Notes on Intuitionistic Fuzzy Sets
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Almost uniformly convergence on MV-algebra of intuitionistic fuzzy sets

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This paper is dedicated to the 20-th anniversary
of the research of intuitionistic fuzzy sets in Slovakia

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Abstract: The aim of this contribution is to formulate some definitions of almost uniformly convergence for a sequence of observables in the MV-algebra of the intuitionistic fuzzy sets. We define a partial binary operation $\ominus$ called difference on MV-algebra of intuitionistic fuzzy sets. As an illustration of the use the almost uniformly convergence we prove a variation of Egorov's theorem for the observables in MV-algebra of intuitionistic fuzzy sets.
Keywords: MV-algebra, ℓ-groups, Intuitionistic fuzzy sets, States, Observables, Difference, Almost uniformly convergence, Egorov's theorem.
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1 Introduction

The year 2023 is the 40-th anniversary of the invention of the concept and theory of intuitionistic fuzzy sets by K. T. Atanassov in the paper [1]. As an **intuitionistic fuzzy set A** on Ω he understands a pair (μ_A, ν_A) of mappings $\mu_A, \nu_A : \Omega \to [0, 1]$ such that $\mu_A + \nu_A \leq 1_\Omega$. The concept of the intuitionistic fuzzy sets is the generalization of the concept of the fuzzy sets introduced by L. Zadeh (see [13, 14]). Namely if $\mu_A : \Omega \longrightarrow [0,1]$ is a fuzzy set, then $\mathbf{A} = (\mu_A, 1 - \mu_A)$ is the corresponding intuitionistic fuzzy set. Sometimes we need to work with intuitionistic fuzzy events. An **intuitionistic fuzzy event** is an intuitionistic fuzzy set $\mathbf{A} = (\mu_A, \nu_A)$ such that $\mu_A, \nu_A : \Omega \to [0,1]$ are $\mathcal{S}$-measurable (see [2, 3, 8]). The family of all IF-events on $(\Omega, \mathcal{S})$ will be denoted by $\mathcal{F}$.

In papers [7, 9] Riečan constructed the suitable MV-algebra $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ to the intuitionistic fuzzy space $(\mathcal{F}, \mathbf{m})$. In this paper we study an almost uniformly convergence for a sequence of observables on mentioned MV-algebra and we formulate some definitions of this convergence. As an example of the use of almost uniformly convergence we prove a variation of the Egorov's theorem for MV-algebra of intuitionistic fuzzy sets. This theorem says about a connection between almost everywhere convergence and almost uniformly convergence. We define a partial binary operation $\ominus$ called difference on MV-algebra of intuitionistic fuzzy sets. We are inspired by the results of B. Riečan in paper [6]. There he studied an almost uniformly convergence in D-posets.

Remark that in a whole text we use a notation "IF" in short as the phrase "intuitionistic fuzzy".

2 MV-algebra of intuitionistic fuzzy sets

In this section we study the properties of the MV-algebra of IF-sets. In papers [7, 9] B. Riečan showed that any IF-space $\mathcal{F}$ can be embedded to a convenient MV-algebra. Now we recall the basic notions about MV-algebras. By the Mundici theorem any MV-algebra can be defined by the help of an l-group (see [11]).

Definition 2.1 ([11]). *By an ℓ-group we shall mean the structure $(G, +, \leq)$ such that the following properties are satisfied:*

(i) $(G, +)$ *is an Abelian group;*

(ii) $(G, \leq)$ *is a lattice;*

(iii) $a \leq b \Longrightarrow a + c \leq b + c$.

For each ℓ-group G, an element $u \in G$ is said to be a strong unit of G, if for all $a \in G$ there is an integer $n \geq 1$ such that $nu \geq a$ (nu is the sum $u + \ldots + u$ with n).

Example 2.1. *Let $(\Omega, \mathcal{S})$ be a measurable space, $\mathcal{S}$ be a σ-algebra. Consider*

$$\begin{aligned} \mathcal{G} &= \{\mathbf{A} = (\mu_A, \nu_A); \mu_A, \nu_A : \Omega \to R \textit{ are } \mathcal{S} - \textit{measurable functions}\}, \\ \mathbf{A} + \mathbf{B} &= (\mu_A + \mu_B, \nu_A + \nu_B - 1_\Omega), \\ \mathbf{A} \leq \mathbf{B} &\iff \mu_A \leq \mu_B, \nu_A \geq \nu_B. \end{aligned}$$

Then $(\mathcal{G}, +, \leq)$ is an ℓ-group with the neutral element $\mathbf{0} = (0_\Omega, 1_\Omega)$,

$$\mathbf{A} - \mathbf{B} = (\mu_A - \mu_B, \nu_A - \nu_B + 1_\Omega)$$

and the lattice operations

$$\begin{aligned}\mathbf{A} \vee \mathbf{B} &= (\mu_A \vee \mu_B, \nu_A \wedge \nu_B),\\ \mathbf{A} \wedge \mathbf{B} &= (\mu_A \wedge \mu_B, \nu_A \vee \nu_B).\end{aligned}$$

Definition 2.2 ([11]). *An MV-algebra is an algebraic system $(M, \oplus, \odot, \neg, 0, u)$, where $\oplus, \odot$ are binary operations, $\neg$ is a unary operation, $0, u$ are fixed elements, which can be obtained by the following way: there exists a lattice group $(G, +, \leq)$ such that $M = \{x \in G; 0 \leq x \leq u\}$, where 0 is the neutral element of G, u is a strong unit of G, and*

$$\begin{aligned}a \oplus b &= (a + b) \wedge u,\\ a \odot b &= (a + b - u) \vee 0,\\ \neg a &= u - a.\end{aligned}$$

Here $\vee, \wedge$ are the lattice operations with respect to the order and $\neg a$ is the opposite element of the element a with respect to the operation of the group.

Example 2.2. *Let $(\Omega, \mathcal{S})$ be a measurable space, $\mathcal{M}$ the family of all pairs $\mathbf{A} = (\mu_A, \nu_A)$, where $\mu_A, \nu_A : \Omega \to [0, 1]$ are $\mathcal{S}$-measurable functions,*

$$\begin{aligned}\mathbf{A} \oplus \mathbf{B} &= \big((\mu_A + \mu_B) \wedge 1_\Omega, (\nu_A + \nu_B - 1_\Omega) \vee 0_\Omega\big),\\ \mathbf{A} \odot \mathbf{B} &= ((\mu_A + \mu_B - 1_\Omega) \vee 0_\Omega, (\nu_A + \nu_B) \wedge 1_\Omega)),\\ \neg\mathbf{A} &= (1_\Omega - \mu_A, 1_\Omega - \nu_A).\end{aligned}$$

Then the system $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ is an MV-algebra. Here the corresponding group is $(\mathcal{G}, +, \leq)$ considered in Example 1.

Definition 2.3 ([11]). *An MV-algebra M is said to be σ-complete if its underlying lattice is σ-complete, i.e., every non-empty countable subset of M has a supremum in M.*

Every finite MV-algebra M is σ-complete - indeed, M is complete, in the sense that every non-empty subset of M has a supremum in M.

Definition 2.4 ([10]). *Let $(M, \oplus, \odot, \neg, 0, u)$ be an MV-algebra. By a finitely additive state on an MV-algebra M is considered each monotone mapping (i.e. $a \leq b \Rightarrow m(a) \leq m(b)$) $m : M \to [0, 1]$ satisfying the following conditions:*

(i) $m(u) = 1, m(0) = 0$;

(ii) $a \odot b = 0 \Rightarrow m(a \oplus b) = m(a) + m(b)$.

A finitely additive state is a state, if moreover

(iii) $a_n \nearrow a \Rightarrow m(a_n) \nearrow m(a)$.

We say that m is faithful (also called, strictly positive) if $m(x) \neq 0$ whenever $x \neq 0$, $x \in M$.

Example 2.3. *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2. By a state on an* MV*-algebra* $\mathcal{M}$ *we understand each monotone mapping* $m : \mathcal{M} \to [0,1]$ $\big($*i.e.* $\mathbf{A} \leq \mathbf{B} \Rightarrow m(\mathbf{A}) \leq m(\mathbf{B})\big)$ *satisfying the following conditions:*

(i) $m((1_\Omega, 0_\Omega)) = 1, m((0_\Omega, 1_\Omega)) = 0$*;*

(ii) $\mathbf{A} \odot \mathbf{B} = (0_\Omega, 1_\Omega) \Rightarrow m(\mathbf{A} \oplus \mathbf{B}) = m(\mathbf{A}) + m(\mathbf{B})$*;*

(iii) $\mathbf{A}_n \nearrow \mathbf{A} \Rightarrow m(\mathbf{A}_n) \nearrow m(\mathbf{A})$

for all $\mathbf{A}, \mathbf{A}_n, \mathbf{B} \in \mathcal{M}$*,* $n \in N$*.*

Following proposition says about the properties of a state m on the MV-algebra M.

Proposition 2.1 ([11])**.** *Let* m *be a finitely additive state on an MV-algebra* M*. Then we have:*

(i) $m(\neg a) = 1 - m(a)$ *for all* $a \in M$*;*

(ii) m *is a valuation:* $m(a) + m(b) = m(a \oplus b) + m(a \odot b)$ *for all* $a, b \in M$*;*

(iii) if m *is faithful, then* m *is strictly monotone: if* $a < b$*, then* $m(a) < m(b)$*;*

(iv) m *is also a valuation with respect to the underlying lattice order of* M*; stated otherwise, for all* $a, b \in M$*, we have* $m(a) + m(b) = m(a \vee b) + m(a \wedge b)$*;*

(v) m *is subadditive, in the sense that* $m(a \vee b) \leq m(a \oplus b) \leq m(a) + m(b)$*.*

Each state on MV-algebra is sub-σ-additive.

Lemma 2.1 ([5])**.** *Let* m *be a state on MV-algebra* M*. Then*

$$m\left(\bigvee_{n=1}^{\infty} a_n\right) \leq \sum_{n=1}^{\infty} m(a_n)$$

for each sequence $(a_n)_1^\infty$*,* $a_n \in M$*.*

Now we recall the definition of n-dimensional observable in MV-algebras.

Definition 2.5 ([11])**.** *Let* M *be an MV-algebra. An* n*-dimensional observable of* M *is a map* $x : \mathcal{B}(R^n) \to M$ *satisfying the following conditions:*

(i) $x(R^n) = u$*;*

(ii) whenever $A, B \in \mathcal{B}(R^n)$ *and* $A \cap B = \emptyset$*, then* $x(A \cup B) = x(A) \oplus x(B)$*;*

(iii) for all $A, A_i \in \mathcal{B}(R^n)$*,* $i \in N$ *, if* $A_i \nearrow A$*, then* $x(A_i) \nearrow x(A)$*.*

When $n = 1$ *we say that* x *is an observable.*

The condition (ii) above states that, whenever $A \cap B = \emptyset$, then $x(A \cup B) = x(A) + x(B)$ in the ℓ-group with strong unit corresponding to M.

Example 2.4. *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2. An* n*-dimensional observable of MV-algebra* $\mathcal{M}$ *is a map* $x : \mathcal{B}(R^n) \to \mathcal{M}$ *satisfying the following conditions:*

(i) $x(R^n) = (1_\Omega, 0_\Omega)$*;*

(ii) whenever $A, B \in \mathcal{B}(R^n)$ *and* $A \cap B = \emptyset$*, then* $x(A \cup B) = x(A) \oplus x(B)$*;*

(iii) for all $A, A_i \in \mathcal{B}(R^n)$*,* $i \in N$ *, if* $A_i \nearrow A$*, then* $x(A_i) \nearrow x(A)$*.*

When $n = 1$ *we say that* x *is an observable.*

3 Almost uniformly convergence in MV-algebra of IF-sets

In this section we study an almost uniformly convergence of observables in MV-algebra of IF-sets constructed in Example 2.2. We show some definitions of this convergence.

Definition 3.1. *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega,1_\Omega),(1_\Omega,0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2 and* m *be a state. We say that the sequence* $(x_n)_1^\infty$ *of the observables converges* m*-almost uniformly to* 0*, if*

$$\forall \alpha > 0 \;\; \exists\, \mathbf{A} \in \mathcal{M} : \; m(\neg \mathbf{A}) < \alpha,$$
$$\forall \beta > 0 \;\; \exists\, k \in N \;\; \forall n \geq k : \; \mathbf{A} \leq x_n\big((-\beta, \beta)\big).$$

The Definition 3.1 can be rewritten in the following form.

Definition 3.2. *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega,1_\Omega),(1_\Omega,0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2 and* m *be a state. We say that the sequence* $(x_n)_1^\infty$ *of the observables converges* m*-almost uniformly to* 0*, if*

$$\forall \alpha > 0 \;\; \exists\, \mathbf{A} \in \mathcal{M} : \; m(\mathbf{A}) > 1 - \alpha,$$
$$\forall \beta > 0 \;\; \exists\, k \in N \;\; \forall n \geq k : \; \mathbf{A} \leq x_n\big((-\beta, \beta)\big).$$

Now we define a partial binary operation $\ominus$ called **difference** on the MV-algebra of IF-sets and we formulate a definition of almost uniformly convergence using this partial binary operation. We are inspired by paper [6]. There B. Riečan studied an almost uniformly convergence in D-posets.

Let $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ be the MV-algebra constructed in *Example 2.2*. If $\mathbf{A} = (\mu_A, \nu_A) \in \mathcal{M}$, $\mathbf{B} = (\mu_B, \nu_B) \in \mathcal{M}$ and $\mathbf{B} \leq \mathbf{A}$, then we define a partial binary operation $\ominus$ on $\mathcal{M}$ by

$$\mathbf{A} \ominus \mathbf{B} = ((\mu_A - \mu_B) \vee 0_\Omega, (\nu_A - \nu_B + 1_\Omega) \wedge 1_\Omega).$$

It is easy to see, that $\mathbf{A} \ominus \mathbf{B} = \mathbf{A} \odot \neg \mathbf{B}$. Really

$$\begin{aligned}
\mathbf{A} \odot \neg \mathbf{B} &= (\mu_A, \nu_A) \odot (1_\Omega - \mu_B, 1_\Omega - \nu_B) \\
&= \big((\mu_A + 1_\Omega - \mu_B - 1_\Omega) \vee 0_\Omega, (\nu_A + 1_\Omega - \nu_B) \wedge 1_\Omega\big) \\
&= ((\mu_A - \mu_B) \vee 0_\Omega, (\nu_A - \nu_B + 1_\Omega) \wedge 1_\Omega) = \mathbf{A} \ominus \mathbf{B}.
\end{aligned}$$

Definition 3.3. *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega,1_\Omega),(1_\Omega,0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2 and* m *be a state. We say that the sequence* $(x_n)_1^\infty$ *of the observables converges* m*-almost uniformly to* 0*, if*

$$\forall \alpha > 0 \,\exists\, \mathbf{C} \in \mathcal{M} : \ m(\mathbf{C}) > 1 - \alpha,$$
$$\forall \beta > 0 \,\exists\, k \in N \ \forall n \geq k \ \exists\, \mathbf{C}_n \in \mathcal{M}, m(\mathbf{C}_n) < \alpha, \ \mathbf{C}_n \leq \mathbf{C}_{n+1} \leq \mathbf{C} : \mathbf{C} \ominus \mathbf{C}_n \leq x_n\big((-\beta, \beta)\big).$$

In [12] F. Chovanec proved that every MV-algebra M is a D-poset, where $b \ominus a = b \odot \neg a$. Recall that D-poset is partially ordered set D with the greatest element 1_D and with a partial binary operation $\ominus$ such that $b \ominus a$ is defined if and only if $a \leq b$ and satisfying the following conditions (see [12]):

(i) if $a \leq b$, then $b \ominus a \leq b$ and $b \ominus (b \ominus a) = a$;

(ii) if $a \leq b \leq c$, then $b \ominus a \leq c \ominus a$ and $(c \ominus a) \ominus (c \ominus b) = b \ominus a$;

In paper [5] we formulated an almost uniformly convergence for a family of IF-events $\mathcal{F}$. We proved a variation of the Egorov's theorem, too. The results were the generalization of the results in [4], because if $\mu_A : \Omega \longrightarrow [0,1]$ is a fuzzy set, then $\mathbf{A} = (\mu_A, 1 - \mu_A) : \Omega \to [0,1]^2$ is the corresponding intuitionistic fuzzy set. Next theorem shows a connection between m-almost everywhere convergence and m-almost uniformly convergence of the observables in the MV-algebra $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ constructed in *Example 2.2* with respect to the state m.

Theorem 3.1. (A variation of Egorov's Theorem) *Let* $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega, 1_\Omega), (1_\Omega, 0_\Omega))$ *be the* MV*-algebra constructed in Example 2.2 and* m *be a state. If a sequence* $(x_n)_1^\infty$ *of the observables converges* m*-almost everywhere to* 0*, then the sequence* $(x_n)_1^\infty$ *converges* m*-almost uniformly to* 0*.*

Proof. Let a sequence of the observables $(x_n)_1^\infty$ converges m-almost everywhere to 0. By Definition 2.13 in [11] we have

$$m\Big(\bigwedge_{p=1}^{\infty} \bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} x_n\Big(\Big(-\frac{1}{p}, \frac{1}{p}\Big)\Big)\Big) = 1.$$

Put

$$\mathbf{A}_k^p = \bigwedge_{n=k}^{\infty} x_n\Big(\Big(-\frac{1}{p}, \frac{1}{p}\Big)\Big).$$

Then $\mathbf{A}_k^p \leq \mathbf{A}_{k+1}^p$ and

$$m\Big(\bigvee_{k=1}^{\infty} \mathbf{A}_k^p\Big) = m\Big(\bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} x_n\Big(\Big(-\frac{1}{p}, \frac{1}{p}\Big)\Big)\Big) = 1 \tag{1}$$

for every p, i.e. $\lim\limits_{p \to \infty} m\big(\mathbf{A}_k^p\big) = 1$.

By (1) we have that for every $\alpha > 0$ and every p there exists $\mathbf{A}_{k(p)}^p \in \mathcal{M}$ such that

$$m\Big(\neg \mathbf{A}_{k(p)}^p\Big) < \frac{\alpha}{2^p}. \tag{2}$$

Put

$$\mathbf{A} = \bigwedge_{p=1}^{\infty} \mathbf{A}_{k(p)}^{p},$$

then using De Morgan rules we have

$$\neg\mathbf{A} = \bigvee_{p=1}^{\infty} \neg\mathbf{A}_{k(p)}^{p}.$$

Therefore using sub-σ-additivity of state m (see Lemma 2.1) and using the inequality (2) we obtain

$$m(\neg\mathbf{A}) = m\Big(\bigvee_{p=1}^{\infty} \neg\mathbf{A}_{k(p)}^{p}\Big) \leq \sum_{p=1}^{\infty} m\Big(\neg\mathbf{A}_{k(p)}^{p}\Big) < \sum_{p=1}^{\infty} \frac{\alpha}{2^p} = \alpha.$$

To every $\beta > 0$ choose p such that $\frac{1}{p} < \beta$. Then

$$\mathbf{A} = \bigwedge_{p=1}^{\infty} \mathbf{A}_{k(p)}^{p} \leq \mathbf{A}_{k(p)}^{p} = \bigwedge_{n=k(p)}^{\infty} x_n\Big(\Big(-\frac{1}{p},\frac{1}{p}\Big)\Big) \leq x_n\Big(\Big(-\frac{1}{p},\frac{1}{p}\Big)\Big) \leq x_n(-\beta,\beta),$$

i.e. by Definition 3.1 the sequence $(x_n)_1^{\infty}$ of observables converges m-almost uniformly to 0. $\square$

4 Conclusion

The paper is concerned in a probability theory on the MV-algebra $(\mathcal{M}, \oplus, \odot, \neg, (0_\Omega,1_\Omega),(1_\Omega,0_\Omega))$ constructed in Example 2.2. We formulated three definitions of m-almost uniformly convergence for a sequence of observables in the MV-algebra $\mathcal{M}$. We defined a partial binary operation $\ominus$ called difference on mentioned MV-algebra $\mathcal{M}$. Therefore the MV-algebra $\mathcal{M}$ is a D-poset $(\mathcal{M}, \leq, \ominus, (1_\Omega, 0_\Omega))$. We proved the Egorov's theorem and we showed the connection between an almost everywhere convergence and an almost uniformly convergence of observables in MV-algebra $\mathcal{M}$.

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