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THE HOMOMORPHISM AND ANTI–HOMOMORPHISM OF LOWER LEVEL SUBGROUPS OF AN INTUITIONISTIC ANTIFUZZY SUBGROUP

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ABSTRACT.

In this paper, we introduce some properties of an intuitionistic antifuzzy subgroup of a group with homomorphism and anti-homomorphism.

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INTRODUCTION.

After the introdution of fuzzy sets by L.A.Zadeh[11], several researchers explored on the generalization of the notion of fuzzy set.The concept of IFS was introduced by K.T.Atanassov [1], as a generalization of the notion of fuzzy set. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S.[2] defined a fuzzy subgroups and fuzzy homomorphism. We introduce some properties of an intuitionistic antifuzzy subgroup of a group with homomorphism and anti-homomorphism.

1. PRELIMINARIES :

1.1 Definition :

An intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form $A=\{ \langle x, \mu_A(x), \nu_A(x) \rangle / x\in X \}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x\in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.2 Definition:

Let G be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic antifuzzy subgroup(AIFSG) of G if the following conditions are satisfied:

(i) $\mu_A(xy^{-1}) \leq \max\{ \mu_A(x), \mu_A(y) \}$,

(ii) $\nu_A(xy^{-1}) \geq \min\{ \nu_A(x), \nu_A(y) \}$, for all x and $y\in G$.

1. 3 Definition :

Let A be an intuitionistic fuzzy subset of a set X. For $t \in [0, 1]$, the lower level subset of A is the set,

$$A_t = \{ x \in X : \mu_A(x) \leq t \text{ and } \nu_A(x) \geq t \}.$$

This is called an intuitionistic fuzzy lower level subset of A .

1.4 Definition :

Let A be an intuitionistic antifuzzy subgroup of a group G. The subgroup A_t of G, for $t \in [0,1]$ such that $t \geq \mu_A(e)$ and $t \leq \nu_A(e)$ is called a lower level subgroup of A.

1.5 Definition :

If (G, **.**) and (G^l, **.**) are any two groups, then the function $f : G \rightarrow G^l$ is called a **group homomorphism** if $f(xy) = f(x)f(y)$, for all x and $y \in G$.

1.6 Definition :

If (G, **.**) and (G^l, **.**) are any two groups, then the function $f : G \rightarrow G^l$ is called a **group anti-homomorphism** if $f(xy) = f(y)f(x)$, for all x and $y \in G$.

1.7 Definition :

Let X and X^1 be any two sets. Let $f : X \rightarrow X^1$ be any function and let A

be an intuitionistic fuzzy subset in X, V be an intuitionistic fuzzy subset in

$f(X)= X^1$, defined by $\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \sup_{x \in f^{-1}(y)} \nu_A(x)$, for all $x \in X$ and $y \in X^1$.

A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.1 Theorem :

Let G, G^l be any two groups with identity. Let $f : G \rightarrow G^l$ be a homomorphism. Then,

(i) $f(1) = 1^l$ where 1 and 1^l are the identities of G and G^l respectively.

(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

Proof: It is trivial.

1.2 Theorem :

Let G, G^l be any two group with identity. Let $f : G \rightarrow G^l$ be an anti-homomorphism. Then,

(i) $f(1)=1^l$ where 1 and 1^l are the identitie of G and G^l respectively, and

(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

Proof : It is trivial.

SOME PROPOSITIONS :

1.1 Proposition :

Let A be an intuitionistic antifuzzy subgroup of a group G. Then for $t \in [0,1]$ such that $t \geq \mu_A(e)$ and $t \leq \mu_A(e)$, A_t is a subgroup of G.

1.2 Proposition:

The homomorphic image of an intuitionistic antifuzzy subgroup of a group G is an intuitionistic antifuzzy subgroup of a group G^l.

1.3 Proposition :

The homomorphic pre-image of an intuitionistic antifuzzy subgroup of a group G^l is an intuitionistic antifuzzy subgroup of a group G.

1.4 Proposition :

The anti-homomorphic image of an intuitionistic antifuzzy subgroup of a group G is an intuitionistic antifuzzy subgroup of a group G^l.

1.5 Proposition :

The anti-homomorphic pre-image of an intuitionistic antifuzzy subgroup of a group G^l is an intuitionistic antifuzzy subgroup of a group G.

1.6 Proposition:

The homomorphic image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G^l.

Proof:

Let G and G^l be any two groups.

Let $f : G \rightarrow G^l$ be a homomorphism.

That is $f(xy) = f(x)f(y)$ for all x and $y \in G$.

Let V=f(A), where A is an intuitionistic antifuzzy subgroup of a group G.

Clearly V is an intuitionistic antifuzzy subgroup of a group G^l.

Let x and $y \in G$, implies f(x) and f(y) in G^l.

Clearly A_t is a lower level subgroup of A.

That is $\mu_A(x) \leq t$ and $\nu_A(x) \geq t$; $\mu_A(y) \leq t$ and $\nu_A(y) \geq t$:

$\mu_A(xy^{-1}) \leq t$ and $\nu_A(xy^{-1}) \geq t$.

We have to prove that $f(A_t)$ is a lower level subgroup of V.

Now, $\mu_V(f(x)) \leq \mu_A(x) \leq t$, implies that $\mu_V(f(x)) \leq t$:

$\mu_V(f(y)) \leq \mu_A(y) \leq t$, implies that $\mu_V(f(y)) \leq t$; and

$$\begin{aligned}\mu_V(f(x)(f(y))^{-1}) &= \mu_V(f(x)f(y^{-1})), \quad \text{as f is a homomorphism}\\ &= \mu_V(f(xy^{-1})), \quad \text{as f is a homomorphism}\\ &\leq \mu_A(xy^{-1}) \leq t,\end{aligned}$$

which implies that $\mu_V(f(x)(f(y))^{-1}) \leq t$.

And, $\nu_V(f(x)) \geq \nu_A(x) \geq t$, implies that $\nu_V(f(x)) \geq t$:

$\nu_V(f(y)) \geq \nu_A(y) \geq t$, implies that $\nu_V(f(y)) \geq t$; and

$$\begin{aligned}\nu_V(f(x)(f(y))^{-1}) &= \nu_V(f(x)f(y^{-1})), \quad \text{as f is a homomorphism}\\ &= \nu_V(f(xy^{-1})), \quad \text{as f is a homomorphism}\\ &\geq \nu_A(xy^{-1}) \geq t,\end{aligned}$$

which implies that $\nu_V(f(x)(f(y))^{-1}) \geq t$.

Therefore $\mu_V(f(x)(f(y))^{-1}) \leq t$ and $\nu_V(f(x)(f(y))^{-1}) \geq t$.

Hence $f(A_t)$ is a lower level subgroup of an intuitionistic antifuzzy subgroup V of a group G^l.

1.7 Proposition :

The homomorphic pre-image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G^l is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G.

Proof:

Let G and G^l be any two groups.

Let $f : G \rightarrow G^l$ be a homomorphism.

That is $f(xy) = f(x)f(y)$ for all x and $y \in G$.

Let V=f(A), where V is an intuitionistic antifuzzy subgroup of a group G^l.

Clearly A is an intuitionistic antifuzzy subgroup of a group G.

Let f(x) and $f(y) \in G^l$, implies x and y in G.

Clearly $f(A_t)$ is a lower level subgroup of V.

That is $\mu_V(f(x)) \leq t$ and $\nu_V(f(x)) \geq t$; $\mu_V(f(y)) \leq t$ and $\nu_V(f(y)) \geq t$;

$\mu_V(f(x)(f(y))^{-1}) \leq t$ and $\nu_V(f(x)(f(y))^{-1}) \geq t$.

We have to prove that A_t is a lower level subgroup of A.

Now, $\mu_A(x) = \mu_V(f(x)) \leq t$, implies that $\mu_A(x) \leq t$:

$\mu_A(y) = \mu_V(f(y)) \leq t$, implies that $\mu_A(y) \leq t$; and

$$\begin{aligned} \mu_A(xy^{-1}) &= \mu_V(f(xy^{-1})), \\ &= \mu_V(f(x)f(y^{-1})), \quad \text{as f is a homomorphism} \\ &= \mu_V(f(x)(f(y))^{-1}), \quad \text{as f is a homomorphism} \\ &\leq t, \end{aligned}$$

which implies that $\mu_A(xy^{-1}) \leq t$.

And, $\nu_A(x) = \nu_V(f(x)) \geq t$, implies that $\nu_A(x) \geq t$:

$\nu_A(y) = \nu_V(f(y)) \geq t$, implies that $\nu_A(y) \geq t$; and

$$\begin{aligned} \nu_A(xy^{-1}) &= \nu_V(f(xy^{-1})), \\ &= \nu_V(f(x)f(y^{-1})), \quad \text{as f is a homomorphism} \\ &= \nu_V(f(x)(f(y))^{-1}), \quad \text{as f is a homomorphism} \\ &\geq t, \end{aligned}$$

which implies that $\nu_A(xy^{-1}) \geq t$.

Therefore $\mu_A(xy^{-1}) \leq t$ and $\nu_A(xy^{-1}) \geq t$.

Hence A_t is a lower level subgroup of an intuitionistic antifuzzy subgroup A of a group G.

1.8 Proposition:

The anti-homomorphic image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G^l.

Proof:

Let G and G^l be any two groups.

Let $f : G \rightarrow G^l$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$ for all x and $y \in G$.

Let V=f(A), where A is an intuitionistic antifuzzy subgroup of a group G.

Clearly V is an intuitionistic antifuzzy subgroup of a group G^l.

Let x and $y \in G$, implies f(x) and f(y) in G^l.

Clearly A_t is a lower level subgroup of A.

That is $\mu_A(x) \leq t$ and $\nu_A(x) \geq t$; $\mu_A(y) \leq t$ and $\nu_A(y) \geq t$:

$\mu_A(y^{-1}x) \leq t$ and $\nu_A(y^{-1}x) \geq t$.

We have to prove that $f(A_t)$ is a lower level subgroup of V.

Now, $\mu_V(f(x)) \leq \mu_A(x) \leq t$, implies that $\mu_V(f(x)) \leq t$:

$\mu_V(f(y)) \leq \mu_A(y) \leq t$, implies that $\mu_V(f(y)) \leq t$; and

$$\begin{aligned} \mu_V(f(x)(f(y))^{-1}) &= \mu_V(f(x)f(y^{-1})), \quad \text{as f is an anti-homomorphism} \\ &= \mu_V(f(y^{-1}x)), \quad \text{as f is an anti-homomorphism} \\ &\leq \mu_A(y^{-1}x) \leq t, \end{aligned}$$

which implies that $\mu_V(f(x)(f(y))^{-1}) \leq t$.

And, $\nu_V(f(x)) \geq \nu_A(x) \geq t$, implies that $\nu_V(f(x)) \geq t$:

$\nu_V(f(y)) \geq \nu_A(y) \geq t$, implies that $\nu_V(f(y)) \geq t$; and

$$\begin{aligned} \nu_V(f(x)(f(y))^{-1}) &= \nu_V(f(x)f(y^{-1})), \quad \text{as f is an anti-homomorphism} \\ &= \nu_V(f(y^{-1}x)), \quad \text{as f is an anti-homomorphism} \\ &\geq \nu_A(y^{-1} x) \geq t, \end{aligned}$$

which implies that $\nu_V(f(x)(f(y))^{-1}) \geq t$.

Therefore $\mu_V(f(x)(f(y))^{-1}) \leq t$ and $\nu_V(f(x)(f(y))^{-1}) \geq t$.

Hence $f(A_t)$ is a lower level subgroup of an intuitionistic antifuzzy subgroup V of a group G^l.

1.9 Proposition :

The anti-homomorphic pre-image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G^l is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group G.

Proof:

Let G and G^l be any two groups.

Let $f : G \rightarrow G^l$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$, for all x and $y \in G$.

Let V=f(A), where V is an intuitionistic antifuzzy subgroup of a group G^l.

Clearly A is an intuitionistic antifuzzy subgroup of a group G.

Let f(x) and $f(y) \in G^l$, implies x and y in G.

Clearly $f(A_t)$ is a lower level subgroup of V.

That is $\mu_V(f(x)) \leq t$ and $\nu_V(f(x)) \geq t$; $\mu_V(f(y)) \leq t$ and $\nu_V(f(y)) \geq t$;

$\mu_V((f(y))^{-1}f(x)) \leq t$ and $\nu_V((f(y))^{-1}f(x)) \geq t$.

We have to prove that A_t is a lower level subgroup of A.

Now, $\mu_A(x) = \mu_V(f(x)) \leq t$, implies that $\mu_A(x) \leq t$:

$\mu_A(y) = \mu_V(f(y)) \leq t$, implies that $\mu_A(y) \leq t$; and

$$\begin{aligned} \mu_A(xy^{-1}) &= \mu_V(f(xy^{-1})), \\ &= \mu_V(f(y^{-1})f(x)), \quad \text{as f is an anti-homomorphism} \\ &= \mu_V((f(y))^{-1}f(x)), \quad \text{as f is an anti-homomorphism} \\ &\leq t, \end{aligned}$$

which implies that $\mu_A(xy^{-1}) \leq t$.

And, Now, $\nu_A(x) = \nu_V(f(x)) \geq t$, implies that $\nu_A(x) \geq t$:

$\nu_A(y) = \nu_V(f(y)) \geq t$, implies that $\nu_A(y) \geq t$; and

$$\begin{aligned} \nu_A(xy^{-1}) &= \nu_V(f(xy^{-1})), \\ &= \nu_V(f(y^{-1})f(x)), \quad \text{as f is an anti-homomorphism} \\ &= \nu_V((f(y))^{-1}f(x)), \quad \text{as f is an anti-homomorphism} \\ &\geq t, \end{aligned}$$

which implies that $\nu_A(xy^{-1}) \geq t$.

Therefore $\mu_A(xy^{-1}) \leq t$ and $\nu_A(xy^{-1}) \geq t$.

Hence A_t is a lower level subgroup of an intuitionistic antifuzzy subgroup A of a group G.

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