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A NEW GEOMETRICAL INTERPRETATION OF THE INTUITIONISTIC FUZZY SETS

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All definitions of the concepts of the Intuitionistic Fuzzy Sets (IFSs) are given in [1]. Here we shall note only that the three functions related to a given IFS A satisfy the equality:

$$\mu(x) + \gamma(x) + \pi(x) = 1 \qquad (*)$$

for every $x \in E$, where E is a fixed universum.

In [2-5] four geometrical interpretations of the elements of the IFSs and of the Intuitionistic Fuzzy Logics (IFLs) [6-9] are introduced. All of them are planar interpretations - by points [2, 3], angles [4] and segments [5]. Here we shall introduce the first stereometrical interpretation of the elements of the IFSs and IFLs.

Obviously, the functions μ, γ and π are non-negative and limited. The equality (*) gives the possibility to put:

$$\mu(x) = \sin^2\Theta(x).\cos^2\varphi(x)$$

$$\gamma(x) = \sin^2\Theta(x).\sin^2\varphi(x)$$

$$\pi(x) = \cos^2\Theta(x),$$

where $0 \le \Theta(x) \le \pi/2$ and $0 \le \varphi(x) \le \pi/2$ (here "π" is a constant, unlike the above function "π").

The analogy with the spherical coordinates of the points (see, e.g. [10]) is total, because $m = \sqrt{\mu}$, $n = \sqrt{\gamma}$ and $p = \sqrt{\pi}$ are cos-directories of the vector with a start point <0, 0, 0> and end point belonging to the single spherical surface in the first octant. For the Jacobian J of this transformation it is valid that $J \neq 0$ for all points outside the sizes of the basic spherical triangle (see Fig. 1). $J = 0$ iff $\pi(x) = 0$, i.e., when IFS is reduced to an ordinary fuzzy set (see e.g. [11, 12]).

The relations between Θ and φ, and μ, γ and π are the following

$$\varphi(x) = \text{arctg}\left(\sqrt{\frac{\gamma(x)}{\mu(x)}}\right) \text{ and } \varphi(x) = \text{arctg}\left(\sqrt{\frac{\mu(x) + \gamma(x)}{\pi(x)}}\right)$$

The above defined geometrical interpretation gives the possibility to introduce distance between points, which are geometrical representations of the IFL-truth-values of two given sentences A and B, as the lengths of the catenary which connected them, i.e.,

$$L_{A,B} = M(A, B),$$

where M is an operator determining the length of the catenary $L_{A,B}$ (cf. [13, 14]). Such defined metrics satisfies the conditions:

$$M(A, A) = 0$$

$$M(A, B) = M(B, A)$$

$$M(A, B) + M(B, C) \ge M(A, C).$$

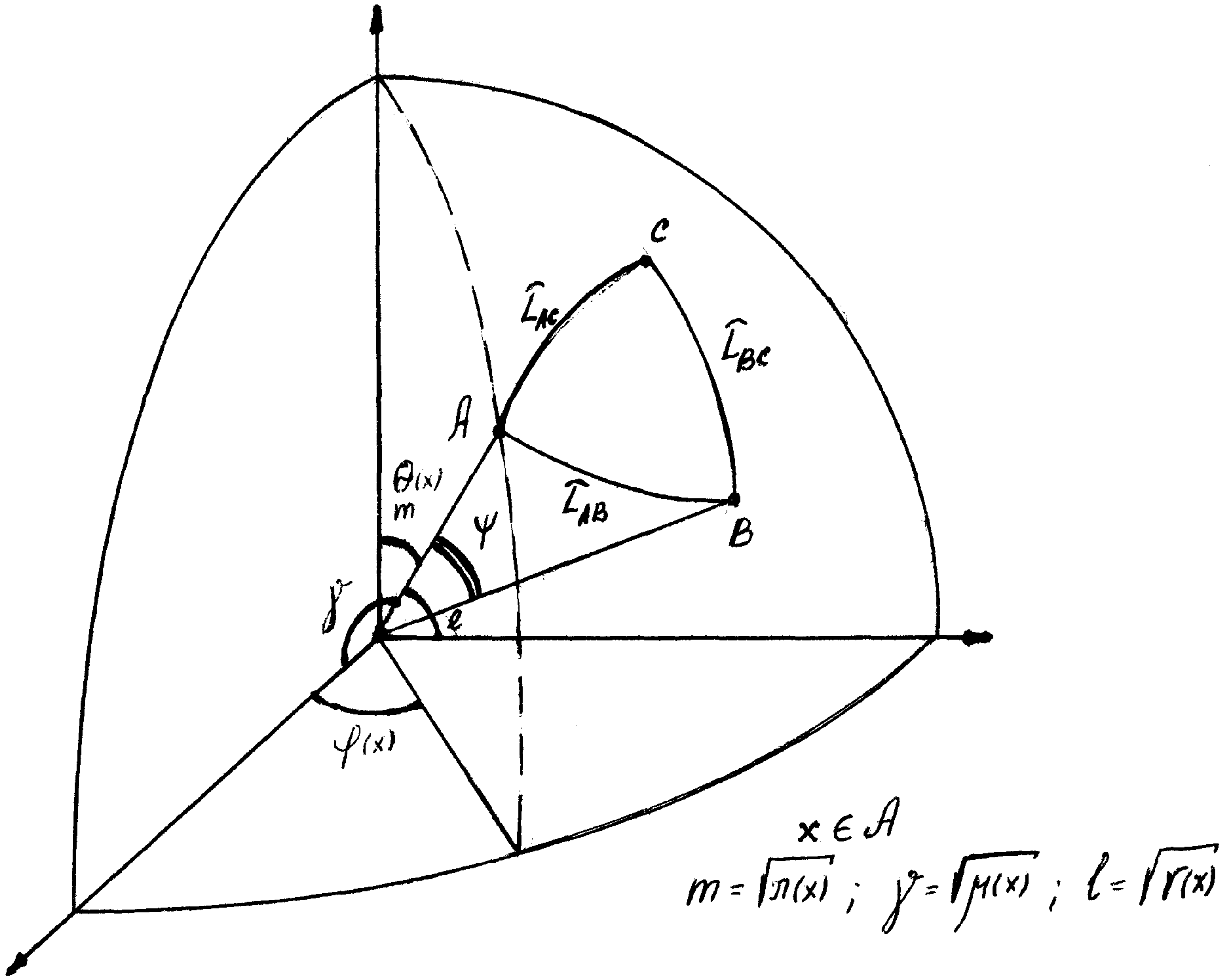
C
$\widehat{L}_{AC}$
$\widehat{L}_{BC}$
A
$\theta(x)$
m
ψ
$\widehat{L}_{AB}$
B
γ
ℓ
$\varphi(x)$
$x \in A$
$m = \sqrt{\pi(x)}$; $\gamma = \sqrt{\mu(x)}$; $\ell = \sqrt{\nu(x)}$

fig. 1

The following equality is valid:

$$L_{A,B} = \arccos\ (\overline{A \sqrt{B}}),$$

where the sign means:

$$\overline{A \sqrt{B}} = |\sqrt{\mu(A).\mu(B)} + \sqrt{\gamma(A).\gamma(B)} + \sqrt{\pi(A).\pi(B)}\ |.$$

These expressions and the geometrical interpretation (see Fig. 1) of the angle φ help determine the form of the metrix operator M as the composition $M = \arccos(\overline{\sqrt{\ }})$.

The properties of this operator for different forms of the function Θ and φ will be an object of a further research.

REFERENCES:

[1] Atanassov K., Intuitionistic fuzzy sets, Fuzzy sets and Systems, Vol. 20 (1986), No. 1, 87-96.

[2] Atanassov K., Geometrical interpretations of the elements of the intuitionistic fuzzy objects, Preprint IM-MFAIS-1-89, Sofia, 1989.

[3] Atanassov K., On the geometrical interpretations of the intuitionistic fuzzy logical objects. Part I., BUSEFAL, Vol. 60, 1994, 48-50.

[4] Atanassov K., On the geometrical interpretations of the intuitionistic fuzzy logical objects. Part II., BUSEFAL, Vol. 60, 1994, 51-54.

[5] Atanassov K., On the geometrical interpretations of the intuitionistic fuzzy logical objects. Part III., BUSEFAL, Vol. 60, 1994, 55-59.

[6] Atanassov K., Two variants of intuitonistic fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.

[7] Atanassov K., Two variants of intuitionistic fuzzy modal logic. Prepriny IM-MFAIS-3-89, Sofia, 1989.

[8] Atanassov K., Gargov G., Intuitionistic fuzzy logic, Compt. rend. Acad. bulg. Sci., Tome 43, N. 3, 1990, 9-12.

[9] Gargov G., Atanassov K., Two results in intuitionistic fuzzy logic, Compt. rend. Acad. bulg. Sci., Tome 45, N. 12, 1992, 29-31.

[10] Korn G., Korn T., Mathematical Handbook for Scientists and Engineers, McGraw-Hill Book Co., New York, 1961.

[11] Dubois D., Prade H.p Theorie des possibilites, Paris, Masson, 1988.

[12] Dubois D., Prade H., An introduction to possibilistic and fuzzy logics, Non-standard logics for automated reasoning, Academic Press, 1988.

[13] Atanassov K., Norms and metrics over intuitionistic fuzzy sets, BUSEFAL, Vol. 55, 1993, 11-20.

[14] Atanassov K., Norms and metrics over intuitionistic fuzzy logics, BUSEFAL, Vol. 59, 1994, 49-58.