Notes on Intuitionistic Fuzzy Sets
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Characterization of level sets of intuitionistic *L*-fuzzy semi filter of lattices

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Abstract: As an extension of intuitionistic L-fuzzy semi filter (ILFSF) [3] we develop some new concepts of an ILFSF. In this paper, we characterize the level sets of ILFSF.

Keywords: Fuzzy subset, L-fuzzy subset, Intuitionistic L-fuzzy subset, Intuitionistic L-fuzzy semi filter, Intuitionistic L-fuzzy semi ideal, Level sets of ILFSF.

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1 Introduction

In 1965, Lofti A. Zadeh [6] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [1] as a generalization of the notion of fuzzy set. Although a lot of studies have been done for fuzzy order structures the lattice-valued sets can be more appropriate to model natural problems. The justification to consider lattice valued fuzzy sets has been widely explained in the literature, since lattices are more richer structure and we can obtain non-comparable values of fuzzy sets. They can be applied, for instance, in image processing. Also the concepts of *p*-cuts (level sets) play a principal role in the relationship between fuzzy sets and crisp sets. They can be viewed as a bridge by which fuzzy sets and crisp sets are connected.

2 Preliminaries

In this section, some well-known definitions are recalled. It will be necessary in order to understand the new concepts and theorems introduced in this paper.

Definition 2.1. Let $(L, \leq)$ be a complete lattice with least element 0 and greatest element 1 and an involutive order reversing operation $N : L \to L$. Then an Intuitionistic L-fuzzy subset (ILFS) A in a non-empty set X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \mid x \in X\rangle\}$$

where $\mu_A : X \to L$ is the degree membership and $\nu_A : X \to L$ is the degree of non-membership of the element $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

Definition 2.2. Let $(L, \leq)$ be a complete lattice with least element 0 and greatest element 1 and an involutive order reversing operation $N : L \to L$. Then an Intuitionistic L-fuzzy subset (ILFS) A in a non-empty set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \mid x \in X\rangle\}$ where $\mu_A : X \to L$ is the degree membership and $\nu_A : X \to L$ is the degree of non-membership of the element $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

Definition 2.3. An Intuitionistic L-fuzzy set of a Lattice is called as an Intuitionistic L-fuzzy semifilter whenever $x \leq y$, we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$.

Example: Let $X = \{0, 1, 2, 3\}$ and $L = \{0, a, b, 1\}$. Define $\mu : X \to L$ and $\nu : X \to L$ as follows:

X	**0**	**1**	**2**	**3**
μ	a	a	b	1
ν	1	b	a	a

Here

$$0 \leq 1 \Rightarrow \mu_A(0) \leq \mu_A(1) \;\&\; \nu_A(0) \geq \nu_A(1)$$

$$0 \leq 2 \Rightarrow \mu_A(0) \leq \mu_A(2) \;\&\; \nu_A(0) \geq \nu_A(2)$$

$$0 \leq 3 \Rightarrow \mu_A(0) \leq \mu_A(3) \;\&\; \nu_A(0) \geq \nu_A(3)$$

$$1 \leq 2 \Rightarrow \mu_A(1) \leq \mu_A(2) \;\&\; \nu_A(1) \geq \nu_A(2)$$

$$1 \leq 3 \Rightarrow \mu_A(1) \leq \mu_A(3) \;\&\; \nu_A(1) \geq \nu_A(3)$$

$$2 \leq 3 \Rightarrow \mu_A(2) \leq \mu_A(3) \;\&\; \nu_A(2) \geq \nu_A(3)$$

Hence, whenever $x \leq$ we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$.

Definition 2.4. An Intuitionistic L-fuzzy set of a Lattice is called as an Intuitionistic L-fuzzy semi-ideal whenever $x \leq y$, we have $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$.

Definition 2.5. Any sub poset A of a poset X is said to be a semi-filter if $x \in A$, $y \in X$, and $x \le y$ implies $y \in A$.

Definition 2.6. Let $A : X \to L$ is an Intuitionistic L-fuzzy semi filter on X. Then for $p \in L$, the level set of Intuitionistic L-fuzzy semi filter is defined as the set

$$A_p = \{x \in X \mid \mu_A(x) \ge p \text{ and } \nu_A(x) \le p\}.$$

Definition 2.7. Let $A : X \to L$ is an Intuitionistic L-fuzzy semi ideal on X. Then for $p \in L$, the level set of Intuitionistic L-fuzzy semi ideal is defined as the set

$$A_p = \{x \in X \mid \mu_A(x) \ge p \text{ and } \nu_A(x) \le p\}.$$

3 Properties of level sets of ILFSF

Theorem 3.1. Let $(L, \le)$ be a complete lattice and let A be an ILFS on X. Then A is an ILFSF if and only if the level set A_p is an crisp semi-filter.

Proof. Let A be an Intuitionistic L-fuzzy semi-filter, that is $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ and if $x \le y$, we have $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$.

To prove: $A_p = \{x \in X \mid \mu_A(x) \ge p$ and $\nu_A(x) \le p\}$ is a semi-filter. Let $x \in A_p$ and $x \le y$.

By hypothesis $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$. Also $p \le \mu_A(x) \le \mu_A(y)$ and $p > \nu_A(x) \ge \nu_A(y)$, which implies $p \le \mu_A(y)$ and $p \le \nu_A(y)$ that is if $x \in A_p$, $y \in X$ and $x \le y$ implies $y \in A_p$. Therefore, A_p is a semi-crisp filter.

Conversely suppose A_p is a crisp semi-filter. To prove: A is an Intuitionistic L-fuzzy semi-filter. Let $x, y \in A$ and $x \le y$. Let $x \in A_p$, that is $\mu_A(x) \ge p$ and $\nu_A(x) \le p$.

By hypothesis $y \in A_p$. That is $\mu_A(y) \ge p$ and $\nu_A(y) \le p$. Clearly $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$. Thus A is an ILFSF.

Therefore, A is an ILFSF *iff* the level set A_p is a crisp semi-filter. □

Corollary 3.1 Let $(L, \le)$ be a complete lattice and let A be an ILFSI on X. Then A is an ILFSI if and only if the level set A_p is an crisp semi-ideal.

The proof of this theorem is very much similar to the above theorem (duality).

Theorem 3.2. Let $A : X \to L$ be an Intuitionistic L-fuzzy set. Let p_1 , $p_2 \in L$ such that $p_1 < p_2$. Then $A_{p2} \subseteq A_{p1}$.

Proof. Let A: $X \to L$ be an ILFS

$$A_{p1} = \{x \in X \mid \mu_A(x) \ge p_1 \text{ and } \nu_A(x) \le p_1\}$$
$$A_{p2} = \{x \in X \mid \mu_A(x) \ge p_2 \text{ and } \nu_A(x) \le p_2\}$$

Let $x \in A_{p2}$, then $\mu_A(x) \ge p_2 > p_1$ and $\nu_A(x) \le p_2$, thus $\mu_A(x) \ge p_1$.
Also $\nu_A(x) \le p_1$, which implies $x \in A_{p1}$.
Therefore $A_{p2} \subseteq A_{p1}$. □

Theorem 3.3. Let A be an ILFSF on X. Then the intersection of any two level sets of A is again a level set of A.

Proof. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an Intuitionistic L-fuzzy semi-filter, that is and if $x \leq y$, we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$. Let A_p and A_q be two level sets of A.

To prove: $A_P \cap A_q$ is a level set of A.

Let $x \in A_p \cap A_q$

$\Rightarrow x \in A_P$ and $x \in A_q$

$\Rightarrow \mu_A(x) \geq p$, $\mu_A(x) \geq q$ and $\nu_A(x) \leq p$, $\nu_A(x) \leq q$

Let $m = p \wedge q$.

Now $\mu_A(x) \geq p \geq m$, $\mu_A(x) \geq q \geq m$ and also $\nu_A(x) \leq m$, $\nu_A(x) \leq m$

$\Rightarrow x \in A_m$.

Thus $A_p \cap A_q = \{x \in X \mid \mu_A(x) \geq m$ and $\nu_A(x) \leq m\}$ where $m = p \wedge q$.

Thus the intersection of two level sets of an ILFSF is again a level set of A. ☐

Theorem 3.4. Let A be an ILFSI on X. Then the intersection of any two level sets of A is again a level set of A.

The proof of this theorem is analogous to the above theorem.

Theorem 3.5. Let A and B are two ILFSF on X. If $A \subseteq B$ then $A_p \subseteq B_p$.

Proof. Let A and B are Intuitionistic L-fuzzy semi filters and $A \subseteq B$

that is and if $x \leq y$, we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$.

Also, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.

Let $x \in A_p \Rightarrow \mu_A(x) \geq p$ and $\nu_A(x) \leq p$

Now $\mu_B(x) \geq \mu_A(x) \geq p$, and $p > \nu_A(x) \geq \nu_B(x)$.

Thus, $\mu_B(x) \geq p$, and $\nu_B(x) \leq p$. Which implies $x \in B_p$.

Thus, $x \in A_p \Rightarrow x \in B_p$.

Therefore, $A_p \subseteq B_p$. ☐

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