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Shannon approach to intuitionistic fuzzy information definition. Part 2

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Abstract: A probabilistic approach to intuitionistic fuzzy information is presented. It is based on Shannon's definition of information.

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1 Introduction

Since 1984, when K. Atanassov [1] introduced Intuitionistic Fuzzy Sets (IFS) the interest in studying this theory has been permanently growing. Respectively, different approaches to the information definition of these sets are still continuing to be proposed.

In 1996, Burillo and Bustince [2] introduced the probabilistic entropy measure on IFS, based on intuitionistic index. Later, Szmidt and Kacprzyk proposed in 2002 a non-probabilistic approach.

In 2006, Wen-Liang and Miin-Shen [6] proposed an improved version of the above entropy measure.

A new approach has been used by Wenyi and others [7] to determine the entropy by similarity measure. And recently Vlahos and Sergiadis [5] introduced a new non-probabilistic approach to information definition in 2007.

In 2008 in [3], we proposed an information metrics, based on the Shanon probablistic approach. We consider that it is more close to the native of intuitionistic fuzzy sets than the non-probablistic approaches.

In the article below, a futher attempt to improve our Shannon approach to intuitionistic fuzzy information is presented.

2 Probabilistic approach to the intuitionistic fuzzy information definition

We propose our information definition to be based on the Shannon probabilistic way of information measure, [4]. The main point of this choice is the fact that the functions of membership and non-membership, determining intuitionistic fuzzy sets, are naturally probabilistic.

For that purpose we will consider X as a set of probabilistic events x_i ($i = 1, \ldots, N$), with respective probabilities of occurrence p_i, where

$$\sum_{i=1}^{N} p_i = 1 \tag{1}$$

and the information in it:

$$I(X) = H(X) = -\sum_{i=1}^{N} p_i \log_2 p_i \tag{2}$$

Then we will decompose X into the following subsets:

$$\{x_1, \ldots, x_s\} \cup \{x_{s+1}, \ldots, x_q\} \cup \{x_{q+1}, \ldots, x_N\} \tag{3}$$

and respective probabilities:

$$\{p_1, \ldots, p_s\} \cup \{p_{s+1}, \ldots, p_q\} \cup \{p_{q+1}, \ldots, p_N\}, \tag{4}$$

where $1 \le s \le q \le N$.

Next, we will consider a subset $\{x_1, \ldots, x_s\}$ as one, which is probable to occur, while subset $\{x_{s+1}, \ldots, x_q\}$ – as one, which is not probable to occur.

Following the Shannon approach, we will determine normalized information on each subset in following way:

$$\mu = \frac{\sum_{i=1}^{s} p_i \log_2 p_i}{I(X)} \tag{5}$$

and

$$\nu = \frac{\sum_{i=s+1}^{q} p_i \log_2 p_i}{I(X)} \tag{6}$$

Now, we will consider the normalized information of both subsets as a degree of membership μ and a degree of non-membership v, respectively, of a global set X, i.e. we can reconsider now this set as an intuitionistic one. It is clear that

$$\mu + v \le 1 \tag{7}$$

is satisfied.

And both indexes are source of knowledge about the behaviour of X, i.e. could it occur or not. In that sense we can apply again the Shannon information definition and calculate the information of the intuitionistic set X, as:

$$I_{\text{ifs}}(X) = -(\mu \log_2 \mu + v \log_2 v) \tag{8}$$

The graphical representation of the defined information quantity in an intuitionistic fuzzy set as a function of membership and non-membership is shown on Figure 1.

As it is seen, the proposed measure of the information quantity in an intuitionistic fuzzy set keeps the behaviour of the Shannon measure of information. The difference is that the intuitionistic fuzzy information is a function of two variables. The elimination of one of these variables makes the intuitionistic fuzzy information definition equal to that of Shannon.

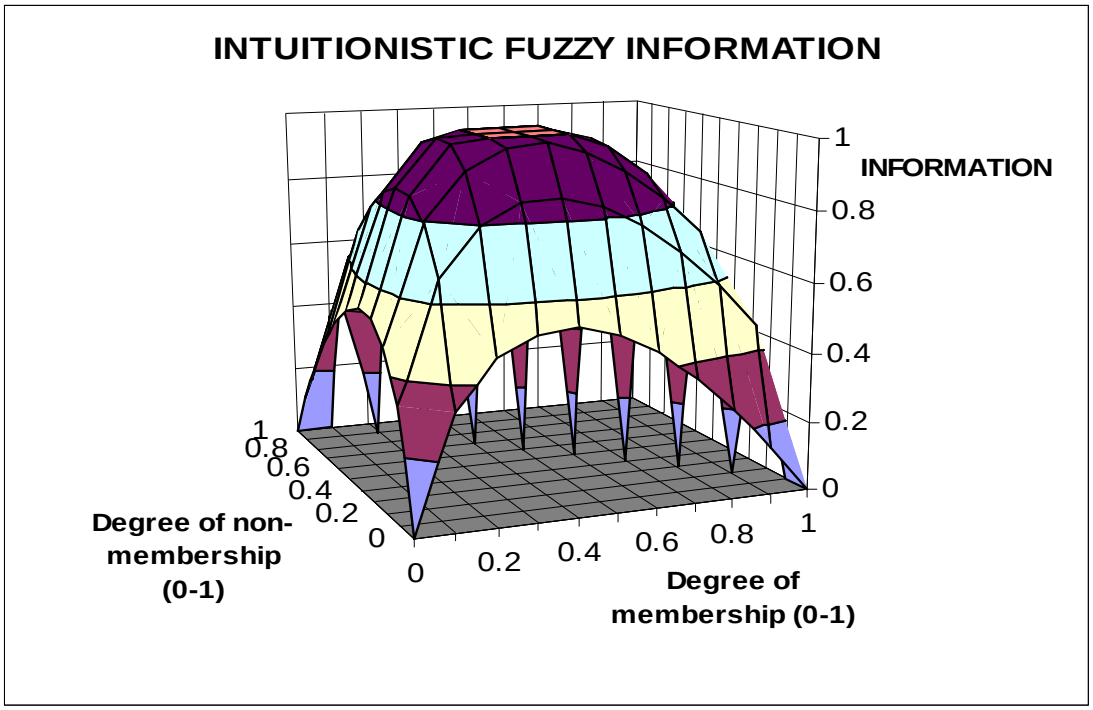

Figure 1: Graphical presentation of the intuitionistic fuzzy information as a function of the degree of membership μ and non-membership ν.

3 Scenarios of the relations between the degrees of membership and non-membership

As the values of the two variables, the degree of membership μ and non-membership ν, are connected with inequality (7), where the degree of uncertainty π is the complementary part to one, there are several scenarios of the relations among them. They will influence on the the view and values of the intuitionistic fuzzy information.

The possible scenarios are presented on Figure 2, as follows:

Scenarios / Degree values	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Degree of membership μ	0	0	0	1/2	1/2	1	random
Degree of non-membership ν	0	1/2	1	0	1/2	0	random
Degree of uncertainty π	1	1/2	0	1/2	0	0	random

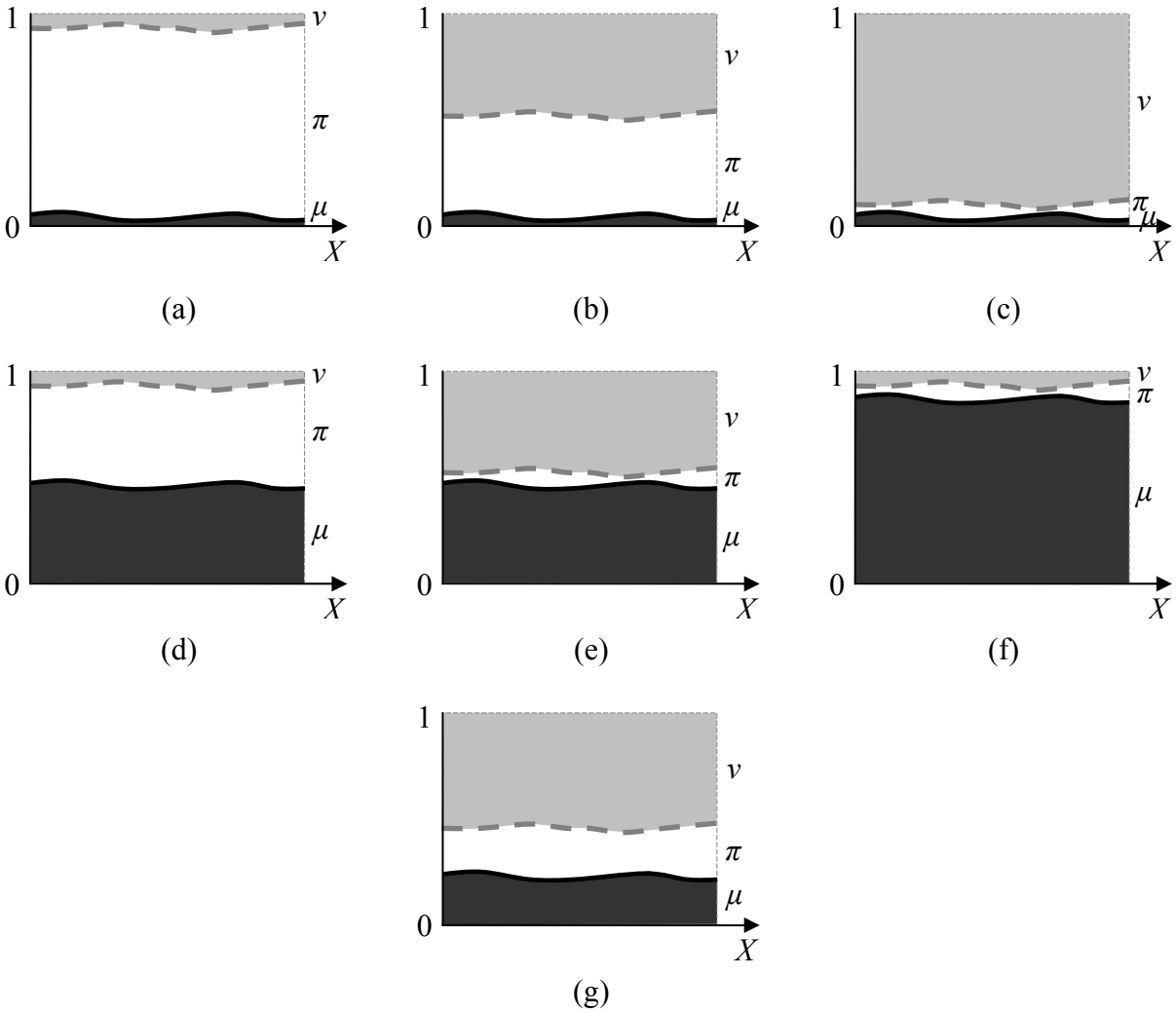

Figure 2: Scenarios of the relations between the degree of membership μ, non-membership ν and uncertainty π

4 Conclusion

The Shannon's approach to intuitionistic fuzzy information definition has been proposed because:

1. Shannon's approach provides a way to a quantitative measure.
2. Shannon information definition is well proven and accepted.
3. It satisfies all requirements for easy use (monotonic, additive etc.).
4. Shannon information can be derived from the proposed intuitionistic fuzzy information definition.

Thus, the proposed measure for intuitionistic fuzzy information can be applied directly to the analysis and evaluation of information system and processes.

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