Notes on Intuitionistic Fuzzy Sets
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A system for medical diagnosis based on intuitionistic fuzzy relation

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Abstract: The aim of this paper is to propose a new approach for medical diagnosis by using trapezoidal valued intuitionistic fuzzy relations. First, we develop trapezoidal valued intuitionistic fuzzy relations and then use it to solve medical diagnosis decision making problem. We study Sanchez's method of medical diagnosis with the notion of trapezoidal valued intuitionistic fuzzy sets. To further elaborate our method an example of medical diagnosis is given assuming that there is a database, i.e. description of a set of symptoms and a set of diagnoses.
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1 Introduction

In this paper, a system for medical diagnosis based on trapezoidal valued intuitionistic fuzzy relations is proposed. The functions of this system are: first, to collect all the information that is usually analyzed in these cases by the expert, second, to study the relations between the different considered factors and the patients, and third, to offer an automated diagnostic aid. In some situations it is very difficult to use classical logic to model a system with the available knowledge. Numeric values are not suitable for the representation of human preference for real decision problems. Classical logic does not allow working with uncertainty in the information when knowledge about the behaviour of the systems is imprecise. To solve the ambiguity in information from human preference, Zadeh [24] proposed the concept of fuzzy set theory and successfully used it to handle uncertainty in decision making. Fuzzy logic is an extension of classical logic. Fuzzy logic is used as a tool for representing different forms of knowledge about a problem. Due to this property, fuzzy logic principles have been successfully applied to a wide range of problems in different domains for which uncertainty and vagueness emerge in varying ways. Fuzzy set theory has been regarded as a formalism suitable to deal with the imprecision intrinsic to many medical problems [1, 2, 12, 25]. The behavior of fuzzy systems is likely to be closer to medical reality than the behaviour of the classical systems. At the same time, fuzzy sets allow to use symbolic models. Fuzzy sets can bridge the gap between the discrete world of reasoning and the continuity of reality. The fuzzy versions of multi-criteria decision making (MCDM) techniques are more suitable for subjective and qualitative assessments in the evaluation processes than other classical MCDM techniques applying crisp values [16, 22, 23].

Atanassov [4] gave the notion of intuitionistic fuzzy sets (IFS) as a generalization of Zadeh's [24] fuzzy sets. IFS can be useful in situations when description of a problem by a linguistic variable, given in terms of a membership function only, seems too rough. IFS has proved to be a very suitable tool to describe the uncertain or imprecise decision information. The concept of interval-valued intuitionistic fuzzy sets (IVIFS) was introduced by Atanassov [6], as a generalization of IFS. The basic characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Some operational laws of the IVIFS are defined in [5]. Shannon et al. [21] were the first to develop an approach using IFS for decision making in medical diagnosis. Recently, IFS has been applied in the field of multicriteria decision making [3, 9, 10, 15]. Furthermore, IFS and IVIFS has also been used in medical diagnosis [11, 13, 14]. Beg and Rashid [8] developed the concept of trapezoidal valued intuitionistic fuzzy sets (TVIFS) and used this concept for decision making. Here, we are going to further develop trapezoidal valued intuitionistic fuzzy relations to solve medical diagnosis decision making problem. We study Sanchez's method [19] of medical diagnosis with the notion of TVIFS.

In this study, Section 2 presents an introduction to fuzzy set and some preliminary concepts to understand this paper. In Section 3, we introduce trapezoidal value intuitionistic fuzzy relations and composition of these relations. In Section 4, we propose a new method for the medical diagnostics and flow chart of this method is given. In Section 5, an example is given to understand this method. Conclusion and further researches is given in the last section of paper.

2 Preliminaries

Some preliminary concepts are given in this section.

Let X be a universe of discourse, a fuzzy set [24] in X is an expression A given by $A = \{\langle x, t_A(x)\rangle \,|x \in X\}$, where $t_A : X \to [0, 1]$ is a membership function which characterizes the degree of membership of the element x to the set A . The main characteristic of fuzzy sets is that: the membership function assigns to each element x in a universe of discourse X a membership degree in interval $[0, 1]$ and the non-membership degree equals one minus the membership degree, i.e., this single membership degree combines the evidence for x and the evidence against x. Atanassov [4] generalized the concept of fuzzy set, and defined the concept of intuitionistic fuzzy set in X, is an expression A given by $A = \{(x, t_A(x), f_A(x))|x \in X\}$, where $t_A : X \to [0,1]$, $f_A : X \to [0,1]$ with the condition: $0 \leq t_A(x) + f_A(x) \leq 1$, for all x in X. The numbers $t_A(x)$ and $f_A(x)$ represent the degree of membership and the degree of non-membership of the element x in the set A, respectively. If $1 - t_A(x) - f_A(x) = 0$, for all $x \in X$. Then the intuitionistic fuzzy set A is reduced to a fuzzy set. Atanassov and Gargov [6] subsequently introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers.

Definition 2.1. [17, p. 330] A function 'A' given by

$$A(x) = \begin{cases} 0 & \textit{if} \quad x < a \textit{ or } x > d, \\ \frac{x-a}{b-a} & \textit{if} \quad a \leq x \leq b, \\ 1 & \textit{if} \quad b \leq x \leq c, \\ \frac{x-d}{c-d} & \textit{if} \quad c \leq x \leq d, \end{cases}$$

where $0 \leq a \leq b \leq c \leq d \leq 1$, is called a trapezoidal fuzzy number. Symbolically, A is denoted by (a, b, c, d).

We denote by $Trap[0,1]$ the set of all trapezoidal values in $[0,1]$.
Let $A = (x_1, x_2, x_3, x_4)$, $B = (y_1, y_2, y_3, y_4)$ be two TFN, 'min' and 'max' operator for any two TFNs are defined [7] as:.

$$\min(A, B) = (\min(x_1, y_1), \min(x_2, y_2), \min(x_3, y_3), \min(x_4, y_4))$$

and

$$\max(A, B) = (\max(x_1, y_1), \max(x_2, y_2), \max(x_3, y_3), \max(x_4, y_4))$$

Let $A = (x_1, x_2, x_3, x_4)$ be a TFN, $\sup(A)$ is defined [7] as: $\sup(A) = x_4$.

Fuzzy data is a data type with imprecision or with a source of uncertainty not caused by randomness, but due to ambiguity. Examples of fuzzy data types can easily be found in natural language. It is generally more convenient and useful in describing fuzzy data to use trapezoidal fuzzy numbers [20]. Beg and Rashid [8] defined trapezoidal-valued intuitionistic fuzzy set (TVIFS). The fundamental characteristic of the TVIFS is that the values of its membership function and non-membership function are trapezoidal number rather than exact numbers or interval-valued. IFS and IVIFS are special case of TVIFS.

Definition 2.2. Let X be a universe of discourse. A trapezoidal-valued intuitionistic fuzzy set A in X is an expression given by $A = \{(x, t_A(x), f_A(x))|x \in X\}$, where $t_A : X \to Trap[0,1]$, $f_A : X \to Trap[0,1]$ with the condition: $0 \leq \sup t_A(x) + \sup f_A(x) \leq 1$, for all x in X. The trapezoidal values $t_A(x)$ and $f_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to the set A.

For any two trapezoidal values (x_1, x_2, x_3, x_4) and (x'_1, x'_2, x'_3, x'_4) with $x_4 + x'_4 \leq 1$ belonging to $Trap[0,1]$, let $t_A(x) = (x_1, x_2, x_3, x_4)$, $f_A(x) = (x'_1, x'_2, x'_3, x'_4)$, so we have a trapezoidal-valued intuitionistic fuzzy set whose value is denoted by

$$A = \{\langle x, ((x_1, x_2, x_3, x_4), (x'_1, x'_2, x'_3, x'_4))\rangle \, | x \in X\}.$$

We call $((x_1, x_2, x_3, x_4), (x'_1, x'_2, x'_3, x'_4))$ a trapezoidal-valued intuitionistic fuzzy value.
Beg and Rashid [8] defined a score function and an accuracy function of trapezoidal-valued intuitionistic fuzzy values for the comparison between two trapezoidal-valued intuitionistic fuzzy values.

Definition 2.3. [8] Let $\tilde{a} = ((a_1, a_2, a_3, a_4), (a'_1, a'_2, a'_3, a'_4))$ be a trapezoidal-valued intuitionistic fuzzy values, if $S(\tilde{a}) = (a_1 + a_2 + a_3 + a_4 - a'_1 - a'_2 - a'_3 - a'_4)/4$, then $S(\tilde{a})$ is called a score function of $\tilde{a}$, where $S(\tilde{a}) \in [-1, 1]$.

Definition 2.4. [8] Let $\tilde{a} = ((a_1, a_2, a_3, a_4), (a'_1, a'_2, a'_3, a'_4))$ be a trapezoidal-valued intuitionistic fuzzy values, if $H(\tilde{a}) = (a_1 + a_2 + a_3 + a_4 + a'_1 + a'_2 + a'_3 + a'_4)/4$, then $H(\tilde{a})$ is called an accuracy function of $\tilde{a}$, where $H(\tilde{a}) \in [0, 1]$.

The score function S and the accuracy function H are, respectively, defined as the difference and the sum of the membership function $t_A(x)$ and the non-membership function $f_A(x)$. The order relation between any two trapezoidal-valued intuitionistic fuzzy values is given by:

Definition 2.5. [8] Let $\tilde{a} = ((a_1, a_2, a_3, a_4), (a'_1, a'_2, a'_3, a'_4))$ and $\tilde{b} = ((b_1, b_2, b_3, b_4), (b'_1, b'_2, b'_3, b'_4))$ be any two trapezoidal-valued intuitionistic fuzzy values.

1. If $S(\tilde{a})$¡$S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$.

2. If $S(\tilde{a})$=$S(\tilde{b})$ and;

   i. if $H(\tilde{a})$¡$H(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$.

   ii. if $H(\tilde{a})$=$H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a} = \tilde{b}$.

3 Trapezoidal-valued intuitionistic fuzzy relations

In this section, we define trapezoidal valued intuitionistic fuzzy relations and the composition of relations.

Definition 3.1. Let X and Y be two sets. A trapezoidal-valued intuitionistic fuzzy relation (TVIFR) R from X to Y is a TVIFS of $X \times Y$ characterized by the membership function t_R and non-membership function f_R, where out put value of t_R and f_R is a trapezoidal fuzzy number. A TVIFR R from X to Y will be denoted by $R(X \to Y)$.

Definition 3.2. If A is n TVIFS of X, the composition of the TVIFR $R(X \to Y)$ with A is a TVIFS B of Y denoted by $B = R \circ A$, and is defined as:

$$R \circ A(y) = (t_{R \circ A}(y), f_{R \circ A}(y)) = B(y),$$

where $t_{R \circ A}(y) = \max_x [t_A(x) \min t_R(x, y)]$ and $f_{R \circ A}(y) = \min_x [f_A(x) \max f_R(x, y)], \; \forall y \in Y$.

Definition 3.3. Let $Q(X \to Y)$ and $R(Y \to Z)$ be two TVIFRs. The composition $R \circ Q$ is the trapezoidal-valued intuitionistic fuzzy relation from X to Z, defined as:

$$R \circ Q(x, z) = (t_{R \circ Q}(x, z), f_{R \circ Q}(x, z)),$$

where

$$t_{R \circ Q}(x, z) = \max_y [t_Q(x, y) \min t_R(y, z)]$$

and

$$f_{R \circ Q}(x, z) = \min_y [f_Q(x, y) \max f_R(y, z)],$$

$\forall (x, z) \in X \times Z$ and $\forall y \in Y$.

De et al. [11, Definitions 2.3, 2.4, 2.5] are special case of Definitions 3.1, 3.2, 3.3.

4 Medical diagnosis

In this section we present an application of TVIFS in Sanchez's approach [18, 19] for medical diagnosis. In a given pathology, suppose S is a set of symptoms, D is a set of diagnoses, and P is a set of patients.

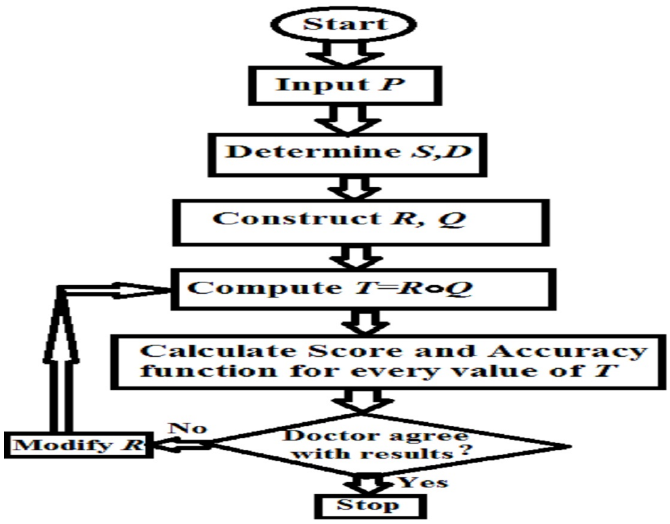

Figure 1: Flow chart of medical diagnosis

Step 1. We establish a trapezoidal valued intuitionistic fuzzy relation Q between set of patients P and the set of symptoms S (i.e., on $P \times S$), which reveals the degree of association and the degree of non-association between patients and symptoms.

Step 2. We define trapezoidal valued intuitionistic medical knowledge as a trapezoidal valued intuitionistic fuzzy relation R from the set of symptoms S to the set of diagnoses D (i.e., on $S \times D$), which reveals the degree of association and the degree of non-association between symptoms and diagnosis.

Step 3. Composition of trapezoidal valued intuitionistic fuzzy relations to get the TVIFR T ($T = R \circ Q$) describes the state of patients in terms of the diagnosis as a TVIFR from P to D.

Step 4. Score and accuracy function is calculated for every entry of T to medical diagnosis for the patients. In case, the doctor is not satisfied with the results, R is modified.

A computer-based diagnostic system can be used for this purpose. For the clear illustration of medical diagnosis a flow chart is given in Figure 1.

5 Illustrative example

To see the application of the method given in Section 4, we discuss an example.
We observe four patients Ali, Taqi, Ahmed and Hasan in a hospital at Lahore. Their symptoms are stomach pain, fever, headache, cough, and vomiting. So, the set of patients P = {Ali, Taqi, Ahmed, Hasan} and the set of symptoms S = {fever, headache, vomiting, cough, stomach pain}. The set of medical diagnosis D = {Dengue, Malaria, Gastritis, Diarrhea, Hepatitis}.

Step 1. The TVIFR $Q(P \to S)$ is given as in Table 1.

Table 1

Q	Ali
Fever	((0.7,0.75,0.8,0.82),(0.07,0.1,0.14,0.15))
Headache	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Vomiting	((0.17,0.19,0.2,0.21),(0.7,0.75,0.78,0.79))
Cough	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Stomach pain	((0.07,0.1,0.14,0.15),(0.55,0.58,0.62,0.65))
Q	Taqi
Fever	((0,0.01,0.03,0.05),(0.7,0.75,0.8,0.82))
Headache	((0.36,0.38,0.42,0.45),(0.36,0.38,0.42,0.45))
Vomiting	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Cough	((0.07,0.1,0.14,0.15),(0.66,0.68,0.72,0.75))
Stomach pain	((0.07,0.1,0.14,0.15),(0.7,0.75,0.8,0.82))
Q	Ahmed
Fever	((0.7,0.75,0.8,0.82),(0.07,0.1,0.14,0.15))
Headache	((0.7,0.75,0.8,0.82),(0.07,0.1,0.14,0.15))
Vomiting	((0,0.01,0.03,0.05),(0.55,0.58,0.62,0.65))
Cough	((0.1,0.14,0.17,0.19),(0.65,0.68,0.72,0.75))
Stomach pain	((0,0.01,0.03,0.05),(0.48,0.5,0.54,0.55))
Q	Hasan
Fever	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Headache	((0.48,0.5,0.54,0.55),(0.35,0.39,0.43,0.45))
Vomiting	((0.27,0.29,0.33,0.35),(0.38,0.4,0.44,0.45))
Cough	((0.65,0.68,0.7,0.72),(0.15,0.18,0.22,0.23))
Stomach pain	((0.27,0.29,0.33,0.35),(0.38,0.4,0.44,0.45))

Step 2. The trapezoidal valued intuitionistic fuzzy relation $R(S \to D)$ is given as in Table 2.

Table 2

R	Dengue
Fever	((0.37,0.39,0.43,0.45),(0,0.02,0.04,0.05))
Headache	((0.26,0.29,0.33,0.35),(0.45,0.48,0.54,0.55))
Vomiting	((0.09,0.1,0.13,0.15),(0.66,0.68,0.72,0.73))
Cough	((0.38,0.4,0.43,0.45),(0.27,0.29,0.34,0.35))
Stomach pain	((0.09,0.1,0.13,0.15),(0.66,0.68,0.72,0.73))
R	Malaria
Fever	((0.66,0.68,0.72,0.73),(0,0.02,0.04,0.05))
Headache	((0.16,0.19,0.23,0.25),(0.55,0.58,0.64,0.65))
Vomiting	((0,0.02,0.04,0.05),(0.85,0.88,0.9,0.93))
Cough	((0.66,0.68,0.72,0.73),(0,0.02,0.04,0.05))
Stomach pain	((0.08,0.14,0.15,0.17),(0.75,0.78,0.82,0.83))
R	Gastritis
Fever	((0.25,0.28,0.32,0.35),(0.25,0.28,0.32,0.35))
Headache	((0.55,0.58,0.64,0.65),(0.06,0.09,0.13,0.15))
Vomiting	((0.17,0.2,0.24,0.25),(0.66,0.68,0.72,0.73))
Cough	((0.17,0.2,0.24,0.25),(0.55,0.58,0.64,0.65))
Stomach pain	((0.02,0.04,0.05,0.07),(0.85,0.88,0.9,0.93))
R	Diarrhea
Fever	((0.05,0.08,0.12,0.15),(0.66,0.68,0.72,0.73))
Headache	((0.17,0.2,0.24,0.25),(0.35,0.38,0.44,0.45))
Vomiting	((0.76,0.78,0.82,0.83),(0,0.02,0.05,0.07))
Cough	((0.17,0.2,0.24,0.25),(0.66,0.68,0.72,0.73))
Stomach pain	((0.17,0.2,0.24,0.25),(0.66,0.68,0.72,0.73))
R	Hepatitis
Fever	((0.05,0.08,0.12,0.15),(0.76,0.78,0.82,0.83))
Headache	((0,0.02,0.04,0.05),(0.75,0.78,0.84,0.85))
Vomiting	((0.13,0.16,0.19,0.2),(0.76,0.78,0.79,0.8))
Cough	((0.13,0.16,0.19,0.2),(0.76,0.78,0.79,0.8))
Stomach pain	((0.76,0.78,0.82,0.83),(0.05,0.08,0.12,0.15))

Step 3. The composition $T = R \circ Q$ is as given in Table 3.

Table 3

T	Ali
Dengue	((0.38,0.4,0.43,0.45),(0.07,0.1,0.14,0.15))
Malaria	((0.66,0.68,0.72,0.73),(0.07,0.1,0.14,0.15))
Gastritis	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Diarrhea	((0.17,0.2,0.24,0.25),(0.35,0.38,0.44,0.45))
Hepatitis	((0.13,0.16,0.19,0.2),(0.55,0.58,0.62,0.65))
T	Taqi
Dengue	((0.26,0.29,0.33,0.35),(0.45,0.48,0.54,0.55))
Malaria	((0.16,0.19,0.23,0.25),(0.55,0.58,0.64,0.65))
Gastritis	((0.36,0.38,0.42,0.45),(0.36,0.38,0.42,0.45))
Diarrhea	((0.55,0.58,0.62,0.65),(0.07,0.1,0.14,0.15))
Hepatitis	((0.13,0.16,0.19,0.2),(0.7,0.75,0.79,0.8))
T	Ahmed
Dengue	((0.37,0.39,0.43,0.45),(0.07,0.1,0.14,0.15))
Malaria	((0.66,0.68,0.72,0.73),(0.07,0.1,0.14,0.15))
Gastritis	((0.55,0.58,0.64,0.65),(0.07,0.1,0.14,0.15))
Diarrhea	((0.17,0.2,0.24,0.25),(0.35,0.38,0.44,0.45))
Hepatitis	((0.1,0.14,0.17,0.19),(0.48,0.5,0.54,0.55))
T	Hasan
Dengue	((0.38,0.4,0.43,0.45),(0.07,0.1,0.14,0.15))
Malaria	((0.65,0.68,0.7,0.72),(0.07,0.1,0.14,0.15))
Gastritis	((0.48,0.5,0.54,0.55),(0.25,0.28,0.32,0.35))
Diarrhea	((0.27,0.29,0.33,0.35),(0.35,0.39,0.44,0.45))
Hepatitis	((0.27,0.29,0.33,0.35),(0.38,0.4,0.44,0.45))

Step 4. We calculate score and accuracy (def 2.3 and 2.4) as given in Table 4.

Table 4

Score, Accuracy	Ali	Taqi	Ahmed	Hasan
Dengue	0.3,0.53	-0.1975,0.8125	0.295,0.525	0.3,0.53
Malaria	0.5825,0.8125	-0.3975,0.8125	0.5825,0.8125	0.5725,0.8025
Gastritis	0.485,0.715	0,0.805	0.49,0.72	0.2175,0.8175
Diarrhea	-0.19,0.62	0.485,0.715	-0.19,0.62	-0.0975,0.7175
Hepatitis	-0.43,0.77	-0.59,0.93	-0.3675,0.6675	-0.1075,0.7275

From the score values of Table 4 it is obvious that, if the doctor agrees, then Ahmed, Ali and Hasan suffer from Malaria and Taqi faces Diarrhea as according to the order define in definition 2.5.

6 Conclusion

In this study, Sanchez's approach for medical diagnosis is generalized by the application of TVIFS theory. We propose a method in medical diagnosis based on TVIFS. As a consequence, a study of Sanchez's approach for medical diagnosis has been made with a generalized notion (i.e., TVIFS theory). The non-membership functions have more important roles in case of medical diagnosis. And then we apply the score and accuracy functions to order trapezoidal valued intuitionistic fuzzy numbers. The score and accuracy function method of TVIFS is more comprehensive and flexible than the conventional IFS and IVIFS method.

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