Notes on Intuitionistic Fuzzy Sets
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Non-linear arithmetic operation on generalized triangular intuitionistic fuzzy numbers

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Abstract: In this paper we discussed some nonlinear arithmetic operation on generalized triangular intuitionistic fuzzy numbers. Some examples and an application are given.
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1 Introduction

Zadeh [1] and Dubois and Prade [2] were the first who introduced the conception based on fuzzy number and fuzzy arithmetic. Generalizations of fuzzy sets theory [1] is considered to be one of Intuitionistic fuzzy set (IFS). Out of several higher-order fuzzy sets, IFS was first introduced by Atanassov [3] have been found to be suitable to deal with unexplored areas. The fuzzy set considers only the degree of belongingness and non-belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e., degree of non-determinacy defined as, 1 − sum of membership function and non-membership function. To handle such situations, Atanassov [4] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory. The degree of acceptance in Fuzzy Sets is only considered, otherwise IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [4].

Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [5] using membership and non-membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by Mahapatra and Roy in [6], by considering the six tuple number itself and division by A. Nagoorgani and K. Ponnalagu [7].

Now-a-days, IFSs are being studied extensively and being used in different fields of Science and Technology. Amongst the all research works mainly on IFS we can include Atanassov [4, 8–11], Atanassov and Gargov [12], Szmidt and Kacprzyk [13], Buhaescu [14], Ban [15], Deschrijver and Kerre [16], Stoyanova [17], Cornelis et al. [18], Buhaesku [19], Gerstenkorn and Manko [20], Stoyanova and Atanassov [21], Stoyanova [22], Mahapatra and Roy [23], Hajeeh [24], Persona et al. [25], Prabha et al. [26], Nikolaidis and Mourelatos [27], Kumar et al.[28] and Wang [29], Shaw and Roy [30], Adak et al.[31], A.Varghese and S. Kuriakose [32].

2 Preliminary concepts

Definition 2.1: Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows
i) an intuitionistic fuzzy subject of real line
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$)
iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\big) \quad \forall\, x_1, x_2 \in R, \lambda \in [0,1]$
iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,
$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq max\big(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)\big) \quad \forall\, x_1, x_2 \in R, \lambda \in [0,1]$.

Definition 2.2: Triangular Intuitionistic Fuzzy number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \vartheta_{\tilde{A}^i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{'}} & for\ a_1^{'} \leq x \leq a_2 \\ \frac{x-a_2}{a_3^{'}-a_2} & for\ a_2 \leq x \leq a_3^{'} \\ 1 & otherwise \end{cases}$$

Where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and TIFN is denoted by $\tilde{A}^i{}_{TIFN} = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$

Definition 2.3: Generalized Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows
i) an intuitionistic fuzzy subject of real line
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = \omega$ (so $\vartheta_{\tilde{A}^i}(x_0) = \sigma$) for $0 < \omega + \sigma \leq 1$.
iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\big) \quad \forall\, x_1, x_2 \in R, \lambda \in [0, \omega]$
iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,
$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq max\big(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)\big) \quad \forall\, x_1, x_2 \in R, \lambda \in [\sigma, 1]$.
v) $\mu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, \omega]$ and $\vartheta_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[\sigma, 1]$ and for $x_0 \in R$, the relation
$0 \leq \mu_{\tilde{A}^i}(x_0) + \vartheta_{\tilde{A}^i}(x_0) \leq 1$ holds.

Definition 2.4: Generalized Triangular Intuitionistic Fuzzy number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \omega\frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2, \\ \omega & for\ x = a_2 \\ \omega\frac{a_3-x}{a_3-a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \quad \text{and}$$

$$\vartheta_{\tilde{A}^i}(x) = \begin{cases} \sigma\frac{a_2-x}{a_2-a_1^{'}} & for\ a_1^{'} \leq x \leq a_2, \\ \sigma & for\ x = a_2 \\ \sigma\frac{x-a_2}{a_3^{'}-a_2} & for\ a_2 \leq x \leq a_3^{'} \\ 1 & otherwise \end{cases}$$

Where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and GTIFN is denoted by
$\tilde{A}^i{}_{GTIFN} = \big((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\big)$

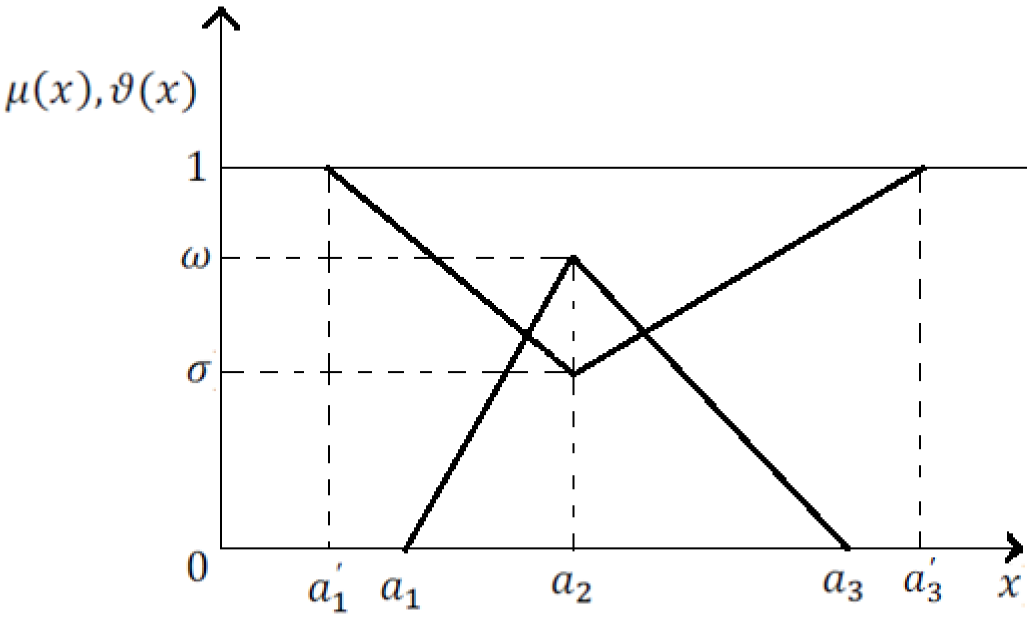

Figure: Generalized triangular intuitionistic fuzzy number

With subject to the condition $0 \leq \omega \leq 1$, $0 \leq \sigma \leq 1$ and $0 \leq \omega + \sigma \leq 1$.

Definition 2.5: A GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\right)$ is said to be non-negative iff $a_1^{'} \geq 0$.

Definition 2.6: Two GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1)\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2)\right)$ are said to be equal iff $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_1^{'} = b_1^{'}, a_3^{'} = b_3^{'}, \omega_1 = \omega_2$ and $\sigma_1 = \sigma_2$.

Definition 2.7: α-cut set: A α-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\right)$ is a crisp subset of R which is defined as follows

$$A_\alpha = \{x: \mu_{\tilde{A}^i}(x) \geq \alpha\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \frac{\alpha}{\omega}(a_2 - a_2), a_3 - \frac{\alpha}{\omega}(a_3 - a_2)]$$

Definition 2.8: β-cut set: A α-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega\right)$ is a crisp subset of R which is defined as follows

$$A_\alpha = \{x: \vartheta_{\tilde{A}^i}(x) \leq \beta\} = [A_1'(\beta), A_2'(\beta)] = [a_2 - \frac{\beta}{\sigma}(a_2 - a_1^{'}), a_2 + \frac{\beta}{\sigma}(a_3^{'} - a_2)]$$

Definition 2.9: (α, β)-cut set: A (α, β)-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\right)$ is a crisp subset of R which is defined as follows

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1'(\beta), A_2'(\beta)]\}, \alpha + \beta \leq \min(\omega, \sigma), \alpha \in [0, \omega], \beta \in [\sigma, 1]$$

Definition 2.10: Addition of two GTIFN: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1)\right)$ and $\tilde{B}^{i}{}_{GTIFN} = \left((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2)\right)$ be two GTIFN, then the addition of two GTIFN is given by

$$\tilde{A}^{i}{}_{GTIFN} \oplus \tilde{B}^{i}{}_{GTIFN} = \left((a_1 + b_1, a_2 + b_2, a_3 + b_3; \omega), (a_1^{'} + b_1^{'}, a_2 + b_2, a_3^{'} + b_3^{'}; \sigma)\right)$$

where $0 < \omega, \sigma \leq 1, \omega = \max(\omega_1, \omega_2)$ and $\sigma = \min(\sigma_1, \sigma_2)$.

Definition 2.11: Subtraction of two GTIFN: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1)\right)$ and $\tilde{B}^{i}{}_{GTIFN} = \left((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2)\right)$ be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^{i}{}_{GTIFN} \ominus \tilde{B}^{i}{}_{GTIFN} = \left((a_1 - b_3, a_2 - b_2, a_3 - b_1; \omega), (a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'}; \sigma)\right)$$

where $0 < \omega, \sigma \leq 1, \omega = \max(\omega_1, \omega_2)$ and $\sigma = \min(\sigma_1, \sigma_2)$.

Definition 2.12: Multiplication by a scalar: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\right)$ and k is a scalar then $k\tilde{A}^{i}{}_{GTIFN}$ is also a GTIFN and is defined as

$$k\tilde{A}^{i}{}_{GTIFN} = \begin{cases} \left((ka_1, ka_2, ka_3; \omega), (ka_1^{'}, ka_2, ka_3^{'}; \sigma)\right), & if\ k > 0 \\ \left((ka_3, ka_2, ka_1; \omega), (ka_3^{'}, ka_2, ka_1^{'}; \sigma)\right), & if\ k < 0 \end{cases}$$

where $0 < \omega, \sigma \leq 1$.

Definition 2.13: Multiplication of two GTIFN: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1)\right)$ and $\tilde{B}^{i}{}_{GTIFN} = \left((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2)\right)$ be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^{i}{}_{GTIFN} \otimes \tilde{B}^{i}{}_{GTIFN} = \left((a_1 b_1, a_2 b_2, a_3 b_3; \omega), (a_1^{'} b_1^{'}, a_2 b_2, a_3^{'} b_3^{'}; \sigma)\right)$$

where $0 < \omega, \sigma \leq 1, \omega = \max(\omega_1, \omega_2)$ and $\sigma = \min(\sigma_1, \sigma_2)$.

Definition 2.14: Division of two GTIFN: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1)\right)$ and $\tilde{B}^{i}{}_{GTIFN} = \left((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2)\right)$ be two GTIFN, then the division of two GTIFN is given by

$$\tilde{A}^{i}{}_{GTIFN} \div \tilde{B}^{i}{}_{GTIFN} = \left(\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}; \omega\right), \left(\frac{a_1^{'}}{b_3^{'}}, \frac{a_2}{b_2}, \frac{a_3^{'}}{b_1^{'}}; \sigma\right)\right)$$

where $0 < \omega, \sigma \leq 1, \omega = \max(\omega_1, \omega_2)$ and $\sigma = \min(\sigma_1, \sigma_2)$.

Definition 2.15: Inverse of a GTIFN: Let $\tilde{A}^{i}{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma)\right)$ be a GTIFN, then its inverse is given by

$$\frac{1}{\tilde{A}^{i}{}_{GTIFN}} = \left(\left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}; \omega\right), \left(\frac{1}{a_3^{'}}, \frac{1}{a_2}, \frac{1}{a_1^{'}}; \sigma\right)\right)$$

3 Non-linear operation of GTIFN

3.1 Modulus of a GTIFN

Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)$ be a GTIFN, then its modulus is given by

$$|\tilde{A}^i{}_{GTIFN}| = \left|\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)\right|$$
$$= \begin{cases} \left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right) & \tilde{A}^i{}_{GTIFN} \geq 0 \\ \left((-a_3, -a_2, -a_1; \omega), (a_1', a_2, a_3'; \sigma)\right) & \tilde{A}^i{}_{GTIFN} < 0 \end{cases}$$

3.2 Square root of a GTIFN

Square root of a GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right) > 0$ is obtained as follows

$$\sqrt{\tilde{A}^i{}_{GTIFN}} = \sqrt{\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)} = \left((d, e, f; \omega), (l, m, n; \sigma)\right)$$

Or, $\left(\left((d, e, f; \omega), (l, m, n; \sigma)\right)\right)^2 = \left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)$

Now applying the multiplication rule we get

$$\sqrt{\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)} = \left((\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}; \omega), \left(\sqrt{a_1'}, \sqrt{a_2}, \sqrt{a_3'}; \sigma\right)\right)$$

3.3 A general recursive formula for $\left(\left((\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}), (\mathbf{a_1'}, \mathbf{a_2}, \mathbf{a_3'}); \boldsymbol{\omega}\right)\right)^{\mathbf{n}}$

Using the multiplication of two positive generalized Intutionistic fuzzy number we have

$$\left(\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)\right)^n = \left(((a_1)^n, (a_2)^n, (a_3)^n; \omega), \left((a_1')^n, (a_2)^n, (a_3')^n; \sigma\right)\right)$$

Now we find a general recursive formulae for $\left(-\tilde{A}^i{}_{GTIFN}\right)^n$:

$$\left(-\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)\right)^n$$
$$= \begin{cases} \left(((a_3)^n, (a_2)^n, (a_1)^n; \omega), \left((a_3')^n, (a_2)^n, (a_1')^n; \sigma\right)\right), & n \text{ is even} \\ \left((-(a_3)^n, -(a_2)^n, -(a_1)^n; \omega), \left(-(a_3')^n, -(a_2)^n, -(a_1')^n; \sigma\right)\right) & , n \text{ is odd} \end{cases}$$

3.4 Exponential of a non-negative GTIFN

We use the taylor series expansion method.

We know that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, -\infty < x < \infty$

For $x = \tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)$ we have

$$e^{\tilde{A}^i{}_{GTIFN}} = 1 + \frac{\tilde{A}^i{}_{GTIFN}}{1!} + \frac{\tilde{A}^i{}_{GTIFN}{}^2}{2!} + \frac{\tilde{A}^i{}_{GTIFN}{}^3}{3!} + \cdots, x \geq 0$$

Now, $\left(\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)\right)^n = \left(((a_1)^n, (a_2)^n, (a_3)^n; \omega), \left((a_1')^n, (a_2)^n, (a_3')^n; \sigma\right)\right)$

Therefore, $e^{\tilde{A}^i{}_{GTIFN}} = \left((1,0,0; \omega), (0,0,0; \sigma)\right) + \sum_{i=1}^{\infty} \frac{\left(\left((a_1,a_2,a_3;\omega),(a_1',a_2,a_3';\sigma)\right)\right)^i}{i!}$

i.e., $e^{\tilde{A}^i{}_{GTIFN}} = \left(\left(\left(1 + \frac{a_1}{1!} + \frac{a_1{}^2}{2!} + \cdots\right), \left(\frac{a_2}{1!} + \frac{a_2{}^2}{2!} + \cdots\right), \left(\frac{a_3}{1!} + \frac{a_3{}^2}{2!} + \cdots\right); \omega\right), \left(\left(\frac{a_1'}{1!} + \frac{a_1'^2}{2!} + \cdots\right), \left(\frac{a_2}{1!} + \frac{a_2{}^2}{2!} + \cdots\right), \left(\frac{a_3'}{1!} + \frac{a_3'^2}{2!} + \cdots\right); \sigma\right)\right)$

Or, $e^{\tilde{A}^i{}_{GTIFN}} = \left((e^{a_1}, e^{a_2} - 1, e^{a_3} - 1; \omega), \left(e^{a_1'} - 1, e^{a_2} - 1, e^{a_3'} - 1; \sigma\right)\right)$

3.5 Inverse Exponential of a non-negative GTIFN

$$e^{-\tilde{A}^i{}_{GTIFN}} = \frac{1}{e^{\tilde{A}^i{}_{GTIFN}}} = \left(\left(\frac{1}{e^{a_1}}, \frac{1}{e^{a_2-1}}, \frac{1}{e^{a_3-1}}; \omega\right), \left(\frac{1}{e^{a_1'-1}}, \frac{1}{e^{a_2-1}}, \frac{1}{e^{a_3'-1}}; \sigma\right)\right)$$

Corollary: $e^{\tilde{A}^i{}_{GTIFN}} . e^{\tilde{B}^i{}_{GTIFN}} = e^{\tilde{A}^i{}_{GTIFN} + \tilde{B}^i{}_{GTIFN}}$ if $\tilde{A}^i{}_{GTIFN}, \tilde{B}^i{}_{GTIFN} \geq 0$

Corollary: $\left(e^{\tilde{A}^i{}_{GTIFN}}\right)^a = e^{a\tilde{A}^i{}_{GTIFN}}$ if $\tilde{A}^i{}_{GTIFN} \geq 0$ and $a \in R^+$

Corollary: $\frac{e^{\tilde{A}^i{}_{GTIFN}}}{\tilde{B}^i{}_{GTIFN}} = e^{\tilde{A}^i{}_{GTIFN} - \tilde{B}^i{}_{GTIFN}}$ if $\tilde{A}^i{}_{GTIFN}, \tilde{B}^i{}_{GTIFN} \geq 0$.

3.6 Logarithm of a non-negative GTIFN

Let $\log_e \left(\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right)\right) = \left((x_1, x_2, x_3; \omega), (x_1', x_2, x_3'; \sigma)\right)$

Therefore, $\left((a_1, a_2, a_3; \omega), (a_1', a_2, a_3'; \sigma)\right) = e^{\left((x_1,x_2,x_3;\omega),(x_1',x_2,x_3';\sigma)\right)} = \left((e^{x_1}, e^{x_2} - 1, e^{x_3} - 1; \omega), \left(e^{x_1'} - 1, e^{x_2} - 1, e^{x_3'} - 1; \sigma\right)\right)$

$e^{x_1} = a_1$ or, $x_1 = \log_e a_1$

$e^{x_2} - 1 = a_2$ or, $x_2 = \log_e(1 + a_2)$

$e^{x_3} - 1 = a_3$ or, $x_3 = \log_e(1 + a_3)$

$e^{x_1'} - 1 = a_1'$ or, $x_1' = \log_e(1 + a_1')$

$e^{x_3'} - 1 = a_3'$ or, $x_3' = \log_e(1 + a_3')$

Hence,

$\log_e\left(\left((a_1,a_2,a_3;\omega),(a_1^{'},a_2,a_3^{'};\sigma)\right)\right)=\left((\log_e a_1,\log_e(1+a_2),\log_e(1+a_3);\omega),\left(\log_e(1+a_1^{'}),\log_e(1+a_2),\log_e(1+a_3^{'});\sigma\right)\right)$

Corollary: $\log_e \tilde{A}^i{}_{GTIFN}+\log_e \tilde{B}^i{}_{GTIFN}=\log_e\left(\tilde{A}^i{}_{GTIFN}\tilde{B}^i{}_{GTIFN}\right)$ if $\tilde{A}^i{}_{GTIFN},\tilde{B}^i{}_{GTIFN}>0$

Corollary: $\log_e \tilde{A}^i{}_{GTIFN}-\log_e \tilde{B}^i{}_{GTIFN}=\log_e\left(\frac{\tilde{A}^i{}_{GTIFN}}{\tilde{B}^i{}_{GTIFN}}\right)$ if $\tilde{A}^i{}_{GTIFN}\geq\tilde{B}^i{}_{GTIFN}>0$

Corollary: $\log_e\left(\tilde{A}^i{}_{GTIFN}\right)^a=a\log_e \tilde{A}^i{}_{GTIFN}$ if $\tilde{A}^i{}_{GTIFN}>0, a\in I^+$

3.7 Positive solution of $\left(\left((\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}),(\mathbf{a_1^{'}},\mathbf{a_2},\mathbf{a_3^{'}});\boldsymbol{\omega}\right)\right)^{\frac{1}{\mathbf{n}}}$

By using multiplication rule we also find the n^{th} $(n>0)$ positive root of a fuzzy number as

$$\left(\left((a_1,a_2,a_3;\omega),(a_1^{'},a_2,a_3^{'};\sigma)\right)\right)^{\frac{1}{n}}\simeq\left(\left((a_1)^{\frac{1}{n}},(a_2)^{\frac{1}{n}},(a_3)^{\frac{1}{n}};\omega\right),\left((a_1^{'})^{\frac{1}{n}},(a_2)^{\frac{1}{n}},(a_3^{'})^{\frac{1}{n}};\sigma\right)\right)$$

3.8 $\widetilde{\mathbf{A}}^{\mathbf{i}}{}_{\mathbf{GTIFN}}{}^{\widetilde{\mathbf{B}}^{\mathbf{i}}{}_{\mathbf{GTIFN}}}$ if $\widetilde{\mathbf{A}}^{\mathbf{i}}{}_{\mathbf{GTIFN}}>\mathbf{0},\widetilde{\mathbf{B}}^{\mathbf{i}}{}_{\mathbf{GTIFN}}\geq\mathbf{0}$

Let $\tilde{A}^i{}_{GTIFN}=\left((a_1,a_2,a_3;\omega_1),(a_1^{'},a_2,a_3^{'};\sigma_1)\right)$ and $\tilde{B}^i{}_{GTIFN}=\left((b_1,b_2,b_3;\omega_2),(b_1^{'},b_2,b_3^{'};\sigma_2)\right)$

Therefore, $\left((a_1,a_2,a_3;\omega_1),(a_1^{'},a_2,a_3^{'};\sigma_1)\right)^{\left((b_1,b_2,b_3;\omega_2),(b_1^{'},b_2,b_3^{'};\sigma_2)\right)}$

$=e^{\left((b_1,b_2,b_3;\omega),(b_1^{'},b_2,b_3^{'};\sigma)\right)\ln\left((a_1,a_2,a_3;\omega),(a_1^{'},a_2,a_3^{'};\sigma)\right)}$

$=e^{\left((b_1,b_2,b_3;\omega),(b_1^{'},b_2,b_3^{'};\sigma)\right)\left((\ln a_1,\ln(1+a_2),\ln(1+a_3);\omega),(\ln(1+a_1^{'}),\ln(1+a_2),\ln(1+a_3^{'});\sigma)\right)}$

$=e^{\left((b_1\ln a_1,b_2\ln(1+a_2),b_3\ln(1+a_3);\omega),(b_1^{'}\ln(1+a_1^{'}),b_2\ln(1+a_2),b_3^{'}\ln(1+a_3^{'});\sigma)\right)}$

$=\left(\left(e^{b_1\ln a_1},e^{b_2\ln(1+a_2)}-1,e^{b_3\ln(1+a_3)}-1;\omega\right),\left(e^{b_1^{'}\ln(1+a_1^{'})}-1,e^{b_2\ln(1+a_2)}-1,e^{b_3^{'}\ln(1+a_3^{'})}-1;\sigma\right)\right)$

$=\left(\left({a_1}^{b_1},(1+a_2)^{b_2}-1,(1+a_3)^{b_3}-1;\omega\right),\left((1+a_1^{'})^{b_1^{'}}-1,(1+a_2)^{b_2},(1+a_3^{'})^{b_3^{'}}-1;\sigma\right)\right)$

where $0<\omega,\sigma\leq 1,\omega=\max(\omega_1,\omega_2)$ and $\sigma=\min(\sigma_1,\sigma_2)$.

3.9 $\boldsymbol{a^{\tilde{A}^i{}_{GTIFN}}}$ where $\boldsymbol{a\geq 1}$ and $\boldsymbol{\tilde{A}^i{}_{GTIFN}\geq 0}$

Let $\tilde{A}^i{}_{GTIFN}=\left((a_1,a_2,a_3;\omega),(a_1^{'},a_2,a_3^{'};\sigma)\right)$

$a^{\tilde{A}^i{}_{GTIFN}}=e^{\tilde{A}^i{}_{GTIFN}\ln a}=\exp\left(\left((a_1\ln a,a_2\ln a,a_3\ln a;\omega),(a_1^{'}\ln a,a_2\ln a,a_3^{'}\ln a;\sigma)\right)\right)$

$=\left((a^{a_1},a^{a_2}-1,a^{a_3}-1;\omega),\left(a^{a_1^{'}}-1,a^{a_2}-1,a^{a_3^{'}}-1;\sigma\right)\right)$

4 Numerical examples

Example 4.1. Find the value of $\sqrt{\tilde{x}+\sqrt{\tilde{x}+\sqrt{\tilde{x}+\cdots\cdots\cdots}}}$ where $\tilde{x}=\big((3,4,6;0.8),(2,4,7;0.7)\big)$

Solution: Let the value of the above is $\tilde{p}=\big((a,b,c;0.8),(r,b,t;0.7)\big)$. Therefore $\sqrt{\tilde{x}+\tilde{p}}=\tilde{p}$

Or, $\sqrt{\big((3+a,4+b,6+c;0.8),(2+r,4+b,7+t;0.7)\big)}=\big((a,b,c;0.8),(r,b,t;0.7)\big)$

This implies,

$\sqrt{3+a}=a$ or, $a^2-a-3=0$ or, $a=2.30$

$\sqrt{4+b}=b$ or, $b^2-b-4=0$ or, $b=2.56$

$\sqrt{6+c}=c$ or, $c^2-c-6=0$ or, $c=3$

$\sqrt{2+r}=r$ or, $r^2-r-2=0$ or, $r=2$

$\sqrt{7+t}=t$ or, $t^2-t-7=0$ or, $t=3.14$

Hence the value of the above is $\big((2.30,2.56,3;0.8),(2,2.56,3.14;0.7)\big)$

Example 4.2. Find all the positive solution of $e^{\tilde{x}}=\sqrt{\big((3.24,4,4.84;0.9),(2.56,4,5.76;0.8)\big)}$

Solution:

$e^{\tilde{x}}=\sqrt{\big((3.24,4,4.84;0.9),(2.56,4,5.76;0.8)\big)}=$
$\Big(\big(\sqrt{3.24},\sqrt{4},\sqrt{4.84};0.9\big),\big(\sqrt{2.56},\sqrt{4},\sqrt{5.76};0.8\big)\Big)$

Or, $e^{\tilde{x}}=\big((1.8,2,2.2;0.9),(1.6,2,2.4;0.8)\big)$

Or, $\tilde{x}=\big((\ln 1.8,\ln 3,\ln 3.2;0.9),(\ln 2.6,\ln 3,\ln 3.4\,;0.8)\big)$

Or, $\tilde{x}=\big((0.58,1.09,1.16;0.9),(0.95,1.09,1.22;0.8)\big)$

Example 4.3. Find all the solutions of the equation $\sqrt{|\tilde{x}|}=\big((4,6,7;0.7),(3,6,8;0.6)\big)$

Solution: $|\tilde{x}|=\big((4,6,7;0.7),(3,6,8;0.6)\big)^2=\big((16,36,49;0.7),(9,36,64;0.6)\big)$

The equation has two solutions as $\big((16,36,49;0.7),(9,36,64;0.6)\big)$ and $\big((-16,-36,-49;0.7),(9,36,64;0.6)\big)$

Example 4.4. Evaluate the fuzzy solution of $\big((2.197,3.375,4.913;1),(1.728,3.375,5.832;0.9)\big)^{\frac{1}{3}}$

Solution: The positive fuzzy solution is $\big((1.3,1.5,1.7;1),(1.2,1.5,1.8;0.9)\big)$

Example 4.5. Compute the value of $\big((3,4,5;0.9),(2,4,6;0.7)\big)^{\big((4,5,7;0.8),(3,5,8;0.6)\big)}$

Solution: The value of above is given by $\big((81,3124,279935;0.9),(26,3124,5764800;0.6)\big)$

5 Application

Bank Account Problem, [33]

The Balance $B(t)$ of a bank account grows under continuous process given by $\frac{dB}{dt}=rB$, where r the constant of proportionality is the annual interest rate. If there are initially $B(t)=B_0$ balance, solve the above problem in fuzzy environment when $\widetilde{B_0}=\big((\$850,\$1000,\$1100;0.9),(\$800,\$1000,\$1200;0.8)\big)$ and $\tilde{r}=\big((3.7,4,4.5;0.8),(3.5,4,5;0.6)\big)\%$. Find the solution after $t=3$ years.

Solution: The solution is given by $B(t=3) = \widetilde{B_0} e^{\frac{3}{100}\tilde{r}}$

$$= \big((\$850, \$1000, \$1100; 0.9), (\$800, \$1000, \$1200; 0.8)\big) e^{\big((3.7,4,4.5;0.8),(3.5,4,5;0.6)\big)}$$

$$= \big((\$850, \$1000, \$1100; 0.9), (\$800, \$1000, \$1200; 0.8)\big).$$

$$\big((1.1174, 0.1275, 0.1445; 0.8), (0.1107, 0.1275, 0.1618; 0.6)\big)$$

$$= \big((\$949.79, \$127.50, \$158.95; 0.9), (\$88.56, \$127.50, \$194.16; 0.6)\big)$$

6 Conclusion

In this paper we discussed some nonlinear operation (such as logarithm, exponential) on generalized triangular intuitionistic fuzzy number. Some example and an application are given. An imprecise bank account problem is given in generalized triangular intuitionistic fuzzy environment.

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