Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 23, 2017, No. 3, 88–95

Intuitionistic fuzzy generalized net model of adolescent idiopathic scoliosis classification and the curve progression probability

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Received: 5 June 2017 **Accepted:** 20 July 2017

Abstract: By far the most common type of scoliosis in the adolescent period is one in which the cause is not known and is called idiopathic or adolescent idiopathic scoliosis (AIS). The objective of the present work is to propose a novel approach for classification and evaluation of the curve progression probability in patients with confirmed AIS, using the apparatus of Generalized Nets (GN) Theory and the Intuitionistic fuzzy sets (IFS).

Keywords: Intuitionistic fuzzy generalized net model, Adolescent idiopathic scoliosis.
AMS Classification: 68Q85, 03E72.

To our good friend and colleague Prof. Janusz Kacprzyk
for his 70th anniversary!

1 Introduction

Adolescent idiopathic scoliosis (AIS) is one of the most common deformities of the spine. Clinical evaluation of AIS is based on a detailed history, physical examination and a plan for image testing. The investigation should focus on several main points: exclusion of other causes of spinal deformity, pubertal status and estimation of remaining growth potential, determination of the degree of the spinal curve and the curve pattern. Since scoliosis curves grow larger during rapid growth, the best predictors of curve progression are growth potential and growth velocity, both of which are functions of skeletal maturity. The potential for growth is evaluated taking into consideration the chronological age, skeletal age, menarchal status, and stage of development of the iliac crest apophysis (Risser stage). The Risser grading system [7] is often used to determine a child's skeletal maturity on the pelvis, which correlates with how much spine growth is left. The Risser grading system rates a child's skeletal maturity on a scale of 0 to 5. Higher Risser grades indicate greater skeletal ossification, hence less potential for growth and curve progression.

Accurate measurement of the initial curve is of paramount importance to follow progression. The Cobb angle is used as a standard measurement to determine the degree of the spinal curve and track the progression of AIS. The method remains "gold standard" diagnostic tool of choice due to being well understood within the orthopedic community as well as due to the fact that facilities capable of acquiring full spinal radiographs is readily available in most clinics. Cobb [4] suggested that the angle of curvature may be measured by drawing lines parallel to the upper border of the upper vertebral body and the lower border of the lowest vertebra of the structural curve, then erecting perpendiculars from these lines to cross each other, the angle between these perpendiculars being the "angle of curvature". A Cobb angle greater than 10 degrees establishes a diagnosis of scoliosis. Scoliosis is considered mild at 10-25 degrees, moderate at 25-50 degrees and severe at greater than 50 degrees. The Cobb angle and Risser grading system can be combined to predict the likelihood of curve progression. Taking into consideration the curve pattern, magnitude and flexibility of the scoliosis deformity, King et al. [5] described their classification system in 1983, commonly known as the King classification system for adolescent idiopathic scoliosis. King et al. defined five curve types:

- **Type 1**: An S-shape deformity, in which both curves are structural and cross the center sacral vertical line (CSVL), with the lumbar curve being larger than the thoracic one.
- **Type 2**: An S-shape deformity, in which both curves are structural and cross the CSVL, with the thoracic curve being larger or equal to the lumbar one.
- **Type 3**: Major thoracic curve in which only the thoracic curve is structural and crosses the CSVL.
- **Type 4**: Long C-shape thoracic curve in which the fifth lumbar vertebra is centered over the sacrum and the forth lumbar vertebra is tilted into the thoracic curve.
- **Type 5**: Double thoracic curve.

Described in [6] reduced GN-model represents the diagnostic algorithm for patient with possible diagnosis of AIS. After the final diagnosis is made it is extremely important to define the curve pattern and to estimate the risk for curve progression which will help to categories idiopathic scoliosis for easier communication, prognosticate the disease and guide the treatment strategy. However the presented approach assumes deterministic evaluation, while, in practice, some inaccuracy is always present. Accordingly, here we will construct a reduced IFGN1-model of AIS classification and the curve progression probability. The proposed model (Fig. 1) is based on Cobb angle measurements, King Classification system and the Risser grading system.

2 IFGN model of AIS classification and the curve progression probability

In general the *Generalized Nets* (GNs) [1, 2] may or may not have some of the components in their definition. GNs which do not have some of the components form special class called reduced GNs [2]. In ordinary reduced GNs each transition condition predicate $W_{i,j}$ corresponds to the i-th input and j-th output places. When its truth value is "*true*", a token from the i-th input place transfers to j-th output place, otherwise this is not possible. When each transition predicate $W_{i,j}$ is estimated not by values "*true*" and "*false*" (0,1), but by a couple of real numbers $a_{i,j}$, $b_{i,j}$, such that

$$a_{i,j},\ b_{i,j},\ a_{i,j} + b_{i,j} \in [0,1],$$

then we obtain an *Intuitionistic Fuzzy Generalized Net* (IFGN) of the first type (IFGN1) [2].

All transition condition predicates $W_{i,j}$ have associated degrees of validity ($\mu(W_{i,j})$) and non-validity ($\nu(W_{i,j})$), instead of just a binary truth value. For each of these predicates we can determine two thresholds – lower ($t_l \in [0,1]$) and upper ($1 - t_h \in [0,1]$), such that $t_l + t_h \leq 1$. The token from place l_i will go to place l_j if the following conditions hold:

$$\mu(W_{i,j}) > t_l \text{ and } \nu(W_{i,j}) < 1 - t_h.$$

The GN-model, presented in Figure 1, has 3 transitions and 18 places with the following meaning:

- Transition Y_1 represents the King classification system.
- Transition Y_2 represents the Cobb angle measurements.
- Transition Y_3 represents the Risser grading system.

In order to ease the understandings of the actual formalism in use we shall not describe the transition condition predicates fully formally.

The proposed GN-model contains four types of tokens: α, β, γ and η. The three transitions of the GN-model have three "special places", where a token stays and collects information about the specific parts of the diagnosing process which it represents as follows:

- In place m_7, token β stays permanently and collects information about the curve patterns according to the King classification system.
- In place m_{11}, token γ stays permanently and collects information about the results from the Cobb angle measurements.
- In place m_{18}, token η stays permanently and collects information about the risk for curve progression based on the Risser grading system.

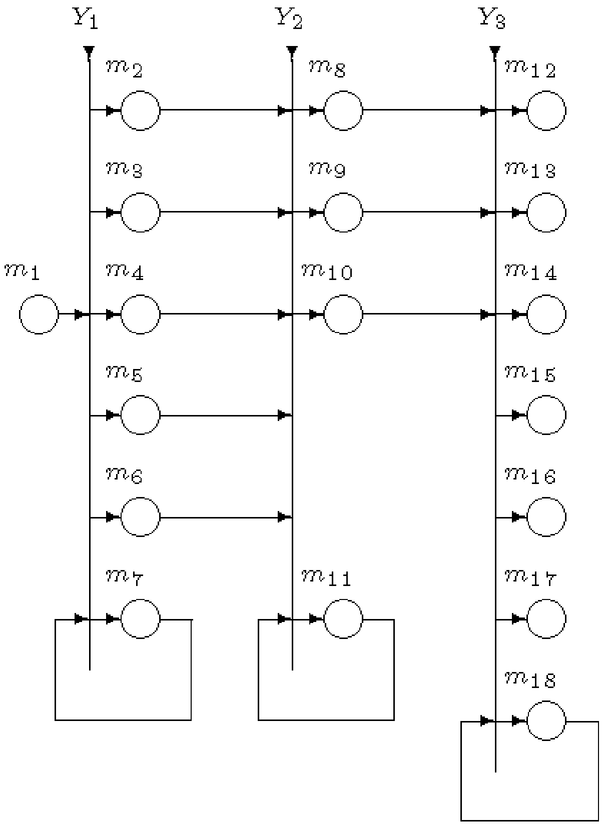

Figure 1. An IFGN-model of AIS classification and the curve progression probability

Token α enters the net with an initial characteristic "*patient with confirmed AIS*" in place m_1. The transition Y_1 of the GN-model has the following form:

$$Y_1 = \langle \{m_1, m_7\}, \{m_2, m_3, m_4, m_5, m_6, m_7\}, r_1 \rangle,$$

where:

$$r_1 = \begin{array}{c|cccccc} & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 \\ \hline m_1 & \mathit{false} & \mathit{false} & \mathit{false} & \mathit{false} & \mathit{false} & \mathit{true} \\ m_7 & W_{7,2} & W_{7,3} & W_{7,4} & W_{7,5} & W_{7,6} & \mathit{true} \end{array}$$

and,

- $W_{7,2}$ = "*there is an S-shape deformity in which both curves are structural and cross the CSVL and the lumbar curve is larger than the thoracic curve*";
- $W_{7,3}$ = "*there is an S-shape deformity, in which both curves are structural and cross the CSVL and the thoracic curve is larger or equal to the lumbar curve*";
- $W_{7,4}$ = "*there is a major thoracic curve in which only the thoracic curve is structural and crosses the CSVL*;
- $W_{7,5}$ = "*there is a long C-shape thoracic curve in which the fifth lumbar vertebra is centered over the sacrum and the forth lumbar vertebra is tilted into the thoracic curve*;
- $W_{7,6}$ = "*there is a double thoracic curve*".

All of these predicates can be estimated by degrees of correctness and non-correctness due to the X-ray image quality, the X-ray positioning and the correct selection of the upper and the lower vertebrae of the spinal curve.

We will give a formal description only for the first predicate "$W_{7,2}$" of transition Y_1, the threshold values for the rest of the predicates in our model can be chosen accordingly to the specific values obtained from the X-ray imaging results.

The token will go to place m_2 if the following condition for predicate "$W_{7,2}$" holds:

$$W_{7,2} = \text{“}u_1 \wedge u_2\text{”}$$

$$u_1 \wedge u_2 = \langle \min(\mu_1, \mu_2), \max(\nu_1, \nu_2) \rangle = \langle \mu_{min}, \nu_{max} \rangle,$$

where u_1 and u_2 correspond to the expressions "*there is an S-shape deformity in which both curves are structural and cross the CSVL*" and "*the lumbar curve is larger than the thoracic curve*", respectively. The same symbols are also used to denote the associated pairs of the membership and non-membership degrees.

In the present work we have chosen the threshold value as $\langle \frac{1}{2}, \frac{1}{2} \rangle$, that is the predicate will evaluate to truth if $\langle \mu_{min}, \nu_{max} \rangle \geq \langle \frac{1}{2}, \frac{1}{2} \rangle$, i.e.

$$\mu_{min} \geq \frac{1}{2}, \nu_{max} \leq \frac{1}{2},$$

The last two conditions ensure that for a predicate be valid the condition:

$$\mu_{min} \geq \nu_{max}$$

have to be satisfied.

The token α on the output of transition T_1 obtains following characteristics:

- "*AIS is of King's Type 1*" in place m_2
- "*AIS is of King's Type 2*" in place m_3
- "*AIS is of King's Type 3*" in place m_4
- "*AIS is of King's Type 4*" in place m_5
- "*AIS is of King's Type 5*" in place m_6

It is important to note that in each time-moment only one of the five predicates is valid. So, there is only one token α on the input of transition Y_2 at any moment.

The transition Y_2 of the GN-model has the following form:

$$Y_2 = \langle \{m_2, m_3, m_4, m_5, m_6, m_{11}\}, \{m_8, m_9, m_{10}, m_{11}\}, r_2 \rangle,$$

where:

$$r_2 = \begin{array}{c|cccc} & m_8 & m_9 & m_{10} & m_{11} \\ \hline m_2 & false & false & false & true \\ m_3 & false & false & false & true \\ m_4 & false & false & false & true \\ m_5 & false & false & false & true \\ m_6 & false & false & false & true \\ m_{11} & W_{11,8} & W_{11,9} & W_{11,10} & true \end{array}$$

and,

- $W_{11,8}$ = "*AIS of King's Type 1, 2, 3, 4, or 5 is with the Cobb measurement of any spinal curve between 10 and 25 degrees*";
- $W_{11,9}$ = "*AIS of King's Type 1, 2, 3, 4, or 5 is with the Cobb measurement of any spinal curve between 25 and 50 degrees*";
- $W_{11,10}$ = "*AIS of King's Type 1, 2, 3, 4, or 5 is with the Cobb measurement of any spinal curve above 50 degrees*".

All of these predicates can be estimated by their intuitionistic fuzzy degrees of accurate measures and non-accurate measures of the Cobb's angle.

The tokens from all input places of transition Y_2 enter place m_{11} and unite with token γ that obtains the above mentioned characteristics. On the next time-moment, token γ splits to two tokens – the same token γ and token α. The token α obtains following characteristics:

- "*AIS is of King's Type 1, 2, 3, 4 or 5 with 10 to 25 degrees curve*" in place m_8
- "*AIS is of King's Type 1, 2, 3, 4 or 5 with 25 to 50 degrees curve*" in place m_9
- "*AIS is of King's Type 1, 2, 3, 4 or 5 with 50 to 100/or above degrees curve*" in place m_{10}

It is important to note that in each time-moment only one of the three predicates is valid. So, there is only one token α on the output of transition T_2 at any moment.

The transition Y_3 has the following form:

$$Y_3 = \langle \{m_8, m_9, m_{10}, m_{18}\}, \{m_{12}, m_{13}, m_{14}, m_{15}, m_{16}, m_{17}, m_{18}\}, r_3 \rangle,$$

where:

$$r_3 = \begin{array}{c|ccccccc} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} & m_{17} & m_{18} \\ \hline m_8 & false & false & false & false & false & false & true \\ m_9 & false & false & false & false & false & false & true \\ m_{10} & false & false & false & false & false & false & true \\ m_{18} & W_{18,12} & W_{18,13} & W_{18,14} & W_{18,15} & W_{18,16} & W_{18,17} & true \end{array}$$

and,

- $W_{18,12}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 10 to 25 degrees curve and a Risser grade from 0 to 1*";
- $W_{18,13}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 10 to 25 degrees curve and a Risser grade from 2 to 4*";
- $W_{18,14}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 25 to 50 degrees curve and a Risser grade from 0 to 1*";
- $W_{18,15}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 25 to 50 degrees curve and a Risser grade from 2 to 4*";
- $W_{18,16}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 50 to 100/or above degrees curve and a Risser grade from 0 to 1*";
- $W_{18,17}$ = "*AIS is of King's Type 1, 2, 3, 4 or 5 with 50 to 100/or above degrees curve and a Risser grade from 2 to 4*".

All of these predicates can be estimated by their intuitionistic fuzzy degrees of accurate measures and non-accurate measures of the Cobb's angle and the Risser grade.

The tokens from all input places of transition Y_3 enter place m_{18} and unite with token η that obtains the characteristic, as mentioned above. On the next time-moment, token η splits to two tokens – the same token η that stays permanently in the place m_{18} and token α. The token α obtains following characteristic:

- "*there is a moderate risk of curve progression*" in place m_{12}
- "*there is a low risk of curve progression*" in place m_{13}
- "*there is a high risk of curve progression*" in place m_{14}
- "*there is a very high risk of curve progression*" in place m_{15}
- "*there is a high risk of curve progression*" in place m_{16}
- "*there is a very high risk of curve progression*" in place m_{17}

It is important to note that in each time-moment only one of the six predicates is valid. So, there is only one token α on the output of transition T_3, and thus, also in the net, at any moment.

When running the developed model with real patients' data, the obtained results may be analyzed by the recently proposed approach of intercriteria analysis [3]. As a multicriteria multiobjective decision making approach, it will permit discovery of new relations as well as improvement of the model accuracy as it has been proven yet in medical object [8].

3 Conclusions

AIS is a serious and important postural disorder commonly found in the population. Early detection of AIS is the key factor for preventing progression of the curve magnitude and implementing an appropriate and successful treatment. Proper diagnosis is extremely important for designing a coordinated exercise programs and reliably monitoring progress during treatment.

In this paper an IFGN-model is presented which describes the categorisation and further evaluation of established diagnosis of AIS. The IF estimations implemented in the proposed model, allows us to take into account the inevitable inacuracy at the different stages during the investigation of a patient with AIS and thus provides for a more adequate modelling of particular symptoms accounting separately for pros and cons of the contribution of particular aspects of the diagnosis and their aggregation. The so described IFGN-model may provide a framework that can be used by primary care practitioners to guide diagnostic processes for patient diagnosed with AIS, enabling more accurate and efficient estimation of the curve progression and would assist in optimizing patient outcomes and more effective treatment.

Acknowledgment

Work presented here is partially supported by the "Program career development of young scientists, BAS", Grant number DFNP-142/2016 and the National Science Fund of Bulgaria under Grant No. DFNI-I-02-5 "InterCriteria Analysis: A new approach to decision making".

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