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## On intuitionistic fuzzy histograms

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In this short note we shall introduce the concept of Intuitionistic Fuzzy Histogram (IFH) and three derivatives of its, by discussing two illustrative examples. All notations related to Intuitionistic Fuzzy Sets (IFSs) are used from [1].

## Example 1.

Let us take a sudoku puzzle that was being solved, no matter correctly or not, and let some of its cells be vacant. For instance, the sudoku puzzle on Fig. 1 contains a lot of mistakes and is not complete, but it serves us well as illustration.

| 6 | 1 |  | 8 | 3 |  | 5 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 7 | 9 | 2 | 1 |  | 6 | 3 |  |
| 3 | 1 | 5 |  | 9 |  |  | 2 | 7 |
| 1 | 9 |  | 6 | 3 |  |  | 8 | 6 |
| 7 | 5 |  | 3 | 7 | 9 | 2 | 1 | 5 |
| 2 | 6 | 3 | 1 |  | 5 | 4 |  | 9 |
| 9 | 8 | 7 | 4 | 2 | 8 | 3 | 9 | 1 |
| 5 | 3 |  | 7 | 9 | 3 | 1 | 5 |  |
| 1 | 2 | 4 | 3 | 5 | 6 |  | 7 |  |

Fig. 1

Let us separate the $9 \times 9$ grid into nine $3 \times 3$ sub-grids, and let us arrange vertically these sub-grids one over another (see Fig. 2) Let the rows and columns of each sub-grid be denoted by " $i$ " and " $j$ ", and let us each of the 9 cells in a sub-grid be denoted by the pair " $(i ; j)$ ", where $i, j=1,2,3$.

Let us design the following table from Fig. 3, in which over the " $(i ; j)$ " indices, that corresponds to a cell, nine fields are placed, coloured respectively in white if the digit in the cell is even; black - if the digit is odd; and half-white half-black if no digit is entered in the cell.


Fig. 2


Fig. 3

Now let us rearrange the table fields in a way that the black ones are shifted to the bottom positions, the black-and-white cells are placed in the middle and the white fields stay on top. Thus we obtain Fig. 4.

This new table has the appearance of a histogram and we can juxtapose to its columns the pair of real numbers $\left\langle\frac{p}{9}, \frac{q}{9}\right\rangle$ where $p$ and $q$ are respectively the numbers of the white and the black sudoku cells, while $9-p-q$ is the number of the empty cells. Let us call this object an intuitionistic fuzzy histogram. It gives us an idea of the kinds of numbers in the sudoku placeholders, and a clearer one that the one we would have if used a standard histogram. Back to the example from Fig. 4, the values of the individual columns will be, respectively: $\left\langle\left\langle\frac{3}{9}, \frac{3}{9}\right\rangle,\left\langle\frac{3}{9}, \frac{4}{9}\right\rangle,\left\langle\frac{4}{9}, \frac{3}{9}\right\rangle,\left\langle\frac{5}{9}, \frac{4}{9}\right\rangle,\langle 1,0\rangle,\left\langle\frac{4}{9}, \frac{2}{9}\right\rangle,\left\langle\frac{4}{9}, \frac{4}{9}\right\rangle,\left\langle\frac{5}{9}, \frac{4}{9}\right\rangle,\left\langle\frac{1}{9}, \frac{4}{9}\right\rangle\right.$


Fig. 4

In the case of IFH, several situations may possibly rise:

1. The black-and-white cells count as white cells. Then we obtain the histogram on Figure 5. The histogram from Fig. 5 will be called "N-histogram" by analogy with the modal operator "necessity" from the modal logic [2] and the theory of intuitionistic fuzzy sets [1]).
2. The black-and-white cells will be counted as half cells each, so that two mixed cells yield one black and one white cell. Then we obtain the histogram from Fig. 6. We well call this histogram "A-histogram", meaning that its values are average with respect to Fig. 4.
3. The black-and-white cells count for black cells. Then we obtain the histogram from Fig. 7. This histogram will be called "P-histogram" by analogy with the modal operator "possibility" from the modal logic [2] and the theory of IFS [1].


Fig. 6


Fig. 7

## Example 2.

Now let us consider the chess board, part of which is illustrated on Fig. 8. Each couple of squares on the board are divided by a stripe of non-zero width. The board squares are denoted in the standard way by " $a_{1}$ ", " $a_{2}$ ", ..., " $h_{8}$ " and they have side length of 2 .


Fig. 8

Let us place a coin of surface 1 and let us toss it $n$ times, each time falling on the chess board. After each tossing, we will assign the pairs $\langle a, b\rangle$ to the squares on which the coin has fallen, where $a$ denotes the surface of the coin that belongs to the respective chess square, while $b$ is the surface of the coin that lies on one or more neighbouring squares. Obviously, $a+b<1$.

In this example, it is possible to have two specific cases:

- If the tossed coin falls on one square only, then it will be assigned the pair of values $\langle 1,0\rangle$. This case is possible, because the radius of the coin is $\sqrt{\frac{1}{\pi}}$ while its diameter is $2 \sqrt{\frac{1}{\pi}}<2$.
- If the tossed coin falls on a square and its neighbouring zone, without crossing another board square, then it will be assigned the pair of values $\langle a, 0\rangle$; where $0<a<1$ is the surface of this part of the coin that lies on the respective square.

Obviously, the tossed coin may not fall on more than 4 squares at a time. Let us draw a table, having 64 columns, that will represent the number of the squares on the chess board, and $n$ rows, that will stand for the number of tossings. On every toss we may enter pairs of values in no more than 4 columns at a time. However, despite assigning pairs of numbers to each chess square, we may proceed by colouring the square in black (rectangle with width $a$ ), white (rectangle of width $b$ ) and leave the rest of the square white, as shown on Fig. 9:


Fig. 9
After the $n$ tossings of the coin, we may rearrange the squares by placing the black and/or grey squares to the bottom of the table. Thus we will obtain a histogram that is analogous to the one from the first example. We can also build a histogram of necessity, a histogram of possiblity and an average histogram.

The so constructed histograms can be easily further investigated so that their modes and medians be calculated. Let us note, for instance, that the mode of the histogram on Figure 5 is the column indexed by " $(2,2)$ ", whilst on Fig. 7 we will have two modes of the histograms represented by the columns indexed by " $(2,2)$ " and " $(2,3)$ ".

The term histogram is fundamental in the mathematical statistics and is widely applied in areas like biology, medicine, physics, engineering sciences and others. So far, the histograms have been used in their traditional form, in which it is assumed that a given event oc-
curs or does not occur. The present paper proposes the first attempt for generalization of the concept of histogram on the basis of the idea of intuitionistic fuzziness. This would allow for the more precise analysis of various processes and phenomena.

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