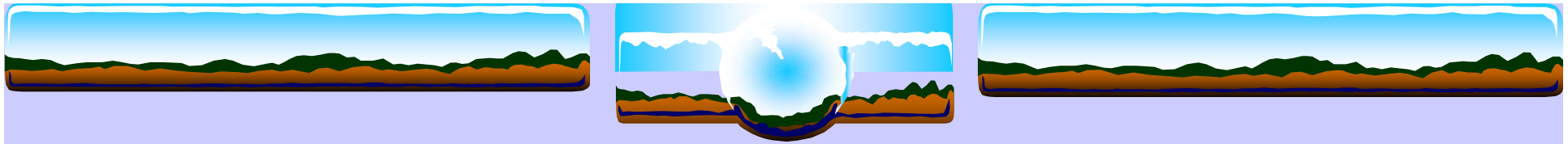

INTUITIONISTIC FUZZY SETS within Multicriteria Decision Making

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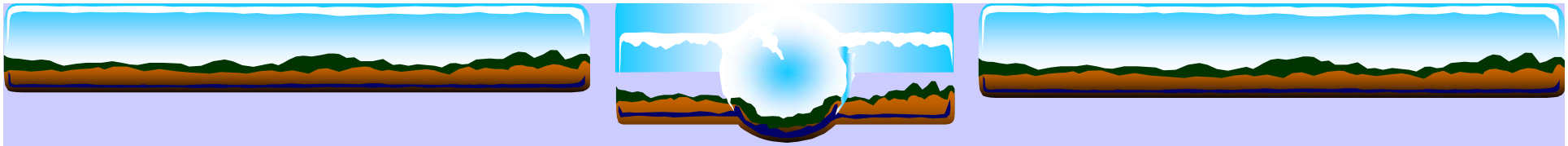
Complutense University of Madrid, Spain

Warsaw, October 17, 2008



Outline

- 1. *An alternative view to Atanassov's IFS*
- 2. *MCDM problems with linear criteria*
- 3. *Dual valuation approaches to MCDM*
- 4. *An A-IFS approach to MCDM*
- 5. *Some direct consequences*
- 7. *Key references*

An alternative view to Atanassov's IFS

Atanassov (1986): for each object

- Degree of membership $\mu_A(x)$
- Degree of non-memebship $\nu_A(x)$

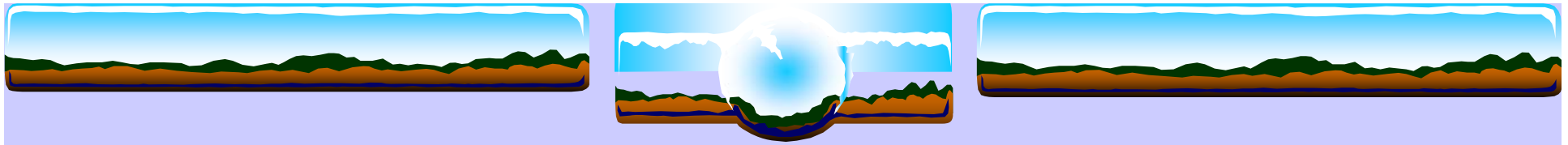
$$X^A : X \rightarrow [0,1]^2$$

$$x \rightarrow (\mu_A(x), \nu_A(x))$$

$$s.t. \quad \mu_A(x) + \nu_A(x) \leq 1$$

- Degree of "non-determinacy"

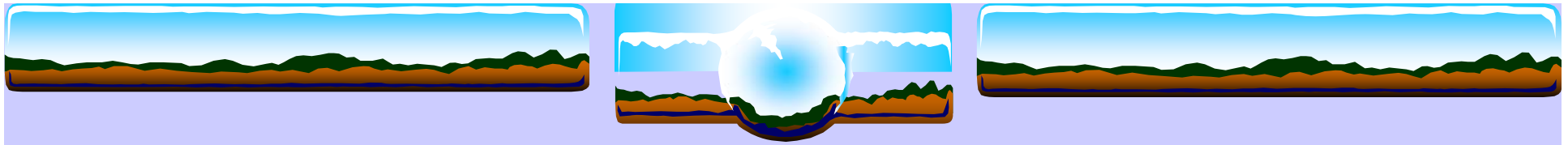
$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$



An alternative view to Atanassov's IFS

Serious problems and criticism:

- Meaning: what "non-determinancy" means?
- Estimation: how non-membership degree is assigned?
- Model fitness: how the sum up to 1 is assured in practice?
- Name: Atanassov´s model is not intuitionistic!
  (Takeuti-Titani, 1984)
- Isomorphism with "Interval Valued Fuzzy Sets"!

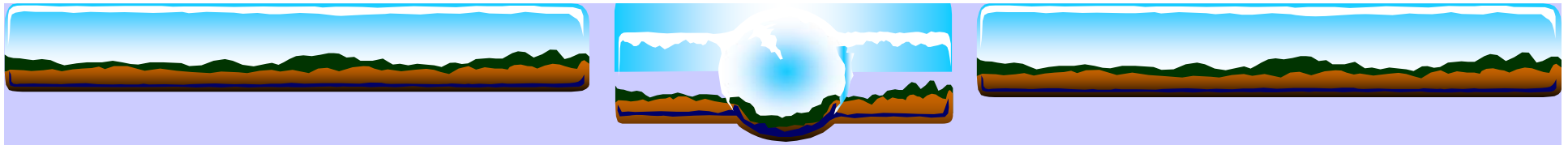


An alternative view to Atanassov's IFS

Montero-Gómez-Bustince (IWIFSGN'07, FSS 2007):

The binary heritage of Atanassov's "global" view

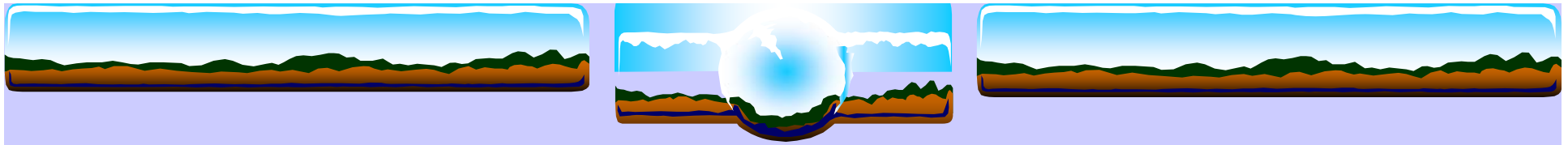
- Every concept comes with its negation?
  - "Tall" and "Non-tall" do not leave much room for anything else...
- Every concept comes with its opposite (dual, antonym)!
    - "Yes" and "No"
    - "Good" and "Bad"
    - "Black" and "White"?
  - In this context there is always room for something else...
    - Ignorance!
    - Contradiction!
    - Multidimensionality!



An alternative view to Atanassov's IFS

- Proposal:
  - "Concept", "Antonym", "Ignorance"
- Consequences
  - Classification approach to Atanassov's model
  - Clear and positive meaning for the three "classes"
  - No sum restriction needed (no Ruspini's fuzzy partition)

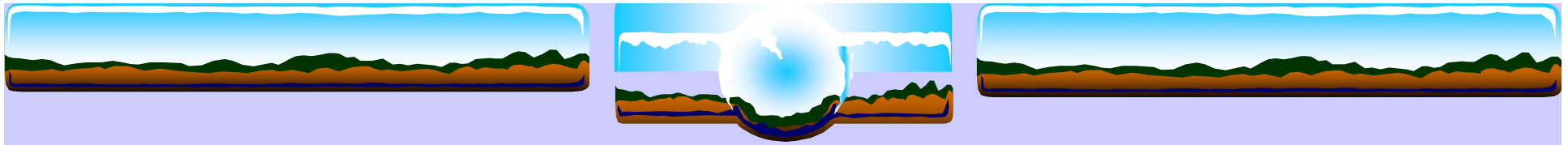
    (orthogonal representation may not exist neither desired)
  - A specific initial state ("ignorance") for learning
  - Direct generalization (into more than 2+1 classes)



An alternative view to Atanassov's IFS

MORE THAN 2+1 CLASSES

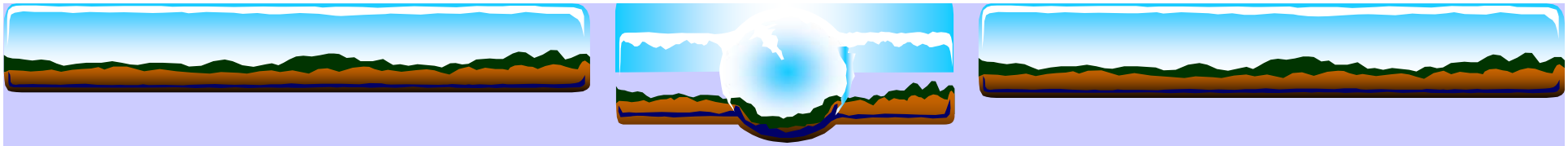
- Simple "linear" extension:
  - "Very good", "Good", "Average", "Poor" plus "Ignorance"
    - Initial stage: "Ignorance" captures all degree of membership
    - "Ignorance" is different in nature to the other 4 states!
- Much more complex structures (valuation spaces)
  - Multidimensional ("colourful" problems)
- Isomorphism with IVFS is avoided!
- Type II fuzzy set with structured states



An alternative view to Atanassov's IFS

MORE THAN 2+1 CLASSES

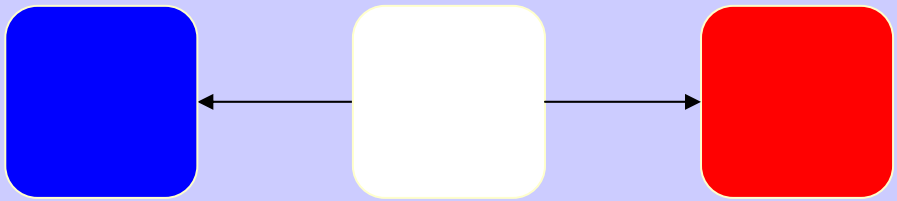
- The natural extension of IVFS (confidence intervals):
  - "Extremely high", "High", "Range", "Short", "Extremely short"
    - Initial stage: "Range" captures the whole interval [0,1]
    - Linear representation of these 5 states!
    - "Ignorance" in Shafer's Probability: P(A) in [0,1]
- Simple "linear" extension to Atanassov's model:



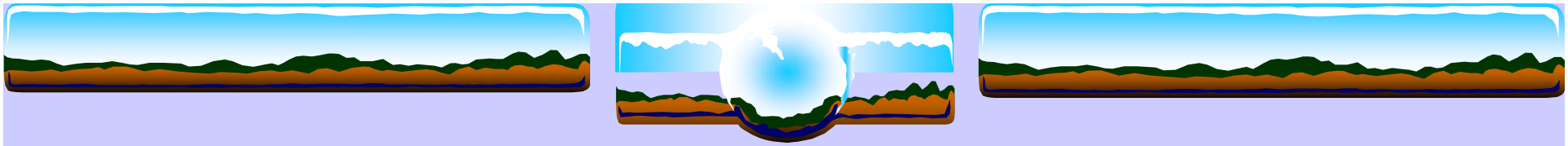
An alternative view to Atanassov's IFS

THE MISSING STRUCTURE

Interval Valued Fuzzy Sets



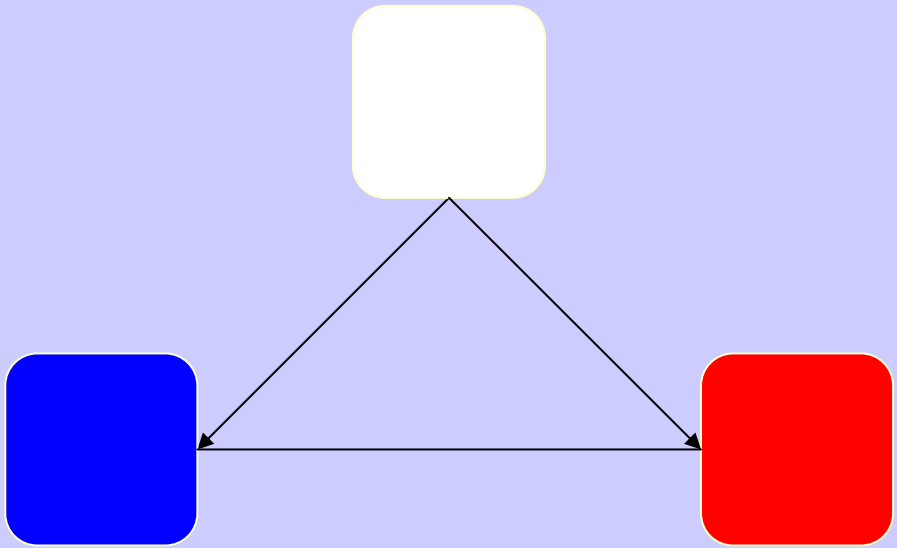
- Middle state is a "frontier" between the other two states

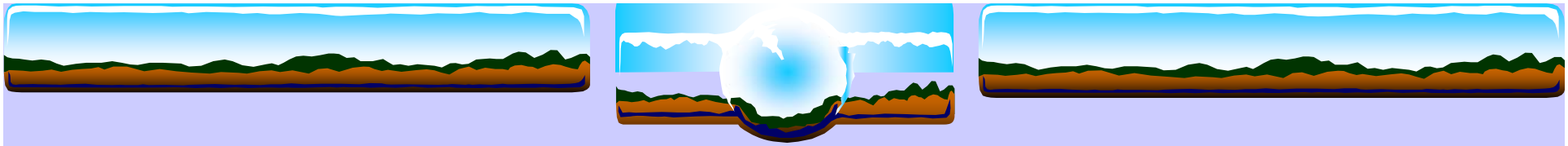


An alternative view to Atanassov's IFS

THE MISSING STRUCTURE

Atanassov's revised model



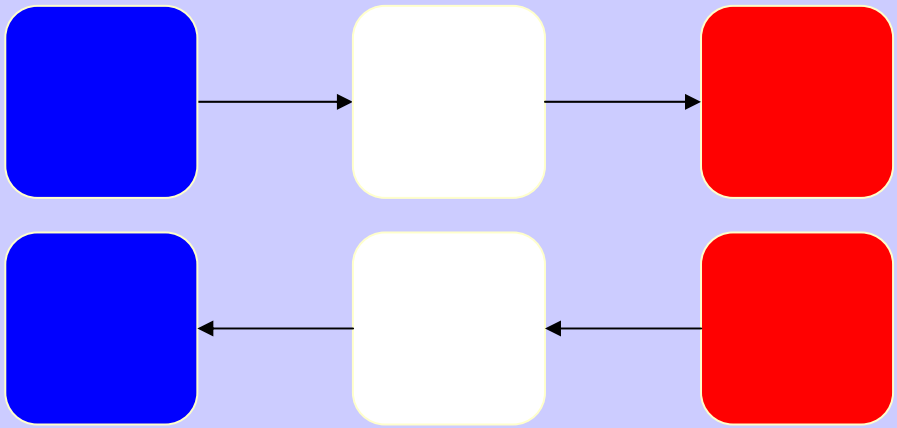


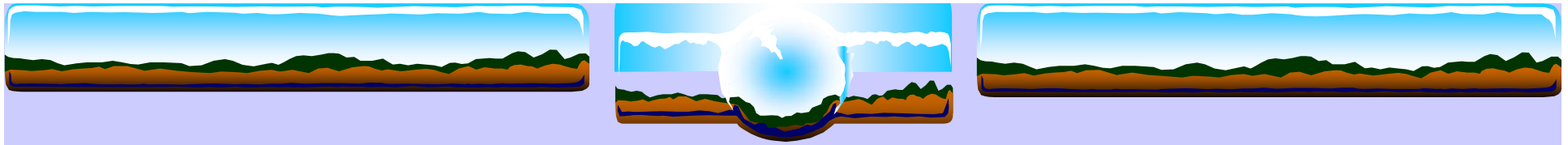
An alternative view to Atanassov's IFS

THE MISSING STRUCTURE

Luckasievich "true/possible/false" logic

(classical Statistics)



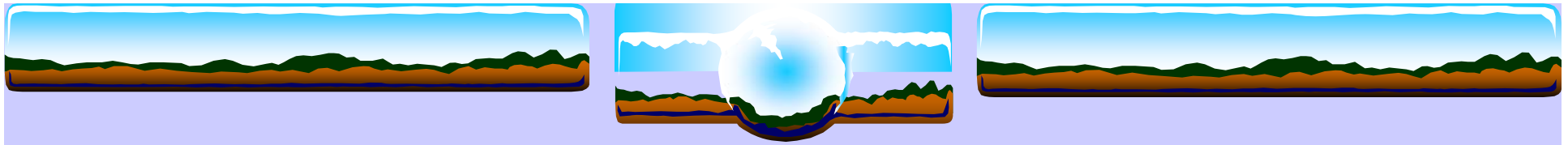


An alternative view to Atanassov's IFS

- Stress classification view of Atanassov's model
- Choose the right structure between states
- Stress the interest of type-2 fuzzy sets

$$\mu : X \rightarrow [0,1]^{|C|}$$

Where X is a family of objects and
C is a family of classes
with an associated (directed) graph
that explains learning procedures
from an "ignorance" state



MCDM problems with linear criteria

CLASSICAL CONSISTENCY ASSUMPTIONS

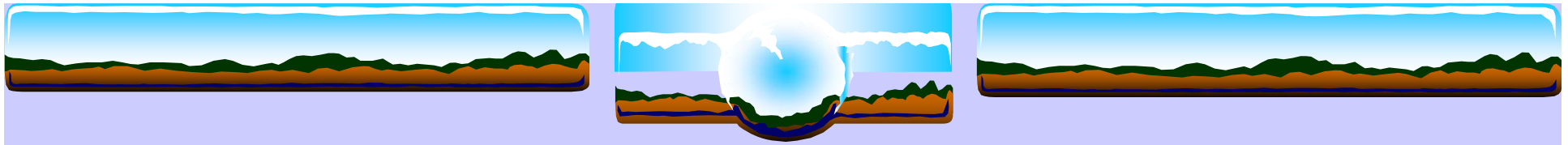
- Strong rationality: linear ordering of alternatives
  (a traditional must for “consistent” decision makers)

- Weak rationality: decomposition in terms of linear orderings

(each criteria is subject to the above strong rationality)

- Optimum search, equilibrium points, satisfying solutions, etc.

- More relevant as modeling tool than real-life decision tool?



Dual valuation approaches to MCDM

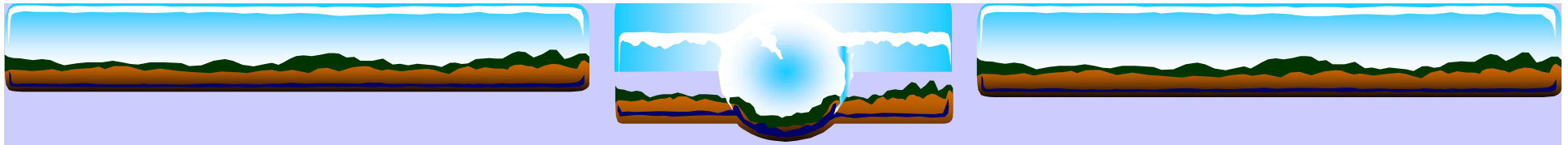
- Do not confuse with dual expression (equivalent): additional information?

- E.g., under a “choice” (IN/OUT) view

SUPPORT

AVERSION

Not to be deduced from a unique preference relation!



An A-IFS approach to MCDM

- Alternative and more sophisticated input structures

(taking into account relations)

- Alternative and more sophisticated aggregation and decomposition procedures

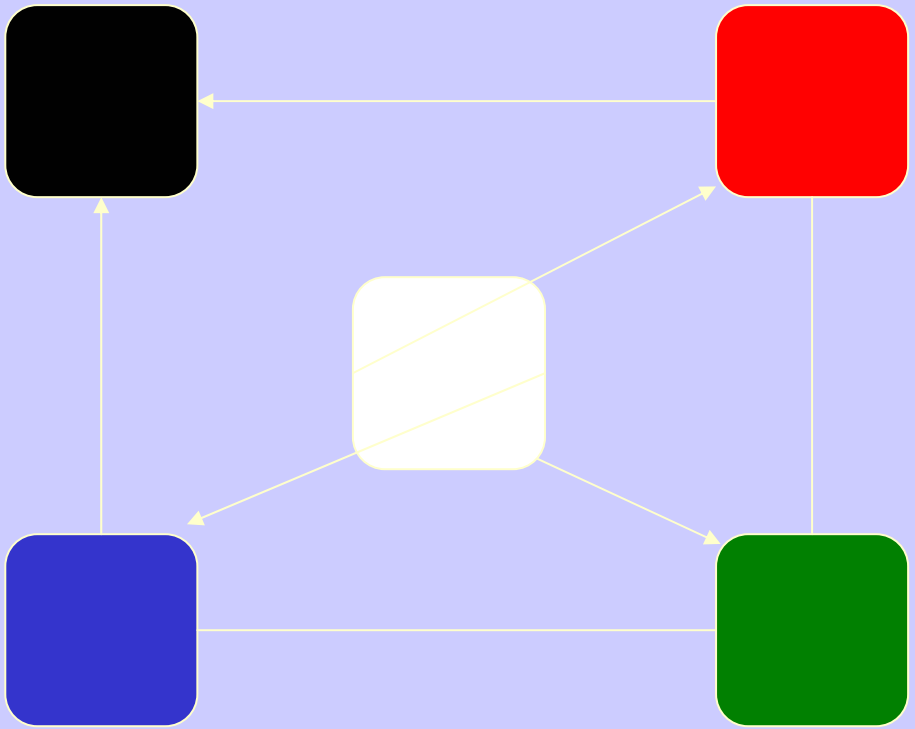
- Alternative and more sophisticated output valuations

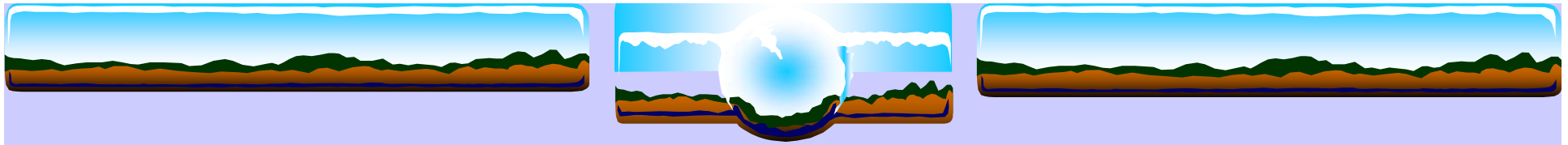
(rather comprehensive than decisional)

- New outranking models!

An A-IFS approach to MCDM

- GDM: voting system with 5 “preference” values





Some direct consequences

- Avoid the binary heritage associated to choice:

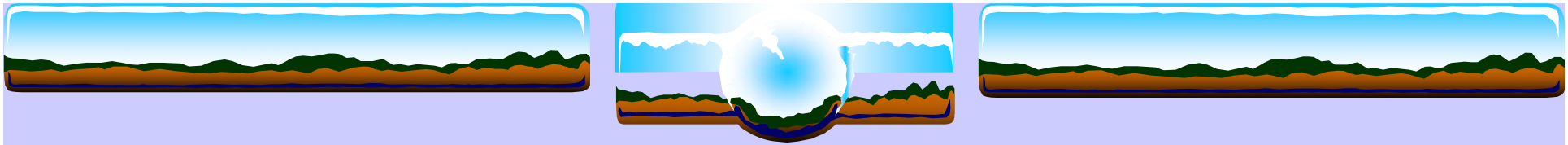
  “knowledge” should be the mathematical objective (description, including relations between objects)

- Redefine aggregation/amalgamation operations

  (several authors)

- Redefine objective of decomposition

  (algorithms for decomposition and final output)



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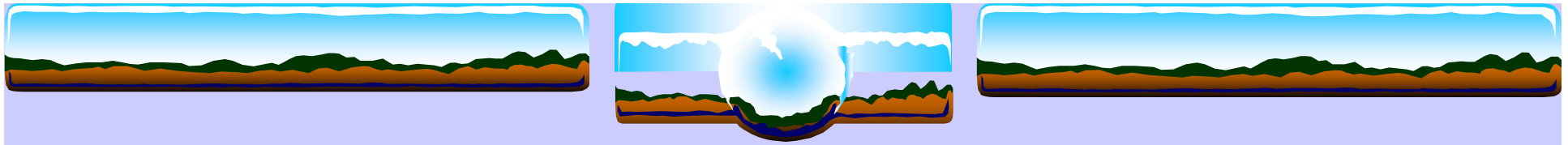

Thanks!