditions:

i) $\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,l}(\gamma)$ is monotonically increasing bounded continuous,

ii) $\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,r}(\gamma)$ is monotonically decreasing bounded continuous,

iii) $\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,l}(\gamma)$ is monotonically increasing bounded continuous,

iv) $\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,r}(\gamma)$ is monotonically decreasing bounded continuous,

v) $\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,l}(\gamma) \leq \mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,r}(\gamma)$ and $\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,l}(\gamma) \leq \mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,r}(\gamma)$, for every $0 \leq \gamma \leq 1$.

Example. A Triangular Intuitionistic Fuzzy Number (TIFN) $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ is an intuitionistic fuzzy set in $\mathbb{R}$ with there membership function Φ and non-membership function Ψ respectively given by:

$$\Phi(z) = \begin{cases} \dfrac{z-b_1}{b_2-b_1} & \text{if } b_1 \leq z \leq b_2, \\ \dfrac{b_3-z}{b_3-b_2} & \text{if } b_2 \leq z \leq b_3, \\ 0. & \text{otherwise,} \end{cases}$$

$$\Psi(z) = \begin{cases} \dfrac{b_2-z}{b_2-b_1'} & \text{if } b_1' \leq z \leq b_2, \\ \dfrac{z-b_2}{b_3'-b_2} & \text{if } b_2 \leq z \leq b_3', \\ 1. & \text{otherwise.} \end{cases}$$

where $b_1' \leq b_1 \leq b_2 \leq b_3 \leq b_3'$, so this TIFN is expressed as $\mathcal{E}_{\langle\Phi,\Psi\rangle} = \left\langle b_1, b_2, b_3; b_1', b_2, b_3' \right\rangle$ and its parametric form is given by

$$\mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,l}(\gamma) = b_1 + \gamma(b_2 - a_1),\ \mathcal{E}^{+}_{\langle\Phi,\Psi\rangle,r}(\gamma) = b_3 - \gamma(b_3 - b_2),$$
$$\mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,l}(\gamma) = b_1' + \gamma(b_2 - b_1'),\ \mathcal{E}^{-}_{\langle\Phi,\Psi\rangle,r}(\gamma) = b_3' - \gamma(b_3' - b_2).$$

Let $\gamma \in [0,1]$ and $\mathcal{E}_{\langle\Phi,\Psi\rangle} \in \mathrm{IF}_1$, the lower and upper γ-cuts of $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ are defined as

$$\left[\mathcal{E}_{\langle\Phi,\Psi\rangle}\right]^{\gamma} = \{z \in \mathbb{R} \mid \Psi(z) \leq 1-\gamma\} \quad \text{and} \quad \left[\mathcal{E}_{\langle\Phi,\Psi\rangle}\right]_{\gamma} = \{z \in \mathbb{R} \mid \Phi(z) \geq \gamma\}.$$

Remark 2.1. *If $\mathcal{E}_{\langle\Phi,\Psi\rangle} \in IF_1$, then we get $\left[\mathcal{E}_{\langle\Phi,\Psi\rangle}\right]_{\gamma}$ as $[\Phi]^{\gamma}$ and $\left[\mathcal{E}_{\langle\Phi,\Psi\rangle}\right]^{\gamma}$ as $[1-\Psi]^{\gamma}$ in the fuzzy set.*

Consider $\mathcal{E}_{\langle\Phi_1,\Psi_1\rangle}$, $\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}$ and $\lambda\in\mathbb{R}$, then the scaler-multiplication and addition are represented as

$$\Big[\mathcal{E}_{\langle\Phi_1,\Psi_1\rangle}\oplus\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]^{\gamma}=\Big[\mathcal{E}_{\langle\Phi_1,\Psi_1\rangle}\Big]^{\gamma}+\Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]^{\gamma},\quad \Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]^{\gamma}=\lambda\Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]^{\gamma},$$
$$\Big[\mathcal{E}_{\langle\Phi_1,\Psi_1\rangle}\oplus\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]_{\gamma}=\Big[\mathcal{E}_{\langle\Phi_1,\Psi_1\rangle}\Big]_{\gamma}+\Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]_{\gamma},\quad \Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]_{\gamma}=\lambda\Big[\mathcal{J}_{\langle\Phi_2,\Psi_2\rangle}\Big]_{\gamma}.$$

Definition 2.3. Consider $\mathcal{E}_{\langle\Phi,\Psi\rangle}$ an element of IF_1 and $\gamma\in[0,1]$, then we introduce the subsequent sets:

$$\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_l^+(\gamma)=\inf\{z\in\mathbb{R}\mid\Phi(z)\geq\gamma\},\qquad \Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_r^+(\gamma)=\sup\{z\in\mathbb{R}\mid\Phi(z)\geq\gamma\}$$
$$\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_l^-(\gamma)=\inf\{z\in\mathbb{R}\mid\Psi(z)\leq 1-\gamma\},\qquad \Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_r^-(\gamma)=\sup\{z\in\mathbb{R}\mid\Psi(z)\leq 1-\gamma\}$$

Remark 2.2. *It is worth noting that the above defined subsequent sets represent the left and right extremities of the lower and upper γ-cuts expressed as the real intervals below:*

$$\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_{\gamma}=\bigg[\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_l^+(\gamma),\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_r^+(\gamma)\bigg],$$
$$\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]^{\gamma}=\bigg[\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_l^-(\gamma),\Big[\mathcal{E}_{\langle\Phi,\Psi\rangle}\Big]_r^-(\gamma)\bigg].$$

Definition 2.4. [12] Consider an intuitionistic fuzzy valued function $\mathcal{K}_{\langle\Phi,\Psi\rangle}:\mathbb{I}\to\mathrm{IF}_1$ and $s_0\in\mathbb{I}$. Then $\mathcal{K}_{\langle\Phi,\Psi\rangle}$ is said to be intuitionistic fuzzy continuous in s_0 if and only if:

$$(\forall\epsilon>0)(\exists\eta>0)\Big(\forall s\in\mathbb{I}\text{ such as }\mid s-s_0\mid<\eta\Big)\Rightarrow\mathcal{D}_{\infty}\Big(\mathcal{K}_{\langle\Phi,\Psi\rangle}(s),\mathcal{K}_{\langle\Phi,\Psi\rangle}(s_0)\Big)<\epsilon.$$

Definition 2.5. [12] We say that the function $\mathcal{K}_{\langle\Phi,\Psi\rangle}:\mathbb{I}\to\mathrm{IF}_1$ is differentiable at $s_0\in(b,c)$ if there exists $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(s_0)\in\mathrm{IF}_1$ such that

$$\lim_{h\to0^+}\frac{\mathcal{K}_{\langle\Phi,\Psi\rangle}(t_0+h)\ominus\mathcal{K}_{\langle\Phi,\Psi\rangle}(s_0)}{h}\quad\text{and}\quad\lim_{h\to0^+}\frac{\mathcal{K}_{\langle\Phi,\Psi\rangle}(s_0)\ominus\mathcal{K}_{\langle\Phi,\Psi\rangle}(s_0-h)}{h},$$

exist and they are equal to $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(s_0)$.

If $\mathcal{K}_{\langle\Phi,\Psi\rangle}:\mathbb{I}\to\mathrm{IF}_1$ is differentiable at $s_0\in\mathbb{I}$, then we say that $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(s_0)$ is the intuitionistic fuzzy derivative of $\mathcal{K}_{\langle\Phi,\Psi\rangle}(s)$ at the point s_0.

Definition 2.6. We say that the function $\mathcal{K}_{\langle\Phi,\Psi\rangle}:\mathbb{I}\to\mathrm{IF}_1$ is differentiable of n-th order at $s_0\in(b,c)$ if there exists $\mathcal{K}^{(k)}_{\langle\Phi,\Psi\rangle}(s_0)\in\mathrm{IF}_1$ for all $k=1,2,\ldots,n$ such that

$$\lim_{h\to0^+}\frac{\mathcal{K}^{(k-1)}_{\langle\Phi,\Psi\rangle}(s_0+h)\ominus\mathcal{K}^{(k-1)}_{\langle\Phi,\Psi\rangle}(s_0)}{h}\quad\textit{and}\quad\lim_{h\to0^+}\frac{\mathcal{K}^{(k-1)}_{\langle\Phi,\Psi\rangle}(s_0)\ominus\mathcal{K}^{(k-1)}_{\langle\Phi,\Psi\rangle}(t_0-h)}{h},$$

exist and they are equal to $\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t_0)$.

The limit is taken in the complete metric space $(\mathrm{IF}_1,\mathcal{D}_{\infty})$ [22]. At the end points of $\mathbb{I}$, we take only the one-sided derivatives.

Theorem 2.1. *Consider that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$, $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)$, $\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)$, $\ldots$, $\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)$ are n-th order differentiable intuitionistic fuzzy-valued functions and let γ-cut sets of $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ with*

$$[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \quad [\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}^+_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^+_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

Then, we have

$$[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \quad [\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

Proof. Due to the fact that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ is differentiable, we have

$$[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \quad [\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}'^+_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^+_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

Again $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)$ is differentiable, then by Definition 2.6 we get:

$$\Big[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t+h)\ominus\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)\Big]_\gamma = \Big[\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t+h,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t+h,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)\Big],$$

and

$$\Big[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)\ominus\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t-h)\Big]_\gamma = \Big[\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t-h,\gamma), \mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t-h,\gamma)\Big].$$

We apply the multiplicity by $h > 0$ and we get:

$$\begin{aligned}\frac{1}{h}&\Big[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t+h)\ominus\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)\Big]_\gamma \\ &= \Big[\frac{1}{h}(\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t+h,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \frac{1}{h}(\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t+h,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma))\Big]\end{aligned}$$

and

$$\begin{aligned}\frac{1}{h}&\Big[\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)\ominus\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t-h)\Big]_\gamma \\ &= \Big[\frac{1}{h}(\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,l}(t-h,\gamma)), \frac{1}{h}(\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)-\mathcal{K}'^-_{\langle\Phi,\Psi\rangle,r}(t-h,\gamma))\Big]\end{aligned}$$

At last, by evaluating the limit, we conclude that

$$[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)]_\gamma = [\mathcal{K}''^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}''^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)].$$

To prove that $[\mathcal{K}''_{\langle\Phi,\Psi\rangle}(t)]^\gamma = [\mathcal{K}''^+_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}''^+_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]$, we apply the same idea as above.

For the third order, fourth order, and up to the n-th order, i.e., $\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t_0)$, the proof is analogous, i.e., we get:

$$\begin{aligned}&\Big[\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t+h)\ominus\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)\Big]_\gamma \\ &\quad= \Big[\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t+h,\gamma)-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t+h,\gamma), \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t+h,\gamma)-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t+h,\gamma)\Big],\end{aligned}$$

and

$$\begin{aligned}&\Big[\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)\ominus\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t-h)\Big]_\gamma \\ &\quad= \Big[\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t-h,\gamma), \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t-h,\gamma)\Big].\end{aligned}$$

We apply the multiplicity by $h > 0$ and we get:

$$\frac{1}{h}\Big[\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t+h) \ominus \mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)\Big]_{\gamma}$$
$$= \Big[\frac{1}{h}(\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t+h,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \frac{1}{h}(\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t+h,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))\Big],$$

and

$$\frac{1}{h}\Big[\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t) \ominus \mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t-h)\Big]_{\gamma}$$
$$= \Big[\frac{1}{h}(\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(t-h,\gamma)), \frac{1}{h}(\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(t-h,\gamma))\Big]$$

At last, by evaluating the limit, we conclude that $[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)]_{\gamma} = [\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]$.

To prove that $[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t,\gamma)(t)]^{\gamma} = [\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), \mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]$, the proof is analogous. □

Definition 2.7. [13] Consider $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$ an intuitionistic fuzzy value function and continuous on $[0,\infty)$. Assume that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)e^{-st}$ is improper intuitionistic fuzzy Riemann integrable on $[0,\infty)$, then, $\int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(t)e^{-st}dt$ is said to be intuitionistic fuzzy Laplace transform and is given as

$$\mathbf{L}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] = \int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(t)e^{-st}dt, s > 0.$$

We obtain

$$\int_0^\infty \mathcal{K}_{\langle\Phi,\Psi\rangle}(t,\gamma)e^{-st}dt = \Bigg(\int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)e^{-st}dt, \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)e^{-st}dt,$$
$$\int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)e^{-st}dt, \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)e^{-st}dt\Bigg).$$

Also through the definition of the classical Laplace transform

$$\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) = \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)e^{-st}dt,$$
$$\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) = \int_0^\infty \mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)e^{-st}dt,$$
$$\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) = \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)e^{-st}dt,$$
$$\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) = \int_0^\infty \mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)e^{-st}dt.$$

As a result, we obtain:

$$\mathbf{L}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t,\gamma)]$$
$$= \Bigg(\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))\Bigg). \quad (2.1)$$

Theorem 2.2. *Suppose that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$, $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t),\ldots,\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)$ are intuitionistic fuzzy valued functions and continuous on $[0,\infty)$ given that $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)e^{-st}$, $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t)e^{-st},\ldots,\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)e^{-st}$ exist, are continuous and improper intuitionistic fuzzy Riemann integrable on $[0,\infty)$. Then, we get*

$$\begin{aligned}&\boldsymbol{L}[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)]\\&\quad= s^n\boldsymbol{L}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)]\ominus s^{n-1}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0)\ominus s^{n-2}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(0)\ominus s^{n-3}\mathcal{K}''_{\langle\Phi,\Psi\rangle}(0)\ \ominus\cdots\ominus\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(0).\end{aligned}\tag{2.2}$$

Proof. First, we consider

$$\begin{aligned}&\Big(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma),\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)\Big),\\&\Big(\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma),\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)\Big),\ \ldots,\\&\Big(\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma),\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma),\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)\Big),\end{aligned}$$

the PF, respectively, of the $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$, $\mathcal{K}'_{\langle\Phi,\Psi\rangle}(t),\ldots,\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)$.

Through the Definition 2.7 we obtain:

$$\mathbf{L}[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)]=\int_0^\infty\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)e^{-st}dt,\, s>0\tag{2.3}$$

$$=s\int_0^\infty\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)e^{-st}dt\ominus\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(0),\tag{2.4}$$

$$=s\mathbf{L}[\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(t)]\ominus\mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(0).\tag{2.5}$$

Actually, for every fixed $\gamma\in[0,1]$, by means of the PF and the classical Laplace transform, we obtain

$$\begin{cases}\mathcal{L}(\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma))=s.[s^{(n-1)}\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma))-s^{(n-2)}\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)\\-s^{(n-3)}\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)-s^{(n-4)}\mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)\ -\cdots-s\mathcal{K}^{(n-2)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)]-\mathcal{K}^{(n-1)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma),\\ \\ \mathcal{L}(\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))=s.[s^{(n-1)}\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))-s^{(n-2)}\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\\-s^{(n-3)}\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)-s^{(n-4)}\mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\ -\cdots-\mathcal{K}^{(n-2)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)]-\mathcal{K}^{(n-1)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma),\\ \\ \mathcal{L}(\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma))=s.[s^{(n-1)}\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma))-s^{(n-2)}\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)\\-s^{(n-3)}\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)-s^{(n-4)}\mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)\ -\cdots-\mathcal{K}^{(n-2)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)]-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma),\\ \\ \mathcal{L}(\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))=s.[s^{(n-1)}\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))-s^{(n-2)}\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\\-s^{(n-3)}\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)-s^{(n-4)}\mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)\ -\cdots-\mathcal{K}^{(n-2)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)]-\mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma),\end{cases}$$

Therefore,

$$\begin{cases} \mathcal{L}(\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) = s^n\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) - s^{n-1}\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - s^{n-2}\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \\ -s^{n-3}\mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \ - \cdots - s\mathcal{K}^{(n-2)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \mathcal{K}^{(n-1)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \\ \\ \mathcal{L}(\mathcal{K}^{(n)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) = s^n\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) - s^{n-1}\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - s^{n-2}\mathcal{K}'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \\ -s^{n-3}\mathcal{K}''^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \ - \cdots - s\mathcal{K}^{(n-2)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \mathcal{K}^{(n-1)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma), \\ \\ \mathcal{L}(\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) = s^n\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)) - s^{n-1}\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - s^{n-2}\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \\ -s^{n-3}\mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \ - \cdots - s\mathcal{K}^{(n-2)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma), \\ \\ \mathcal{L}(\mathcal{K}^{(n)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) = s^n\mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)) - s^{n-1}\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - s^{n-2}\mathcal{K}'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \\ -s^{n-3}\mathcal{K}''^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \ - \cdots - s\mathcal{K}^{(n-2)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) - \mathcal{K}^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma), \\ \end{cases}$$

Since $\Big(\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))\Big)$ is the parametric form of the intuitionistic fuzzy Laplace transform of $\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)$, that means

$$\begin{aligned} &\mathbf{L}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t,\gamma)] \\ &\qquad = \Big(\mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)), \mathcal{L}(\mathcal{K}^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma))\Big). \end{aligned} \tag{2.6}$$

Consequently, we obtain

$$\begin{aligned} &\mathbf{L}[\mathcal{K}^{(n)}_{\langle\Phi,\Psi\rangle}(t)] \\ &\quad = s^n\mathbf{L}[\mathcal{K}_{\langle\Phi,\Psi\rangle}(t)] \ominus s^{n-1}\mathcal{K}_{\langle\Phi,\Psi\rangle}(0) \ominus s^{n-2}\mathcal{K}'_{\langle\Phi,\Psi\rangle}(0) \ominus s^{n-3}\mathcal{K}''_{\langle\Phi,\Psi\rangle}(0) ... \ominus \mathcal{K}^{(n-1)}_{\langle\Phi,\Psi\rangle}(0). \end{aligned} \tag{2.7}$$

This completes the proof. □

Theorem 2.3. [13] *Consider $\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t), \mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}(t)$ to be two intuitionistic fuzzy-valued functions and continuous on $[b,c]$, and c_1, c_2 to be two constants in $\mathbb{R}$, then*

$$\boldsymbol{L}[c_1\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t) \oplus c_2\mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}] = c_1\boldsymbol{L}[\mathcal{G}_{\langle\Phi_1,\Psi_1\rangle}(t)] \oplus c_2\boldsymbol{L}[\mathcal{K}_{\langle\Phi_2,\Psi_2\rangle}(t)].$$

3 Intuitionistic fuzzy Laplace Adomian decomposition method for n-th order intuitionistic fuzzy differential equations

In this section, we discuss how to apply the intuitionistic fuzzy Laplace Adomian decomposition method (IFLADM) for solving intuitionistic fuzzy differential equations of higher order.

Consider an non-homogeneous nonlinear intuitionistic fuzzy differential equation with initial conditions in the general form given by

$$\mathcal{H}y_{\langle\Phi,\Psi\rangle}(t) \oplus \mathcal{R}y_{\langle\Phi,\Psi\rangle}(t) \oplus \mathcal{N}y_{\langle\Phi,\Psi\rangle}(t) = f_{\langle\Phi_1,\Psi_1\rangle}(t), \tag{3.1}$$

where

- $\mathcal{H}$ is the derivative of the highest order, which is supposed to be invertible,
- $\mathcal{R}$ represents a linear differential operator of order less than $\mathcal{H}$,
- $\mathcal{N}y_{\langle\Phi,\Psi\rangle}$ is the nonlinear terms,
- $f_{\langle\Phi_1,\Psi_1\rangle}(t)$ is the source term.

At the first step, we apply the intuitionistic fuzzy Laplace transform on both sides of (3.1):

$$\mathbf{L}[\mathcal{H}y_{\langle\Phi,\Psi\rangle}(t)] \oplus \mathbf{L}[\mathcal{R}y_{\langle\Phi,\Psi\rangle}(t)] \oplus \mathbf{L}[\mathcal{N}y_{\langle\Phi,\Psi\rangle}(t)] = \mathbf{L}[f_{\langle\Phi_1,\Psi_1\rangle}(t)]. \tag{3.2}$$

By means of the parametric form of (3.2) and the initial conditions we obtain

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}[\mathcal{N}y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}[\mathcal{N}y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}[\mathcal{N}y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}[\mathcal{N}y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)]. \end{cases} \tag{3.3}$$

At the second step, we decompose the solution $y_{\langle\Phi,\Psi\rangle}$ and the nonlinear term $\mathcal{N}y_{\langle\Phi,\Psi\rangle}$ as an infinite series expressed in (3.4) and (3.5), respectively

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma). \end{cases} \tag{3.4}$$

$$\begin{cases} \mathcal{N}y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} \mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ \mathcal{N}y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} \mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma), \\ \mathcal{N}y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} \mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ \mathcal{N}y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} \mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma), \end{cases} \tag{3.5}$$

such that $\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,nl}(t,\gamma)$, $\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,nr}(t,\gamma)$, $\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,nl}(t,\gamma)$ and $\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,nr}(t,\gamma)$ are Adomian polynomials [3] of $y_{\langle\Phi,\Psi\rangle,0}, y_{\langle\Phi,\Psi\rangle,1}, \ldots, \ y_{\langle\Phi,\Psi\rangle,n}$ which can be determined by:

$$\begin{cases} \mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma) = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[\mathcal{N}\left(\sum\limits_{k=0}^{i}\lambda^k y^{+}_{\langle\Phi,\Psi\rangle,kl}(t,\gamma)\right)\right]_{\lambda=0}, & i=0,1,2,\ldots, \\ \mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma) = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[\mathcal{N}\left(\sum\limits_{k=0}^{i}\lambda^k y^{+}_{\langle\Phi,\Psi\rangle,kr}(t,\gamma)\right)\right]_{\lambda=0}, & i=0,1,2,\ldots, \\ \mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma) = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[\mathcal{N}\left(\sum\limits_{k=0}^{i}\lambda^k y^{-}_{\langle\Phi,\Psi\rangle,kl}(t,\gamma)\right)\right]_{\lambda=0}, & i=0,1,2,\ldots, \\ \mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma) = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[\mathcal{N}\left(\sum\limits_{k=0}^{i}\lambda^k y^{-}_{\langle\Phi,\Psi\rangle,kr}(t,\gamma)\right)\right]_{\lambda=0}, & i=0,1,2,\ldots. \end{cases} \tag{3.6}$$

We replace the equations in (3.4) and (3.5) in Eqs. (3.3) we obtain

$$\begin{cases} \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\sum\limits_{i=0}^{\infty}\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\sum\limits_{i=0}^{\infty}\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right] + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\sum\limits_{i=0}^{\infty}\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad - \dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\sum\limits_{i=0}^{\infty}\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right] + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)]. \end{cases} \tag{3.7}$$

We compare both sides of Eq. (3.7), and by using the classical ADM we get

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)] = \dfrac{y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad + \dfrac{1}{s^n}\mathcal{L}[f^{+}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma)}{s^n} \\ \qquad + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)] = \dfrac{y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s} + \dfrac{y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^2} + \cdots + \dfrac{y^{(n-1)-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma)}{s^n} \\ \qquad + \dfrac{1}{s^n}\mathcal{L}[f^{-}_{\langle\Phi_1,\Psi_1\rangle,r}(t,\gamma)]. \end{cases} \tag{3.8}$$

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)\right], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)\right], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma)\right], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma)\right]. \end{cases} \tag{3.9}$$

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)\right], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)\right], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma)\right], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma)\right]. \end{cases} \tag{3.10}$$

In general, the recursive formula is represented as

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right], \quad i \geq 0 \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right], \quad i \geq 0 \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right], \quad i \geq 0 \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma)] = -\dfrac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] - \dfrac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right], \quad i \geq 0. \end{cases} \tag{3.11}$$

Finally, we apply inverse Laplace transform to Eqs. (3.8)–(3.11), and our recursive formula is represented as:

$$
\begin{cases}
y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{F}^{+}_{\langle\Phi_2,\Psi_2\rangle,0l}(t,\gamma), \\
y^{+}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[-\frac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] - \frac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right]\right], \quad i \geq 0 \\
y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{F}^{+}_{\langle\Phi_2,\Psi_2\rangle,0r}(t,\gamma), \\
y^{+}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[-\frac{1}{s^n}\mathcal{L}[\mathcal{R}y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] - \frac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right]\right], \quad i \geq 0 \\
y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{F}^{-}_{\langle\Phi_2,\Psi_2\rangle,0l}(t,\gamma), \\
y^{-}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[-\frac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] - \frac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right]\right], \quad i \geq 0 \\
y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{F}^{-}_{\langle\Phi_2,\Psi_2\rangle,0r}(t,\gamma), \\
y^{-}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[-\frac{1}{s^n}\mathcal{L}[\mathcal{R}y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] - \frac{1}{s^n}\mathcal{L}\left[\mathcal{A}^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right]\right], \quad i \geq 0,
\end{cases}
\tag{3.12}
$$

such that $\mathcal{F}^{+}_{\langle\Phi_2,\Psi_2\rangle,0l}(t,\gamma)$, $\mathcal{F}^{+}_{\langle\Phi_2,\Psi_2\rangle,0r}(t,\gamma)$, $\mathcal{F}^{-}_{\langle\Phi_2,\Psi_2\rangle,0l}(t,\gamma)$ and $\mathcal{F}^{-}_{\langle\Phi_2,\Psi_2\rangle,0r}(t,\gamma)$ express the terms obtained from the source term and the initial conditions.

4 Numerical examples

4.1 First example

We attack the second order intuitionistic fuzzy differential equation of the form

$$
\begin{cases}
y''_{\langle\Phi,\Psi\rangle}(t) \ominus 2y'_{\langle\Phi,\Psi\rangle}(t) \ominus 3y_{\langle\Phi,\Psi\rangle}(t) = 0, \\
y_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(3+\gamma, 5-\gamma, 2+2\gamma, 6-2\gamma\Big), \\
y'_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(2+\gamma, 4-\gamma, 1+2\gamma, 5-2\gamma\Big).
\end{cases}
\tag{4.1}
$$

The exact solution of (4.1) is given by

$$
\begin{cases}
Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{2\gamma+7}{4})e^{-t} + (\frac{5+2\gamma}{4})e^{3t}, \\
Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-2\gamma+11}{4})e^{-t} + (\frac{-2\gamma+9}{4})e^{3t}, \\
Y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\frac{4\gamma+5}{4})e^{-t} + (\frac{4\gamma+3}{4})e^{3t}, \\
Y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (\frac{-4\gamma+13}{4})e^{-t} + (\frac{-4\gamma+11}{4})e^{3t}.
\end{cases}
\tag{4.2}
$$

The PF of (4.1) is represented by

$$
\begin{cases}
y''^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - 2y'^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - 3y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 0, \\
y''^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - 2y'^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - 3y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 0, \\
y''^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - 2y'^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - 3y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 0, \\
y''^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - 2y'^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - 3y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 0.
\end{cases}
\tag{4.3}
$$

Applying the intuitionistic fuzzy Laplace transforms in their parametric form and using Theorem 2.2, we get

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{1}{s^2}y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{s-2}{s^2}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{1}{s^2}y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{s-2}{s^2}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{1}{s^2}y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{s-2}{s^2}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{1}{s^2}y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{s-2}{s^2}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.4}$$

Next we substitute the values of $y'_{\langle\Phi,\Psi\rangle}(0,\gamma)$ and $y_{\langle\Phi,\Psi\rangle}(0,\gamma)$ in (4.4), and we get:

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{-(4+\gamma)+(3+\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{-(6-\gamma)+(5-\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{-(3+2\gamma)+(2+2\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{-(7-2\gamma)+(6-2\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.5}$$

In view of (3.4), we decompose $y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$, $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$, $y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ in the following form:

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma). \end{cases} \tag{4.6}$$

Now, putting the equations of (4.6) in (4.5), we will get

$$\begin{cases} \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] = \frac{-(4+\gamma)+(3+\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right] = \frac{-(6-\gamma)+(5-\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right] = \frac{-(3+2\gamma)+(2+2\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)\right], \\ \mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma\right] = \frac{-(7-2\gamma)+(6-2\gamma)s}{s^2} + \frac{2s+3}{s^2}\mathcal{L}\left[\sum\limits_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)\right]. \end{cases} \tag{4.7}$$

Applying the inverse Laplace transform to the system (4.7), our required recursive relation is given by

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{-(4+\gamma)+(3+\gamma)s}{s^2}\right], \\ y^{+}_{(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0 \\ y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{-(6-\gamma)+(5-\gamma)s}{s^2}\right], \\ y^{+}_{(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{-(3+2\gamma)+(2+2\gamma)s}{s^2}\right], \\ y^{-}_{(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{-(7-2\gamma)+(6-2\gamma)s}{s^2}\right], \\ y^{-}_{(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0. \end{cases} \tag{4.8}$$

Thus,

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = -(4+\gamma)t+(3+\gamma), \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0 \\ y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = -(6-\gamma)t+(5-\gamma), \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = -(3+2\gamma)t+(2+2\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = -(7-2\gamma)t+(6-2\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{2s+3}{s^2}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0. \end{cases} \tag{4.9}$$

For $i = 0$, we have

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) = (6+2\gamma)t+(1+\gamma)\dfrac{t^2}{2!}-(12+3\gamma)\dfrac{t^3}{3!}, \\ y^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) = (10-2\gamma)t+(3-\gamma)\dfrac{t^2}{2!}+(-18+3\gamma)\dfrac{t^3}{3!}, \\ y^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) = (4+4\gamma)t+2\gamma\dfrac{t^2}{2!}-(9+6\gamma)\dfrac{t^3}{3!}, \\ y^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) = (12-4\gamma)t+(4-2\gamma)\dfrac{t^2}{2!}+(-21+6\gamma)\dfrac{t^3}{3!}. \end{cases} \tag{4.10}$$

For $i = 1$, we have

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) = (12+4\gamma)\dfrac{t^2}{2!} + (20+8\gamma)\dfrac{t^3}{3!} - (21+3\gamma)\dfrac{t^4}{4!} - (36+9\gamma)\dfrac{t^5}{5!}, \\ y^{+}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) = -(-20+4\gamma)\dfrac{t^2}{2!} - (-36+8\gamma)\dfrac{t^3}{3!} + (-27+3\gamma)\dfrac{t^4}{4!} + (-54+9\gamma)\dfrac{t^5}{5!}, \\ y^{-}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) = (8+8\gamma)\dfrac{t^2}{2!} + (12+16\gamma)\dfrac{t^3}{3!} - (18+6\gamma)\dfrac{t^4}{4!} - (27+18\gamma)\dfrac{t^5}{5!}, \\ y^{-}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) = -(-24+8\gamma)\dfrac{t^2}{2!} - (-44+16\gamma)\dfrac{t^3}{3!} + (-30+6\gamma)\dfrac{t^4}{4!} + (-63+18\gamma)\dfrac{t^5}{5!}. \end{cases} \tag{4.11}$$

We obtain the approximate solution as follows

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) + y^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) + y^{+}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) + \cdots, \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) + y^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) + y^{+}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) + \cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) + y^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) + y^{-}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) + \cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) + y^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) + y^{-}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) + \cdots. \end{cases} \tag{4.12}$$

Thus,

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) & = -(4+\gamma)t + (3+\gamma) + (6+2\gamma)t + (1+\gamma)\frac{t^2}{2!} - (12+3\gamma)\frac{t^3}{3!} \\ & \quad +(12+4\gamma)\frac{t^2}{2!} + (20+8\gamma)\frac{t^3}{3!} - (21+3\gamma)\frac{t^4}{4!} - (36+9\gamma)\frac{t^5}{5!} + \cdots, \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) & = -(6-\gamma)t + (5-\gamma) + (10-2\gamma)t + (3-\gamma)\frac{t^2}{2!} + (-18+3\gamma)\frac{t^3}{3!} \\ & \quad -(-20+4\gamma)\frac{t^2}{2!} - (-36+8\gamma)\frac{t^3}{3!} + (-27+3\gamma)\frac{t^4}{4!} + (-54+9\gamma)\frac{t^5}{5!} + \cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) & = -(3+2\gamma)t + (2+2\gamma) + (4+4\gamma)t + 2\gamma\frac{t^2}{2!} - (9+6\gamma)\frac{t^3}{3!} \\ & \quad +(8+8\gamma)\frac{t^2}{2!} + (12+16\gamma)\frac{t^3}{3!} - (18+6\gamma)\frac{t^4}{4!} - (27+18\gamma)\frac{t^5}{5!} + \cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) & = -(7-2\gamma)t + (6-2\gamma) + (12-4\gamma)t + (4-2\gamma)\frac{t^2}{2!} + (-21+6\gamma)\frac{t^3}{3!} \\ & \quad -(-24+8\gamma)\frac{t^2}{2!} - (-44+16\gamma)\frac{t^3}{3!} + (-30+6\gamma)\frac{t^4}{4!} + (-63+18\gamma)\frac{t^5}{5!} + \cdots. \end{cases} \tag{4.13}$$

Table 1. Values of $Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ for $0 \le \gamma \le 1$ and $t = 1$ corresponding to the membership degrees of the exact and approximate solutions in Example 3.1

γ	$Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **Exact**	$y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **IFLADM**	$Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **Exact**	$y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **IFLADM**
0	25.7507	25.7507	46.2041	46.2041
0.1	26.7734	26.7734	45.1815	45.1814
0.2	27.7961	27.7960	44.1588	44.1588
0.3	28.8187	28.8187	43.1361	43.1361
0.4	29.8414	29.8414	42.1134	42.1134
0.5	30.8641	30.8640	41.0908	41.0908
0.6	31.8867	31.8867	40.0681	40.0681
0.7	32.9094	32.9094	39.0454	39.0454
0.8	33.9321	33.9321	38.0228	38.0227
0.9	34.9547	34.9547	37.0001	37.0001
1	35.9774	35.9774	35.9774	35.9774

Table 2. Values of $Y^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$, $Y^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$, $y^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ for $0 \leq \gamma \leq 1$ and $t = 1$ corresponding to the non-membership degrees of the exact and approximate solutions in Example 3.1

γ	$Y^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **Exact**	$y^-_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **IFLADM**	$Y^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **Exact**	$y^-_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **IFLADM**
0	15.5240	15.5240	56.4308	56.4308
0.1	17.5693	17.5693	54.3855	54.3855
0.2	19.6147	19.6147	52.3402	52.3401
0.3	21.6600	21.6600	50.2948	50.2948
0.4	23.7054	23.7054	48.2495	48.2494
0.5	25.7507	25.7507	46.2041	46.2041
0.6	27.7961	27.7960	44.1588	44.1588
0.7	29.8414	29.8414	42.1134	42.1134
0.8	31.8867	31.8867	40.0681	40.0681
0.9	33.9321	33.9321	38.0228	38.0227
1	35.9774	35.9774	35.9774	35.9774

The comparison between the approximate and exact solutions at $t = 1$ and $N = 12$ is shown at Figure 1.

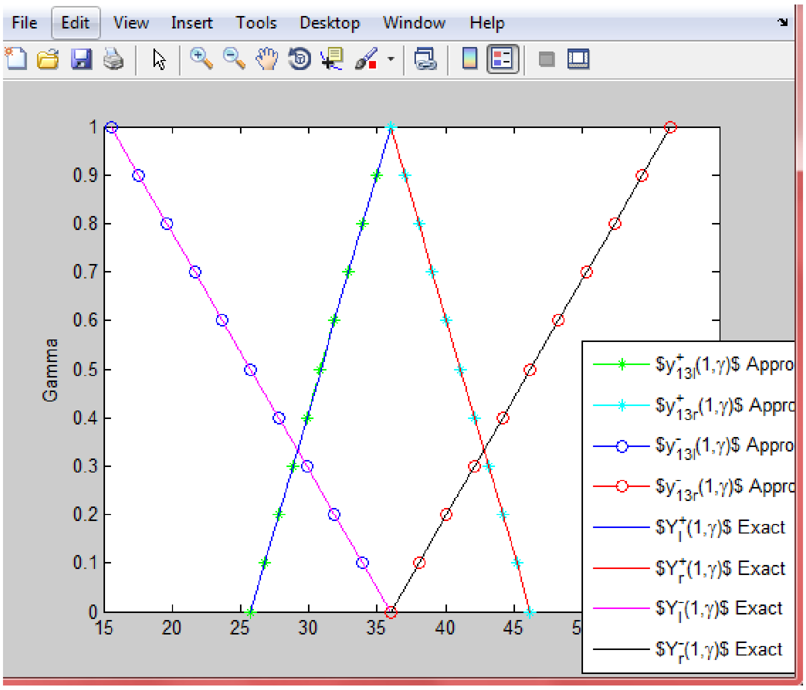

Figure 1. Approximate and exact solutions for membership and nonmembership functions

4.2 Second example

We investigate the fourth order intuitionistic fuzzy ordinary differential equation of the form:

$$\begin{cases} y^{(4)}_{\langle\Phi,\Psi\rangle}(t) \ominus y_{\langle\Phi,\Psi\rangle}(t) = 0, \\ \qquad y_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(-1+\gamma, 1-\gamma, -2+2\gamma, 2-2\gamma \Big), \\ \qquad y'_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(-1+\gamma, 1-\gamma, -2+2\gamma, 2-2\gamma \Big), \\ \qquad y''_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(-1+\gamma, 1-\gamma, -2+2\gamma, 2-2\gamma \Big), \\ \qquad y'''_{\langle\Phi,\Psi\rangle}(0,\gamma) = \Big(-1+\gamma, 1-\gamma, -2+2\gamma, 2-2\gamma \Big). \end{cases} \tag{4.14}$$

The exact solution of (4.14) is given by:

$$\begin{cases} Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\gamma-1)\exp(t), \\ Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (-\gamma+1)\exp(t), \\ Y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (2\gamma-2)\exp(t), \\ Y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (-2\gamma+2)\exp(t). \end{cases} \tag{4.15}$$

The PF of (4.14) is represented as:

$$\begin{cases} y^{(4)+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 0, \\ y^{(4)+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 0, \\ y^{(4)-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) - y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = 0, \\ y^{(4)-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) - y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = 0. \end{cases} \tag{4.16}$$

Applying the intuitionistic fuzzy Laplace transforms in their parametric form and using Theorem 2.2, we get

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{1}{s}y^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^2}y'^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^3}y''^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \\ \qquad + \frac{1}{s^4}y'''^{+}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{1}{s}y^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^2}y'^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^3}y''^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \\ \qquad + \frac{1}{s^4}y'''^{+}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = \frac{1}{s}y^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^2}y'^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^3}y''^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) \\ \qquad + \frac{1}{s^4}y'''^{-}_{\langle\Phi,\Psi\rangle,l}(0,\gamma) + \frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = \frac{1}{s}y^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^2}y'^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^3}y''^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) \\ \qquad + \frac{1}{s^4}y'''^{-}_{\langle\Phi,\Psi\rangle,r}(0,\gamma) + \frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.17}$$

Next we substitute the values of $y_{\langle\Phi,\Psi\rangle}(0,\gamma)$, $y'_{\langle\Phi,\Psi\rangle}(0,\gamma)$, $y''_{\langle\Phi,\Psi\rangle}(0,\gamma)$ and $y'''_{\langle\Phi,\Psi\rangle}(0,\gamma)$ in (4.17), and we get

$$\begin{cases} \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = (-1+\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = (-1+\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)] = (-2+2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)], \\ \mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)] = (2-2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)]. \end{cases} \tag{4.18}$$

In view of (3.4), we decompose $y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$, $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$, $y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ in the following form:

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = \sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma), \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = \sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma). \end{cases} \tag{4.19}$$

We put the equations of (4.19) in Eq. (4.18) and we get

$$\begin{cases} \mathcal{L}[\sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] = (-1+\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[\sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)], \\ \mathcal{L}[\sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] = (-1+\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[\sum_{i=0}^{\infty} y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)], \\ \mathcal{L}[\sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)] = (-2+2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[\sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)], \\ \mathcal{L}[\sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)] = (2-2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})+\frac{1}{s^4}\mathcal{L}[\sum_{i=0}^{\infty} y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]. \end{cases} \tag{4.20}$$

Applying the inverse Laplace transform to Eqs.(4.20), our required recursive relation is given by

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{L}^{-1}\left[(-1+\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})\right], \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0, \\ y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{L}^{-1}\left[(1-\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})\right], \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\frac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0, \\ y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = \mathcal{L}^{-1}\left[(-2+2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})\right], \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i \geq 0, \\ y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = \mathcal{L}^{-1}\left[(2-2\gamma)(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4})\right], \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\frac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i \geq 0. \end{cases} \tag{4.21}$$

Thus,

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = (\gamma-1)\Big(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}\Big), \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i\geq 0 \\ y^{+}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = (-\gamma+1)\Big(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}\Big), \\ y^{+}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{1}{s^4}\mathcal{L}[y^{+}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i\geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0l}(t,\gamma) = (2\gamma-2)\Big(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)l}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,il}(t,\gamma)]\right], \quad i\geq 0 \\ y^{-}_{\langle\Phi,\Psi\rangle,0r}(t,\gamma) = (-2\gamma+2)\Big(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,(i+1)r}(t,\gamma) = \mathcal{L}^{-1}\left[\dfrac{1}{s^4}\mathcal{L}[y^{-}_{\langle\Phi,\Psi\rangle,ir}(t,\gamma)]\right], \quad i\geq 0. \end{cases} \tag{4.22}$$

For $i=0$ we have

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) = (\gamma-1)\Big(\dfrac{t^4}{4!}+\dfrac{t^5}{5!}+\dfrac{t^6}{6!}+\dfrac{t^7}{7!}\Big), \\ y^{+}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) = (-\gamma+1)\Big(\dfrac{t^4}{4!}+\dfrac{t^5}{5!}+\dfrac{t^6}{6!}+\dfrac{t^7}{7!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,1l}(t,\gamma) = (2\gamma-2)\Big(\dfrac{t^4}{4!}+\dfrac{t^5}{5!}+\dfrac{t^6}{6!}+\dfrac{t^7}{7!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,1r}(t,\gamma) = (-2\gamma+2)\Big(\dfrac{t^4}{4!}+\dfrac{t^5}{5!}+\dfrac{t^6}{6!}+\dfrac{t^7}{7!}\Big). \end{cases} \tag{4.23}$$

For $i=1$ we have

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) = (\gamma-1)\Big(\dfrac{t^8}{8!}+\dfrac{t^9}{9!}+\dfrac{t^{10}}{10!}+\dfrac{t^{11}}{11!}\Big), \\ y^{+}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) = (-\gamma+1)\Big(\dfrac{t^8}{8!}+\dfrac{t^9}{9!}+\dfrac{t^{10}}{10!}+\dfrac{t^{11}}{11!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,2l}(t,\gamma) = (2\gamma-2)\Big(\dfrac{t^8}{8!}+\dfrac{t^9}{9!}+\dfrac{t^{10}}{10!}+\dfrac{t^{11}}{11!}\Big), \\ y^{-}_{\langle\Phi,\Psi\rangle,2r}(t,\gamma) = (-2\gamma+2)\Big(\dfrac{t^8}{8!}+\dfrac{t^9}{9!}+\dfrac{t^{10}}{10!}+\dfrac{t^{11}}{11!}\Big). \end{cases} \tag{4.24}$$

We obtain the approximate solution as follows

$$\begin{cases} y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (\gamma-1)\Big(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\frac{t^6}{6!}+\frac{t^7}{7!}+\frac{t^8}{8!}+\frac{t^9}{9!}+\frac{t^{10}}{10!}+\frac{t^{11}}{11!}\Big)+\cdots, \\ y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (-\gamma+1)\Big(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\frac{t^6}{6!}+\frac{t^7}{7!}+\frac{t^8}{8!}+\frac{t^9}{9!}+\frac{t^{10}}{10!}+\frac{t^{11}}{11!}\Big)+\cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma) = (2\gamma-2)\Big(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\frac{t^6}{6!}+\frac{t^7}{7!}+\frac{t^8}{8!}+\frac{t^9}{9!}+\frac{t^{10}}{10!}+\frac{t^{11}}{11!}\Big)+\cdots, \\ y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma) = (-2\gamma+2)\Big(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+\frac{t^6}{6!}+\frac{t^7}{7!}+\frac{t^8}{8!}+\frac{t^9}{9!}+\frac{t^{10}}{10!}+\frac{t^{11}}{11!}\Big)+\cdots. \end{cases} \tag{4.25}$$

Table 3. Values of $Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ for $0 \leq \gamma \leq 1$ and $t = 1$ corresponding to the membership degrees of the exact and approximate solutions in Example 3.2

γ	$Y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **Exact**	$y^{+}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **IFLADM**	$Y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **Exact**	$y^{+}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **IFLADM**
0	-2.718281828459046	-2.718281828459045	2.718281828459046	2.718281828459045
0.1	-2.446453645613141	-2.446453645613141	2.446453645613141	2.446453645613141
0.2	-2.174625462767236	-2.174625462767236	2.174625462767236	2.174625462767236
0.3	-1.902797279921332	-1.902797279921332	1.902797279921332	1.902797279921332
0.4	-1.630969097075428	-1.630969097075427	1.630969097075428	1.630969097075427
0.5	-1.359140914229523	-1.359140914229523	1.359140914229523	1.359140914229523
0.6	-1.087312731383618	-1.087312731383618	1.087312731383618	1.087312731383618
0.7	-0.815484548537714	-0.815484548537714	0.815484548537714	0.815484548537714
0.8	-0.543656365691809	-0.543656365691809	0.543656365691809	0.543656365691809
0.9	-0.271828182845905	-0.271828182845905	0.271828182845905	0.271828182845905
1	0	0	0	0

Table 4. Values of $Y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma), Y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma), y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ and $y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ for $0 \leq \gamma \leq 1$ and $t = 1$ corresponding to the non-membership degrees of the exact and approximate solutions in Example 3.2

γ	$Y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **Exact**	$y^{-}_{\langle\Phi,\Psi\rangle,l}(t,\gamma)$ **IFLADM**	$Y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **Exact**	$y^{-}_{\langle\Phi,\Psi\rangle,r}(t,\gamma)$ **IFLADM**
0	-5.436563656918091	-5.436563656918090	5.436563656918091	5.436563656918090
0.1	-4.892907291226281	-4.892907291226281	4.892907291226281	4.892907291226281
0.2	-4.349250925534473	-4.349250925534473	4.349250925534473	4.349250925534473
0.3	-3.805594559842664	-3.805594559842664	3.805594559842664	3.805594559842664
0.4	-3.261938194150855	-3.261938194150854	3.261938194150855	3.261938194150854
0.5	-2.718281828459046	-2.718281828459045	2.718281828459046	2.718281828459045
0.6	-2.174625462767236	-2.174625462767236	2.174625462767236	2.174625462767236
0.7	-1.630969097075428	-1.630969097075427	1.630969097075428	1.630969097075427
0.8	-1.087312731383618	-1.087312731383618	1.087312731383618	1.087312731383618
0.9	-0.543656365691809	-0.543656365691809	0.543656365691809	0.543656365691809
1	0	0	0	0

The comparison between the approximate and exact solutions at $t = 1$ and $N = 4$ is shown at Figure 2.

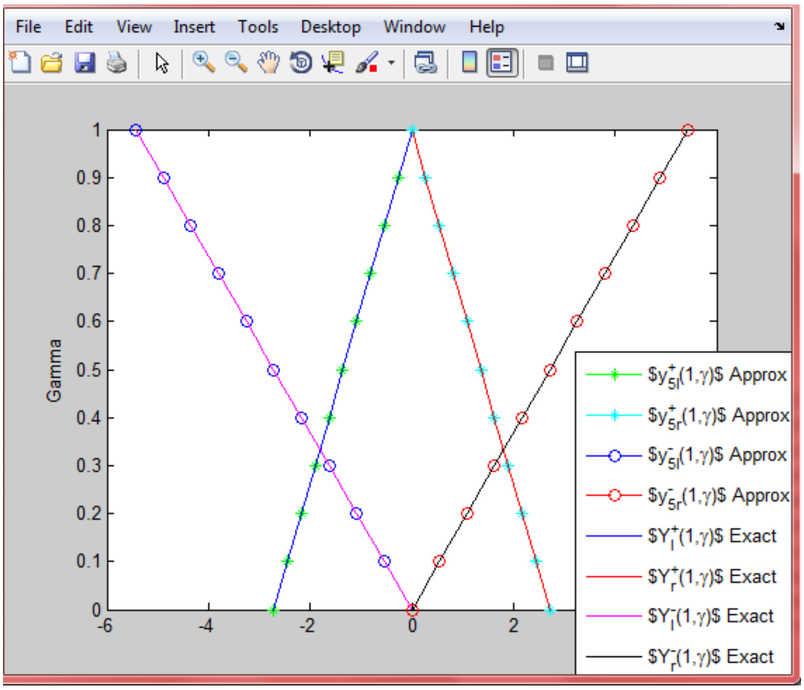

Figure 2. Approximate and exact solutions
for membership and nonmembership functions

5 Conclusion

In this study, we solve the n-th order intuitionistic fuzzy differential equation with intuitionistic fuzzy initial conditions using the intuitionistic fuzzy Laplace Adomian decomposition method. The discussion of the convergence of this method has been applied to intuitionistic fuzzy differential equations of order n. The characteristics and functionality of this analytical-numerical approach were demonstrated through numerical examples.

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