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On similarly structured intuitionistic fuzzy sets

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Abstract: In this paper, we offer a new point of view on intuitionistic fuzzy sets which allows us to introduce new operators in a natural way.
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1 Introduction

Intuitionistic fuzzy sets were introduced by K. Atanassov in 1983 [1]. They are an extension of the fuzzy sets introduced by Zadeh [5].

Further, we remind some basic definitions and notions.

Let X be a universe set, $A \subset X$, $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are mappings reflecting the degree of membership and non-membership of the element $x \in X$ to the set A, respectively, such that for every x it is fulfilled that

$$\mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

Definition 1. *Following [2], we call the set*

$$A^* \stackrel{\text{def}}{=} \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

an intuitionistic fuzzy set (IFS) and the mapping $\pi_A : X \to [0,1]$, *which is given in explicit form by*

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x), \tag{2}$$

is called ***intuitionistic fuzzy index*** *(sometimes also: hesitancy margin or degree of indeterminacy) of the element* x *(cf. [3]).*

Remark 1. *When* $\forall x \in X \ \pi_A(x) \equiv 0$, A^* *is a fuzzy set (FS).*

Definition 2 ([4])**.** *An intuitionistic fuzzy pair (IFP) is an ordered couple of real non-negative numbers* $\langle a, b \rangle$, *with the constraint:*

$$a + b \leq 1. \tag{3}$$

Definition 3. *Let us define for any IFP* $u = \langle a, b \rangle$, *its modulus by:*

$$\tilde{u} \stackrel{\text{def}}{=} a + b. \tag{4}$$

Definition 4. *Let us call an IFP* $u = \langle a, b \rangle$, *fuzzy pair (FP) iff:*

$$\tilde{u} = 1. \tag{5}$$

Remark 2. *Similar definition is possible with the introduction of a parameter* ε, *e.g., we can call* u *a* ε*-fuzzy pair iff*

$$\tilde{u} \geq 1 - \varepsilon.$$

When $\varepsilon = 0$, *we will obtain Definition 4.*

Definition 5. *Let us call an IFP* $u = \langle a, b \rangle$, *proper intuitionistic fuzzy pair (PIFP) iff:*

$$\tilde{u} < 1. \tag{6}$$

2 Similarly structured intuitionistic fuzzy sets

It is easy to see that by using Definitions 1, 2, 4, we can view an IFS as a collection of *two types of* labelled by the elements of the universe set *pairs* - FP and PIFP. Due to the way they are defined there can be no intersection, so our original universe set is split in two non-intersecting subsets - one with PIFP and the other with FP. Of course, it is possible that one of this subsets is empty.

Thus, we can write:

Definition 6. *An intuitionistic fuzzy set* A^* *may be written as:*

$$A^* = \{\langle x, u(x)\rangle | x \in X_{PIFP}\} \cup \{\langle x, v(x)\rangle | x \in X_{FP}\}, \tag{7}$$

where $\cup$ *is to be understood as the standard set-theoretical union,* $u(x) = \langle \mu_A(x), \nu_A(x) \rangle$ *- are proper intuitionistic fuzzy pairs* $\widetilde{u(x)} < 1$, $v(x) = \langle \mu_A(x), \nu_A(x) \rangle$ *- are fuzzy pairs,i.e.* $\widetilde{v(x)} = 1$, *and* $X_{FP} \cup X_{PIFP} = X; X_{FP} \cap X_{PIFP} = \emptyset$.

Definition 7. *We shall call two IFS* A *and* B *structurally similar iff for every* $x \in X$

$$\sigma_{1,A}(x) = 1 \Leftrightarrow \sigma_{1,B}(x) = 1, \tag{8}$$

where $\sigma_{1,A}(x)$, *(cf. [2, p. 134, (7.1)]) for* $x \in X$ *is given by*

$$\sigma_{1,A}(x) = \mu_A(x) + \nu_A(x).$$

Now we are ready to formulate some propositions.

Definition 8. *Let $Q : IFS(X) \to IFS(X)$ be an operator defined over intuitionistic fuzzy sets. Then, we can take the restriction of this operator (Q_r) over all structurally similar IFSs with the same X_{PIFP} (denoted as SSIMIFS) as follows. Let $A \in SSIMIFS(X_{PIFP})$, then*

$$Q_r(A) = Q(\{\langle x, u_A(x)\rangle | x \in X_{PIFP}\}) \cup \{\langle x, v(x)\rangle | x \in X_{FP}\}, \tag{9}$$

where again the union is considered in the set-theoretic sense.

Example 1. *Let us be given an IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$. Then the operator $H^*_{r;\alpha,\beta}$ which is a restriction of the operator $H^*_{\alpha,\beta}$(see [2, p. 83, (5.5)]) may be introduced as follows:*

$$H^*_{r;\alpha,\beta}(A) = \{\langle x, \mu_A * (x), \nu_A * (x)\rangle | x \in X\}$$

where

$$\mu_A* = \begin{cases} \mu_A(x) & \textit{if } \sigma_{1,A}(x) = 1 \\ \alpha\mu_A(x) & \textit{otherwise} \end{cases}$$

$$\nu_A* = \begin{cases} \nu_A(x) & \textit{if } \sigma_{1,A}(x) = 1 \\ \beta(1 - \alpha\mu_A(x) - \nu_A(x)) & \textit{otherwise} \end{cases}$$

For example, the result of the application of this operator with $\alpha = 0.7, \beta = 0.1$ over the set

$$A = \{\langle \textit{apple}, 0.9, 0.1\rangle, \langle \textit{orange}, 0.7, 0.1\rangle, \langle \textit{lemon}, 0.2, 0.75\rangle\}$$

is the following (intuitionistic fuzzy) set:

$$H^*_{r;0.7,0.1}(A) = \{\langle \textit{apple}, 0.9, 0.1\rangle, \langle \textit{orange}, 0.49, 0.041\rangle, \langle \textit{lemon}, 0.14, 0.011\rangle\}$$

Remark 3. *Some operators will coincide with their restricted version but not all. For instance, $F_{\alpha,\beta}$ or F_B coincides with F_r. However, $G_{\alpha,\beta}$ is distinctly different from G_r*

Theorem 1. *Let $A, B \in IFS(X)$. Let*

$$f(x) = 1 - \sigma_{1,A}(x)\sigma_{1,B}(x) \tag{10}$$

Then $A, B \in SSIMIFS(X_{PIFP})$ is equivalent to:

$$\begin{cases} f(x) = 0 \textit{ if } \max(\sigma_{1,A}(x), \sigma_{1,B}(x)) = 1 \\ f(x) \neq 0 \textit{ if } \max(\sigma_{1,A}(x), \sigma_{1,B}(x)) < 1. \end{cases} \tag{11}$$

Proof. The proof is obvious from (5), (6) and (8) and the fact that x runs $X_{FP} \cup X_{PIFP}$. □

3 Conclusion

We have presented a new point of view on intuitionistic fuzzy sets as a collections of fuzzy pairs and proper intuitionistic fuzzy pairs. The idea is to introduce operators which only act on the "points" which have some degree of uncertainty in a natural way. This has an additional benefit that it opens a way for easier implementation of such operators algorithmically, for instance, for improving estimates obtain by certain procedure, e.g., by InterCriteria Analysis. In future research the author will investigate further this approach.

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