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Geometric interpretation and the properties of two new operators over the intuitionistic fuzzy sets

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Abstract: In the present paper are introduced two new operators over the intuitionistic fuzzy sets (*IFS*). Their geometric interpretations are shown. Their properties are studied.

Keywords: intuitionistic fuzzy set, operators

1 Introduction

Pattern recognition is "the act of taking in raw data and taking an action based on the category of the pattern"[1]. Pattern recognition aims to classify data (patterns) in order to facilitate their use. Some of the main applications are in the field of medicine [2], natural language analysis [3], signal analysis [4], computer vision, etc. The theory of *IFS* [5] permits pattern recognition with non-strict membership of the patterns which makes it a useful tool for solving classification problems in various areas of science.

The application of the theory of *IFS* is predetermined by the fact that in practice the data are not strictly separable. This non-strict membership could be captured the three degrees for the theory of *IFS*, which gives a more adequate description of the data and further insight into how to interpret them.

For example, one of the specifics of the data, obtained from medical studies is that one disease may develop with different severity in different patients. This may be expressed with different degrees of membership and non-membership to the class, corresponding to the considered disease.

According to the definition in [5] the set A^* is called intuitionistic fuzzy set (*IFS*), if:

$$A^* = \{\langle x, \mu_A(x), \nu_A(x) / x \in E \rangle\}$$

where the mappings $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define, respectively, the degree of membership and non-membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

The function π_A, which is defined by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

gives the degree of indeterminacy of the membership of the element $x \in E$ to the set A.

One of the geometric interpretations of the element $x \in E$ is a point from the set, bounded by isosceles right triangle with length of the legs equal to 1, Figure 1.

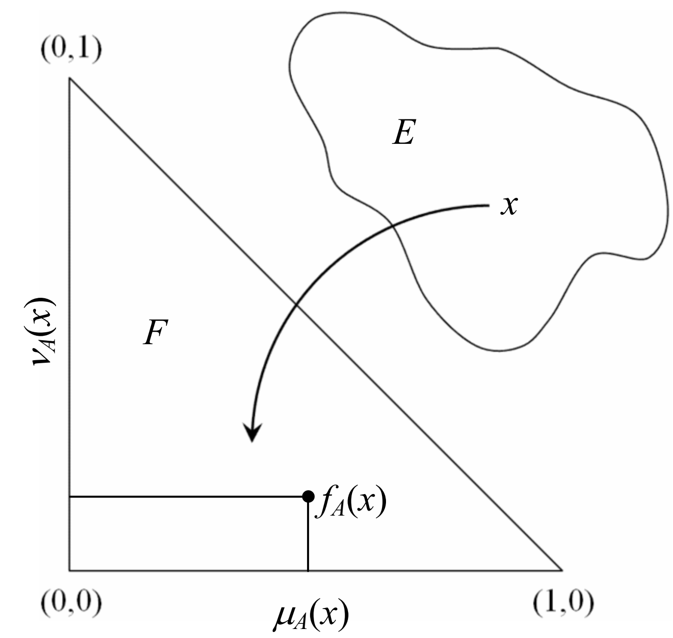

Figure 1

Another geometric interpretation of *IFS* [6] as an equilateral triangle is shown on Figure 2, namely – when the element $x \in E$ is mapped to a point from the set bounded by and equilateral triangle with side equal to $\frac{2\sqrt{3}}{3}$.

In the literature, dedicated to IFS have been defined different operators, operations and relations, as well as their geometric interpretations.

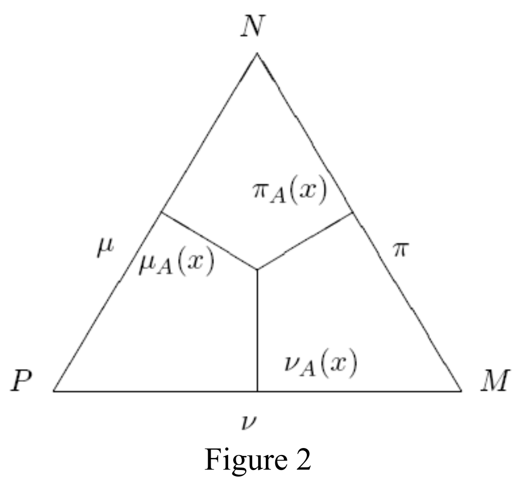

Figure 2

2 Two New Operators

Here we will introduce two new operators, for $\alpha, \beta \in [0,1]$ and $\alpha + \beta \le 1$:

$$P'_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \pi_A(x))\rangle | x \in E\};$$

$$Q'_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \pi_A(x))\rangle | x \in E\}.$$

2.1 Geometric Interpretations

The geometric interpretations of the operator $P'_{\alpha,\beta}(A)$ are shown on Figure 3.

From cases 1, 2, 3 and 4 it can be generalized that the geometric interpretation of the work of the operator $P'_{\alpha,\beta}(A)$ is the movement of all elements in NMOT. The geometric interpretations of the operator $Q'_{\alpha,\beta}(A)$ are shown on Figure 4.

Case 1:
Element $x \in E$ is in the area KLTR: it is preserved.

Case 2:
Element $x \in E$ is in the area LNT: it is moved to a point from the segment LT.

Case 3:
Element $x \in E$ is in the area NMOT: it is moved to the point T.

Case 4:
Element $x \in E$ is in the area TOSR: it is moved to a point from the segment RT.

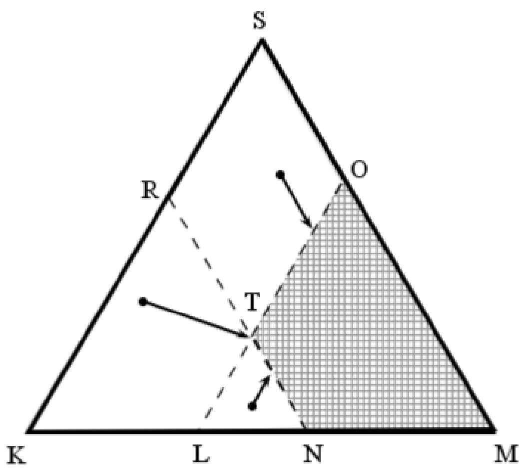

Figure 3.

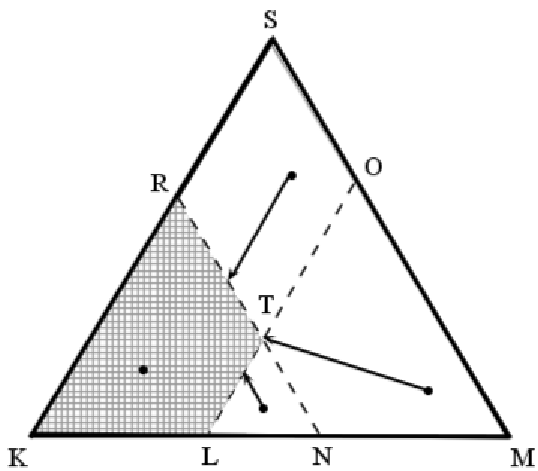

Figure 4.

From cases 1, 2, 3 and 4 it can be generalized that the geometric interpretation of the work of the operator $Q'_{\alpha,\beta}(A)$ is the movement of all elements in TOSR

2.2 Properties

Property: The consecutive application of the two operators (independently on the order) moves every point to the point T $\langle x, \alpha, 1-\alpha-\beta \rangle$.

For the introduced operators a Theorem analogous to Theorem 1.101 from [5] is valid. The Theorem in the case of the new operators is the following:

Theorem: For every IFS A and for every $\alpha, \beta, \gamma, \delta \in [0,1]$, such that $\alpha + \beta \leq 1$, $\gamma + \delta \leq 1$:

(a) $P'_{\alpha,\beta}\left(P'_{\gamma,\delta}(A)\right) = P'_{\max(\alpha,\gamma),\min(\beta,\delta)}(A)$;

(b) $P'_{\alpha,\beta}\left(Q'_{\gamma,\delta}(A)\right) = Q'_{\max(\alpha,\gamma),\min(\beta,\delta)}\left(P'_{\alpha,\beta}(A)\right)$;

(c) $Q'_{\alpha,\beta}\left(P'_{\gamma,\delta}(A)\right) = P'_{\min(\alpha,\gamma),\max(\beta,\delta)}\left(Q'_{\alpha,\beta}(A)\right)$;

(d) $Q'_{\alpha,\beta}\left(Q'_{\gamma,\delta}(A)\right) = Q'_{\min(\alpha,\gamma),\max(\beta,\delta)}(A)$;

Proof:

(a) $P'_{\alpha,\beta}\left(P'_{\gamma,\delta}(A)\right)$

$$= P'_{\alpha,\beta}\left(\left\{\left\langle x, \max(\gamma, \mu_A(x)), 1-\max(\gamma,\mu_A(x)) - \min(\delta, \pi_A(x))\right\rangle \middle| x \in E\right\}\right)$$

$$= \left(\left\{\left\langle x, \max(\alpha, \max(\gamma, \mu_A(x))), 1-\max(\alpha,\max(\gamma,\mu_A(x))) - \min(\beta, \min(\delta, \pi_A(x)))\right\rangle \middle| x \in E\right\}\right)$$

$$= \left(\left\{\left\langle x, \max(\alpha, \gamma, \mu_A(x)), 1-\max(\alpha,\gamma,\mu_A(x)) - \min(\beta, \delta, \pi_A(x))\right\rangle \middle| x \in E\right\}\right)$$

$$= \left(\left\{\left\langle x, \max(\max(\alpha, \gamma), \mu_A(x)), 1-\max(\max(\alpha,\gamma),\mu_A(x)) - \min(\min(\beta, \delta), \pi_A(x))\right\rangle \middle| x \in E\right\}\right)$$

$$= P'_{\max(\alpha,\gamma),\min(\beta,\delta)}(A)$$

The other assertions of the Theorem can be verified in analogous manner.

3 Conclusions

The proposed in the present paper operators permit the adjustment of the parameters α and β. In pattern recognition problems the values of these parameters are determined during the learning stage on the sample set. As a results the values of the degrees of membership and non-membership (and thus indirectly – the values of the degrees of non-membership) are altered. The adjustments to the parameters α and β should continue until the desired recognition accuracy is obtained.

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