Notes on Intuitionistic Fuzzy Sets
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Remark on intuitionistic fuzzy cognitive maps

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Abstract: Since the introduction of the basic model of FCM in 1986 by B. Kosko numerous extensions of FCMs have been developed and tested in various domains of application. Although in the years after 2000 (the period 2001-2012) there is a considerable increase in the number of published research papers on FCMs and the underlying theoretical foundations, only a few formal definitions of the extended models were given. In this work, a new formal definition of Intuitionistic fuzzy cognitive map (IFCM) is proposed combining the model of directed graph and the notion of Intuitionistic fuzzy index matrix.
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1 Introduction

When building different mathematical models of real-world systems scientists and engineers have always to analyze ill-structured and/or weakly-formalized problems. The concept of ill-structured problem (situation) was introduced by H. Simon [14]. It means that the building elements of the system in interest as well as their interrelationships have both quantitative and qualitative nature. In order to cope with this duality the modern decision theory and system analysis utilize various methods of mathematics, cognitive sciences, behavioral sciences, sociology, economics and informatics.

2 Cognitive mapping approach, generic model and extensions

Several new modeling technologies have been proposed in the second half of the past century with the aim to overcome some inherent difficulties of the ill-structured problems. One of the most promising approaches to the analysis and modeling of such problems or systems is based on the notion of cognitive map (CM), and is named cognitive mapping approach to decision making [5]. After the studies conducted by R. Axelrod (1976) in the early 70s cognitive maps (CMs) were introduced as a mathematical and graphical representation of a persons system of beliefs. CMs, also called causal maps, are built up of only two fundamental elements, i.e. concepts (or factors) that represent the variables in the analyzed system and causal beliefs (or relationships) that determine the causal relations among those variables. In general case, each factor represents a variable that characterizes the analyzed system, e.g. factors usually stand for parameters, attributes, states, events, actions, values, goals, trends, components, resources etc. The factors that determine a change are named cause factors, while those that undergo the change are effect factors. The causal relationships between the factors may be positive or negative. To depict these interrelations a signed digraph $G(V, E)$ with vertex set V (nodes of the graph) and arc set E (edges of the graph) was proposed as underlying mathematical model of CMs. The nodes of the signed digraph correspond to the factors that characterize a system or a situation and the edges define causal relationships existing between the factors.

The application of CMs to real-life systems revealed some limitations, e.g. they do not allow the use of fuzzy nodes, fuzzy arcs, or rules, neither they permit feedback, i.e. the arcs could not form closed loops, etc. Hence, Axelrods CM has bounded flexibility to simulate the dynamical behavior of those systems.

Ten years later, in 1986, starting from the generic model of cognitive mapping approach, Axelrods cognitive maps, B. Kosko [10] introduced fuzzy cognitive maps (FCMs) as an extension of the latter. A fuzzy cognitive map (FCM) delineates the whole system to be modeled by a graph representation of system behavior. In general, FCMs are fuzzy signed digraphs with feedback. The functions and powers of cognitive maps were augmented by adding the following properties:

- nodes reflect the degree to which the factors (concepts) are active in the system at a particular time and can take values in the set $\{-1; +1\}$ or in the set $\{0; 1\}$;
- arcs can take any real value in the interval $[-1; +1]$;
- the value of each node is function of time;
- the value of each node at any moment t_{i+1} is a function of the weighted sum of all its incoming node values plus the previous value of the node at time t_i.

The most significant enhancement was the way of assigning fuzzy values to the causal relationships between the nodes. In other words, a weight of causal relationship between two factors quantifies the strength of causal influence from the causal factor to the effect factor.

Although the conventional FCMs are more flexible and reliable than the generic model of CMs, they also have some disadvantages. To overcome them in the past two decades different extensions of FCMs have been proposed and found numerous applications in diverse areas of science, industry and business activities. The first attempt to reflect the advances in their application was made by [1]. Next significant step to shed light on various aspects of the FCMs application was the collection of articles in the book [8] with foreword by B. Kosko. The most comprehensive overview of the evolution in this field is the recent study on FCMs and their applications by E.I. Papageorgiou [11, 12]. Almost all types of FCMs extensions use as a basis the theory of fuzzy sets. Among the variety of those extensions some models proposed in the area of medical diagnosis and decision making have implemented unique approach [7, 9, 13]. Unlike the vast majority of existing models, based on the use of fuzzy sets theory, D.K. Iakovidis, and E. I. Papageorgiou developed two different models of extended FCMs utilizing the notion of Intuitionistic Fuzzy Sets (IFSs, see, e.g. [2, 4]) and called them intuitionistic fuzzy cognitive maps (iFCM-I, iFCM-II). Both models follow the two steps process of construction applied to the conventional model of FCM. The difference between the FCM model and iFCM-I is that to the causal relations (in form of IF-THEN rules) connecting the concepts was included also the hesitation of the experts in expressing these rules. While iFCM-I takes into account only the hesitancy in expression of the influences between concepts (introducing in this way the intuitionistic fuzzy sets in the reasoning process), iFCM-II allows in addition to that the modeling of the concepts as linguistic variables thus giving the model more general mathematical formulation, based entirely on IFS theory. These properties make iFCM-II more capable of modeling both the uncertainties of concept values and those of causal relations among them.

As seen from the previous explanations the mathematical model of directed graph has been gradually extended in order to achieve more precise representation of subjective expert knowledge reflecting the system in interest. That is why the accuracy (precision) of the extended model along with its validity and reliability is very important to support the decision makers in drawing correct inferences, i.e. to derive correct predictions from the extended FCM model. On the other hand the choice of specific model mainly depends on the goals and type of system analysis (static and/or dynamic).

3 Formal definitions

The study of different FCMs extensions revealed that the construction of the models precedes the stage of system analysis. The process of construction and development of FCM may consists of two or three stages [15], but some of them could be combined in one. At the first stage experts are dealing with determination of the conceptual and causal architecture that includes identification of key factors and causal relationships among them. The second stage, parameterization of FCM, encompasses construction of linguistic scales, selection of aggregation functions and assignment of values to nodes and arcs. Namely, the assignment methods for different types of values (signs, numbers, linguistic variables, rules, etc.) to the nodes (factors) and to the arcs (causal relationships) allow the researchers to draw distinction between the various extensions of FCM. This all can be summarized in a preliminary abstract definition of FCM in form of 4-tuple:

$$\mathbf{FCM} = \langle F, W, S_n(f), S_a(w) \rangle,$$

where:

- F - is the set of factors of the system (problem, situation);
- W - stands for the set of causal relationships between factors;

- $S_n(f)$ - is set of scales for the values of system factors;
- $S_a(w)$ - denotes set of scales for the strength of causal relationships.

There are several possible formal definitions of FCMs in the literature. Probably the most commonly used formal definition is in the form given by Chen [6], which respects the original numerical matrix representation proposed by Kosko, where FCM is defined as a 4-tuple,

$$CM = (C, E, \alpha, \beta),$$

where:

- C is finite set of cognitive units (i.e., concepts), $C = \{C_1, C_2, ..., C_n\}$;
- E is a finite set of directed edges between cognitive units, $E = \{e_1, e_2, ..., e_m\}$;
- α is a mapping function from cognitive units to an interval $[a, b]$, where $-1 \leq a \leq b \leq +1$;
- $\beta : E \to [-1, +1]$ is a mapping function from directed edges to real values between -1 and +1.

In this paper we propose a formal definition of Intuitionistic Fuzzy Cognitive Map (IFCM) based on the notion of Intuitionistic Fuzzy Index Matrix (IFIM, see [3]). The new definition is introduced as an extension of Chens formal definition of FCM [6].

4 On intuitionistic fuzzy cognitive maps

Let $\mathcal{C} = \{C_1, C_2, ..., C_n\}$ be a set of cognitive units and for every i ($i \in \{1, 2, ..., n\}$), $\mu_C(C_i)$ and $\nu_C(C_i)$ are degrees of validity and non-validity of the cognitive unit C_i.

Extending Chen's formal definitions of Fuzzy Cognitive Map (FCM, see [6]), we introduce the concept of an Intuitionistic FCM (IFCM) as the pair

$$IFCM = \langle C, E \rangle,$$

where

$$C = \{\langle C_i, \mu_C(C_i), \nu_C(C_i)\rangle | C_i \in \mathcal{C}\}$$

is an IFS and

$$E = [\mathcal{C}, \mathcal{C}, \{\langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle\}],$$

is an Intuitionistic Fuzzy Index Matrix (IFIM, see [3]) of incidence and for every $i, j \in \{1, 2, ..., n\}$, $\mu_E(e_{i,j})$ and $\nu_E(e_{i,j})$ are degrees of validity and non-validity of the oriented edge between neighbouring nodes $C_i, C_j \in \mathcal{C}$.

For every two cognitive units C_i and C_j that are connected with an edge $e_{i,j}$, we can introduce different criteria for correctness, e.g. if C_i is higher than C_j (i.e., $\langle \mu_C(C_i), \nu_C(C_i)\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$), then

1 (top-down-max-min) $\langle \mu_C(C_i), \nu_C(C_i)\rangle \vee \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

2 (top-down-average) $\langle \mu_C(C_i), \nu_C(C_i)\rangle @ \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

3 (top-down-min-max) $\langle \mu_C(C_i), \nu_C(C_i)\rangle \wedge \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \geq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

4 (down-top-max-min) $\langle \mu_C(C_i), \nu_C(C_i)\rangle \wedge \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \leq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

5 (down-top-average) $\langle \mu_C(C_i), \nu_C(C_i)\rangle @ \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \leq \langle \mu_C(C_j), \nu_C(C_j)\rangle$;

6 (down-top-min-max) $\langle \mu_C(C_i), \nu_C(C_i)\rangle \vee \langle \mu_E(e_{i,j}), \nu_E(e_{i,j})\rangle \leq \langle \mu_C(C_j), \nu_C(C_j)\rangle$.

Other criteria also are possible.

If Cr is some one of the above six, or another criterion for correctness, and if all vertices and arcs of a given IFCM satisfy criterion Cr, then this IFSC is called Cr-correct IFCM.

The validity of the following assertion is checked easy on the basis of the above definitions of correctness.

Theorem. If the IFCM is:

(a) (top-down-min-max)-correct, then it is (top-down-average)-correct and (top-down-max-min)-correct;

(b) (top-down-average)-correct, then it is (top-down-max-min)-correct;

(c) (down-top-max-min)-correct, then it is (down-top-average)-correct and (down-top-min-max)-correct;

(d) (down-top-average)-correct, then it is (down-top-min-max)-correct.

5 Conclusion

In near future we will extend the concept of an IFCM is some directions: temporal IFCM, hierarchical IFCM and others, using possibilities, giving by apparatus of the IFIMs.

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