**18th Int. Conf. on IFSs, Sofia, 10–11 May 2014**
**Notes on Intuitionistic Fuzzy Sets**
**ISSN 1310–4926**
**Vol. 20, 2014, No. 2, 100–108**

# Index matrix representation of intuitionistic fuzzy graphs

**R. Parvathi[1], S. Thilagavathi[1], G. Thamizhendhi[1] and M. G. Karunambigai[2]**

[1] Vellalar College for Women,
Erode – 638 009, Tamilnadu, India
e-mail: paarvathis@rediffmail.com
[2] Department of Mathematics, Sri Vasavi College,
Erode, Tamilnadu, India

**Abstract:** In this paper, the operations like addition, vertexwise multiplication, multiplication, structural subtraction on intuitionistic fuzzy graphs using index matrices have been introduced and discussed with suitable illustrations.
**Keywords:** Index matrix representation, intuitionistic fuzzy graph, operations on intuitionistic fuzzy graphs.
**AMS Classification:** 03E72.

## 1 Introduction

The theory of graph plays a vital role for solving combinatorial problems in different areas such as operations research, topology, number theory, computer science. Rosenfeld [7] considered fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs. K.T Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs (IFGs). [4] gives a definition for IFG as a special case of IFGs in [6]. In [5], the operations on IFGs are defined and some of their properties are analyzed. In 1994 K. T Atanassov [2,3] introduced the index matrix representation of IFGs. In [1] he also defined cartesian products of two intuitionistic fuzzy sets (IFSs). In this paper, a revised definition of an IFG is given using IF relations. Further, operations like addition, vertex wise multiplication, multiplication, structural subtraction on IFGs using index matrix (IM) are dfined and studied. Almost all the operations result in different structures.

## 2 Preliminaries

**Definition 2.1** ([1])**.** *Let a set $E$ be fixed. An* Intuitionistic Fuzzy set *(IFS) $A$ in $E$ is an object of the form $A = \{(x, \mu_A(x), \nu_A(x))|x \in E\}$, where the function $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively and for every $x \in E$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.*

**Definition 2.2.** *Let $X$ be an universal set and let $V$ be an IFS over $X$ in the form $V = \{(v_i, \mu_i(v_i), \nu_i(v_i))/v_i \in V\}$ such that $0 \le \mu_i(v_i)+\nu_i(v_i) \le 1$. Six types of Cartesian products (in crisp sense) of $n$ subsets $V_1, V_2, \cdots, V_n$ of $V$ over $X$ are defined as*
$v_i \times_1 v_j = \{\langle\langle v_i, v_j\rangle, \mu_i.\mu_j,\ \nu_i.\nu_j\rangle \mid \langle v_i, v_j\rangle \in V \times V\}$,
$v_i \times_2 v_j = \{\langle\langle v_i, v_j\rangle, \mu_i + \mu_j - \mu_i.\mu_j, \nu_i.\nu_j\rangle \mid \langle v_i, v_j\rangle \in V \times V\}$,
$v_i \times_3 v_j = \{\langle\langle v_i, v_j\rangle, \mu_i.\mu_j, \nu_i + \nu_j - \nu_i.\nu_j\rangle \mid \langle v_i, v_j\rangle \in V \times V\}$,
$v_i \times_4 v_j = \{\langle\langle v_i, v_j\rangle, \min(\mu_i, \mu_j), \max(\nu_i, \nu_j)\rangle \mid \langle v_i, v_j\rangle \in V \times V\}$,
$v_i \times_5 v_j = \{\langle\langle v_i, v_j\rangle, \max(\mu_i, \mu_j), \min(\nu_i, \nu_j)\rangle \mid \langle v_i, v_j\rangle \in V \times V\}$.
$v_i \times_6 v_j = \left\{\left\langle\langle v_i, v_j\rangle, \frac{\mu_i+\mu_j}{2}, \frac{\nu_i+\nu_j}{2}\right\rangle \mid \langle v_i, v_j\rangle \in V \times V\right\}$.
*It must be noted that $v_i \times_t v_j$ is an IFS, where $t = 1, 2, 3, 4, 5, 6$.*

**Definition 2.3.** *An* intuitionistic fuzzy graph *(IFG) is of the form $G = \langle V, E\rangle$ where*
*(i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and*

$$0 \le \mu_i(v_i) + \nu_i(v_i) \le 1 \tag{1}$$

*for every $v_i \in V$, $i = 1, 2, \dots, n$*
*(ii) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $\nu_{ij} : V \times V \to [0,1]$ are such that*

$$\mu_{ij} \le \mu_i \oslash \mu_j; \quad \nu_{ij} \le \nu_i \oslash \nu_j \tag{2}$$

*where $\mu_{ij}$ and $\nu_{ij}$ are the membership and non-membership values of the edge $(v_i, v_j)$ such that $0 \le \mu_{ij} + \nu_{ij} \le 1$, $\oslash \in \{\oslash_1, \oslash_2, \oslash_3, \oslash_4, \oslash_5, \oslash_6\}$ and $\oslash_1, \oslash_2, \oslash_3$, $\oslash_4, \oslash_5, \oslash_6$ are defined as follows:*

$$\begin{array}{lll} (a) & \mu_i \oslash_1 \mu_j = \mu_i.\mu_j; & \nu_i \oslash_1 \nu_j = \nu_i.\nu_j \\ (b) & \mu_i \oslash_2 \mu_j = \mu_i + \mu_j - \mu_i.\mu_j; & \nu_i \oslash_2 \nu_j = \nu_i.\nu_j \\ (c) & \mu_i \oslash_3 \mu_j = \mu_i.\mu_j; & \nu_i \oslash_3 \nu_j = \nu_i + \nu_j - \nu_i.\nu_j \\ (d) & \mu_i \oslash_4 \mu_j = \min(\mu_i, \mu_j); & \nu_i \oslash_4 \nu_j = \max(\nu_i, \nu_j) \\ (e) & \mu_i \oslash_5 \mu_j = \max(\mu_i, \mu_j); & \nu_i \oslash_5 \nu_j = \min(\nu_i, \nu_j) \\ (f) & \mu_i \oslash_6 \mu_j = \frac{\mu_i+\mu_j}{2}; & \nu_i \oslash_6 \nu_j = \frac{\nu_i+\nu_j}{2} \end{array} \tag{3}$$

**Notations.**

1. Hereafter, $\langle\mu(v_i), \nu(v_i)\rangle$ or simply $\langle\mu_i, \nu_i\rangle$ denotes the degrees of membership and non-membership of the vertex $v_i \in V$, such that $0 \le \mu_i + \nu_i \le 1$.

2. $\langle\mu(v_i, v_j), \nu(v_i, v_j)\rangle$ or simply $\langle\mu_{ij}, \nu_{ij}\rangle$ denotes the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$ such that $0 \le \mu_{ij} + \nu_{ij} \le 1$.

**Note.** If $\mu_{ij} = \nu_{ij} = 0$ , for some $i$ and $j$, then there is no edge between $v_i$ and $v_j$, and it is indexed by $\langle 0, 1\rangle$. Otherwise, there exists edge between $v_i$ and $v_j$.

**Example 2.1.** Let $G = \langle V, E\rangle$ be an IFG, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. The edge membership and non-membership values can be determined by using $\oslash_1, \oslash_2, \oslash_3, \oslash_4, \oslash_5, \oslash_6$.

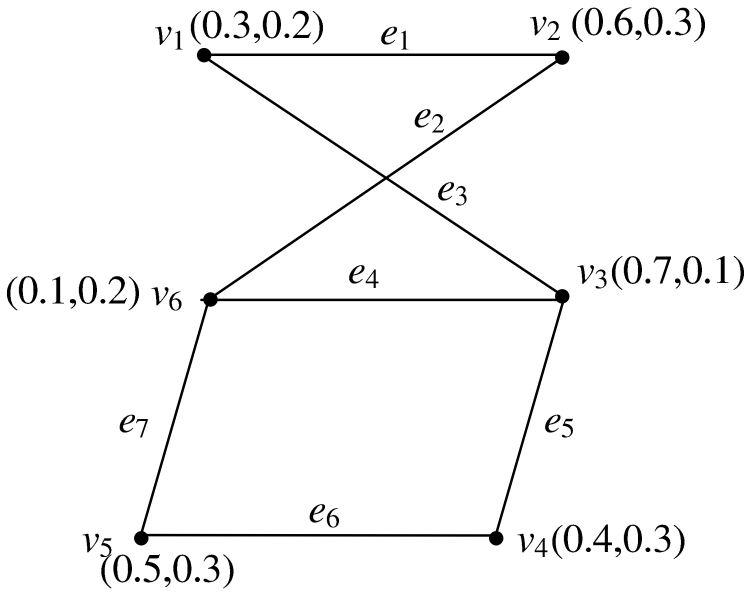


Figure 1: Intuitionistic fuzzy graph $G$

For example, R.H.S of inequalities (2) are calculated using (3) for the IFG $G$ given in Figure 1.

$$\begin{array}{ll}
(a) & \mu_1 \oslash_1 \mu_2 = 0.18; \quad \nu_1 \oslash_1 \nu_2 = 0.06 \\
(b) & \mu_1 \oslash_2 \mu_2 = 0.72; \quad \nu_1 \oslash_2 \nu_2 = 0.06 \\
(c) & \mu_1 \oslash_3 \mu_2 = 0.18; \quad \nu_1 \oslash_3 \nu_2 = 0.44 \\
(d) & \mu_1 \oslash_4 \mu_2 = 0.3; \quad \nu_1 \oslash_4 \nu_2 = 0.3 \\
(e) & \mu_1 \oslash_5 \mu_2 = 0.6; \quad \nu_1 \oslash_5 \nu_2 = 0.2 \\
(f) & \mu_1 \oslash_6 \mu_2 = 0.45; \quad \nu_1 \oslash_6 \nu_2 = 0.25
\end{array}$$

**Definition 2.4** ([2])**.** *Let $K = \{k_1, k_2, ..., k_m\}$ and $L = \{l_1, l_2, ..., l_n\}$ be two arbitrary index sets. The index matrix representation of intuitionistic fuzzy relation (IMIFR) is of the form*

$$[K, L, \{\langle \mu_{ij}, \nu_{ij}\rangle\}] \equiv \begin{array}{c|cccc}
 & l_1 & l_2 & \cdots & l_n \\ \hline
k_1 & \langle \mu_{11}, \nu_{11}\rangle & \langle \mu_{12}, \nu_{12}\rangle & \cdots & \langle \mu_{1n}, \nu_{1n}\rangle \\
k_2 & \langle \mu_{21}, \nu_{21}\rangle & \langle \mu_{22}, \nu_{22}\rangle & \cdots & \langle \mu_{2n}, \nu_{2n}\rangle \\
\vdots & \vdots & \vdots & \cdots & \vdots \\
k_m & \langle \mu_{m1}, \nu_{m1}\rangle & \langle \mu_{m2}, \nu_{m2}\rangle & \cdots & \langle \mu_{mn}, \nu_{mn}\rangle
\end{array}$$

*where for every $1 \le i \le m,\ 1 \le j \le n; 0 \le \mu_{ij} + \nu_{ij} \le 1$.*

**Definition 2.5.** *Let $G = (V, E)$ be an IFG . The IMIFG is of the form $[V, E \subset V \times V]$, where $V = \{v_1, v_2, \ldots v_n\}$ and*

$$E=\{\langle\mu_{ij},\nu_{ij}\rangle\}\equiv\begin{array}{c|cccc} & v_1 & v_2 & \cdots & v_n \\ \hline v_1 & \langle\mu_{11},\nu_{11}\rangle & \langle\mu_{12},\nu_{12}\rangle & \cdots & \langle\mu_{1n},\nu_{1n}\rangle \\ v_2 & \langle\mu_{21},\nu_{21}\rangle & \langle\mu_{22},\nu_{22}\rangle & \cdots & \langle\mu_{2n},\nu_{2n}\rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ v_n & \langle\mu_{n1},\nu_{n1}\rangle & \langle\mu_{n2},\nu_{n2}\rangle & \cdots & \langle\mu_{nn},\nu_{nn}\rangle \end{array}$$

*where* $\langle\mu_{ij},\nu_{ij}\rangle\in[0,1]\times[0,1]$ $(1\le i,j\le n)$*, the edge between two vertices* $v_i$ *and* $v_j$ *is indexed by* $\langle\mu_{ij},\nu_{ij}\rangle$*. The values of* $\langle\mu_{ij},\nu_{ij}\rangle$ *of an IFG* $G=(V,E)$ *can be determined by using one of the Cartesian products* $\times_t$*,* $t=1,2,3,4,5,6$ *from Definition 2.2.*

# 3 Operations on IFGs

Consider the two IFGs $G_1=[V_1,E_1,\mu_{ij},\nu_{ij}]$ and $G_2=[V_2,E_2,\mu_{pq},\nu_{pq}]$, where $V_1$ and $V_2$ are the vertex sets of $G_1$ and $G_2$ and $\mu_{ij},\nu_{ij}$ and $\mu_{pq},\nu_{pq}$ are the edge sets of $G_1$ and $G_2$ respectively.

**Definition 3.1.** *The* addition *of two IFGs* $G_1$ *and* $G_2$*, denoted by* $G=G_1\oplus G_2$*, is defined by* $G_1\oplus G_2=[V_1\cup V_2,V_1\cup V_2,\{\langle\mu_r,\nu_r\rangle\}],[V_1\cup V_2,V_1\cup V_2,\{\langle\mu_{rs},\nu_{rs}\rangle\}]$*, where*

$$\{\langle\mu_r,\nu_r\rangle\}=\begin{cases}\langle\mu_i,\nu_i\rangle & \textit{if } v_r\in V_1-V_2\\ \langle\mu_p,\nu_p\rangle & \textit{if } v_r\in V_2-V_1\\ \langle\max(\mu_i,\mu_p),\min(\nu_i,\nu_p)\rangle & \textit{if } v_r\in V_1\cap V_2\\ \langle 0,1\rangle & \textit{otherwise}.\end{cases}$$

*and*

$$\{\langle\mu_{rs},\nu_{rs}\rangle\}=\begin{cases}\langle\mu_{ij},\nu_{ij}\rangle & \textit{if } v_r=x_i\in V_1 \textit{ and } v_s=v_j\in V_1-V_2\\ & \textit{or } v_r=v_i\in V_1-V_2 \textit{ and } v_s=v_j\in V_1\\ \langle\mu_{pq},\nu_{pq}\rangle & \textit{if } v_r=v_p\in V_2 \textit{ and } v_s=v_q\in V_2-V_1\\ & \textit{or } v_r=v_p\in V_2-V_1 \textit{ and } v_s=v_q\in V_2\\ \langle\max(\mu_{ij},\mu_{pq}),\min(\nu_{ij},\nu_{pq})\rangle & \textit{if } v_r=v_i=v_p\in V_1\cap V_2\\ & \textit{and } v_s=v_j=v_q\in V_1\cap V_2\\ \langle 0,1\rangle & \textit{otherwise}.\end{cases}$$

**Example 3.1.** Consider the graphs $G_1$ and $G_2$ as in Figure 2.

The index matrix of $G_1$ is $G_1=[V_1,V_1,\{\langle\mu_{ij},\nu_{ij}\}\rangle]$, where $V_1=\{v_1,v_2,v_3,v_4,v_5\}$ and

$$\{\langle\mu_{ij},\nu_{ij}\rangle\}\equiv\begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & \langle 0,1\rangle & \langle 0.1,0.5\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.1,0.3\rangle \\ v_2 & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.4,0.5\rangle & \langle 0,1\rangle & \langle 0.1,0.5\rangle \\ v_3 & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.6,0.3\rangle & \langle 0,1\rangle \\ v_4 & \langle 0.1,0.3\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ v_5 & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.1,0.3\rangle & \langle 0.1,0.3\rangle & \langle 0,1\rangle \end{array}$$

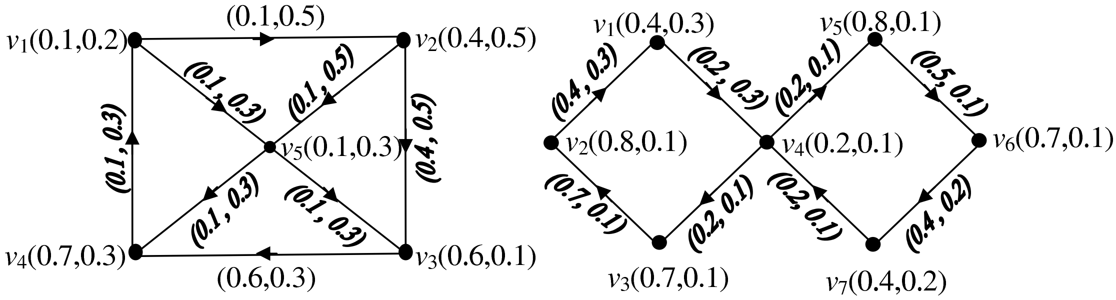


Figure 2: $G_1$ and $G_2$

The index matrix of $G_2$ is $G_2 = [V_2, V_2, \{\langle \mu_{pq}, \nu_{pq} \rangle\}]$, where $V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$$\{\langle \mu_{pq}, \nu_{pq} \rangle\} \equiv$$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_2$ | $\langle 0.4,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_3$ | $\langle 0,1 \rangle$ | $\langle 0.7,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_4$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_5$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.5,0.1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_6$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.4,0.2 \rangle$ |
| $v_7$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |

The index matrix of $G_1 \oplus G_2$ is $[V_1 \cup V_2, V_1 \cup V_2, \{\langle \mu_{rs}, \nu_{rs} \rangle\}]$, where $V_1 \cup V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$$\{\langle \mu_{rs}, \nu_{rs} \rangle\} \equiv$$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | $\langle 0,1 \rangle$ | $\langle 0.1,0.5 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.3 \rangle$ | $\langle 0.1,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_2$ | $\langle 0.4,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.4,0.5 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.1,0.5 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_3$ | $\langle 0,1 \rangle$ | $\langle 0.7,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.6,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_4$ | $\langle 0.1,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_5$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.1,0.3 \rangle$ | $\langle 0.1,0.3 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.5,0.1 \rangle$ | $\langle 0,1 \rangle$ |
| $v_6$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.4,0.2 \rangle$ |
| $v_7$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0.2,0.1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ | $\langle 0,1 \rangle$ |

The graph of $G_1 \oplus G_2$ is shown in Figure 3.

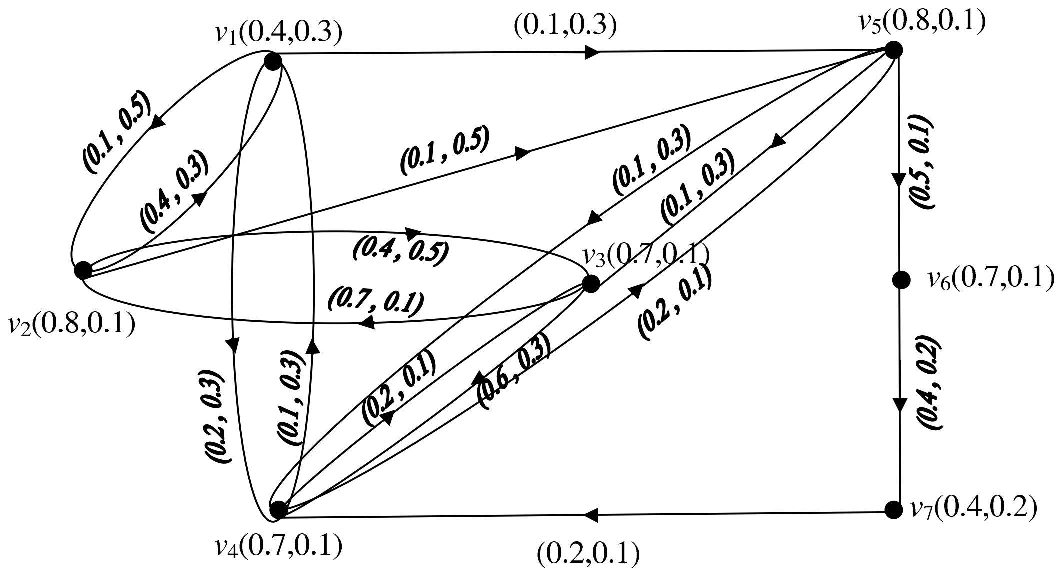


Figure 3: $G_1 \oplus G_2$

## 3.1 Vertexwise multiplication

The *vertexwise multiplication* of two IFDGs $G_1$ and $G_2$, denoted by $G_1 \otimes G_2$, is defined by

$$G_1 \otimes G_2 = \langle [V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_r, \nu_r \rangle\}], [V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_{rs}, \nu_{rs} \rangle\}] \rangle,$$

where

$$\{\langle \mu_r, \nu_r \rangle\} = \langle \text{min}(\mu_i, \mu_p), \text{max}(\nu_i, \nu_p) \rangle \text{ if } v_r \in V_1 \cap V_2;$$

$$\{\langle \mu_{rs}, \nu_{rs} \rangle\} = \langle \text{min}(\mu_{ij}, \mu_{pq}), \text{max}(\nu_{ij}, \nu_{pq}) \rangle ,$$

if $v_r = v_i = v_p \in V_1 \cap V_2$ and $v_s = v_j = v_q \in V_1 \cap V_2$.

**Example 3.2.** The index matrix of $G_1 \otimes G_2$ is $[V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_{rs}, \nu_{rs} \rangle\}]$, where $V_1 \cap V_2 = \{v_1, v_2, v_3, v_4, v_5\}$ and

$\{\langle \mu_{rs}, \nu_{rs} \rangle\} \equiv$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| $v_1$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_2$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_3$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_4$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_5$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |

The graph of $G_1 \otimes G_2$, a null IFG, is displayed in Figure 4.

**Example 3.3.** Consider the two IFGs $G_1$ and $G_2$ as shown in Figure 5.

Figure 6 depicts $G_1 \otimes G_2$, which is not a null IFG.

**Note.** From Example 3.2. and 3.3., it is noteworthy that if there is no common edge between $G_1$ and $G_2$, then $G_1 \otimes G_2$ is a null IFG.

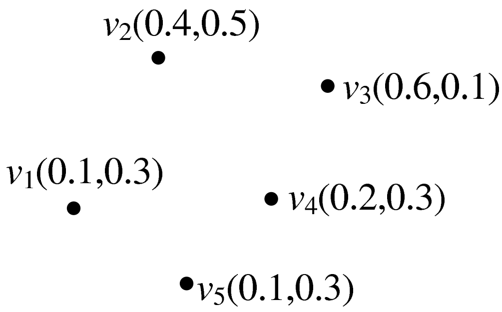


Figure 4: $G_1 \otimes G_2$

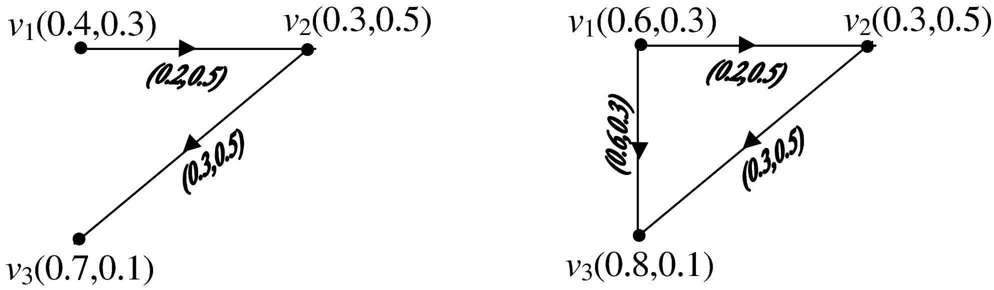


Figure 5: $G_1 \otimes G_2$

## 3.2 Multiplication

The *multiplication* of two IFDGs $G_1$ and $G_2$, denoted by $G_1 \odot G_2$, is defined by $G_1 \odot G_2 = [V_1 \cup (V_2 - V_1), V_2 \cup (V_1 - V_2), \{\langle \mu_r, \nu_r \rangle\}, \{\langle \mu_{rs}, \nu_{rs} \rangle\}]$ where

$$\{\langle \mu_r, \nu_r \rangle\} = \begin{cases} \langle \mu_i, \nu_i \rangle & \text{if } v_r \in V_1 \\ \langle \mu_p, \nu_p \rangle & \text{if } v_r \in V_2 \\ \langle \min(\mu_i, \mu_p), \max(\nu_i, \nu_p) \rangle & \text{if } v_r \in V_1 \cap V_2 \end{cases}$$

In addition, the membership and non-membership values of the loops $\langle v_r, v_r \rangle$ in the resultant graph (if formed) satisfy the following conditions: $\mu_r \leq \mu_i$ or $\mu_r \leq \mu_p$ and $\nu_r \geq \mu_i$ or $\nu_r \geq \mu_p$ . Also,

$$\{\langle \mu_{rs}, \nu_{rs} \rangle\} = \begin{cases} \langle \mu_{ij}, \nu_{ij} \rangle & \text{if } v_r = v_i \in V_1 \text{ and } v_s = v_j \in V_1 - V_2 \\ \langle \mu_{pq}, \nu_{pq} \rangle & \text{if } v_r = v_p \in V_2 - V_1 \text{ and } v_s = v_q \in V_2 \\ \langle \max((\min(\mu_{ij}, \mu_{pq}))), & \\ \min(\max(\nu_{ij}, \nu_{pq})) \rangle & \text{if } v_r = v_i \in V_1 \cap V_2 \text{ and } v_s = v_q \in V_1 \cap V_2 \\ \langle 0, 1 \rangle & \text{otherwise.} \end{cases}$$

**Example 3.4.** Consider the IFDGs given in Figure 2.

The index matrix of $G_1 \odot G_2$ is $[V_1 \cup (V_2 - V_1), V_2 \cup (V_1 - V_2), \{\langle \mu_{rs}, \nu_{rs} \rangle\}]$, where $V_1 \cup (V_2 - V_1) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, $V_2 \cup (V_1 - V_2) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

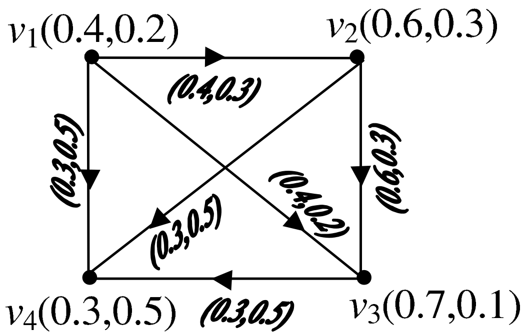


Figure 6: $G_1 \otimes G_2$

$\{\langle \mu_{rs}, \nu_{rs} \rangle\} \equiv$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | $\langle 0.1, 0.5 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_2$ | $\langle 0, 1 \rangle$ | $\langle 0.4, 0.5 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_3$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.2, 0.3 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.2, 0.3 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_4$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.1, 0.3 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_5$ | $\langle 0, 1 \rangle$ | $\langle 0.1, 0.3 \rangle$ | $\langle 0.1, 0.3 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.1, 0.3 \rangle$ | $\langle 0.5, 0.1 \rangle$ | $\langle 0, 1 \rangle$ |
| $v_6$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.4, 0.2 \rangle$ |
| $v_7$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0.2, 0.1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ | $\langle 0, 1 \rangle$ |

Figure 7 displays the graph of $G_1 \odot G_2$.

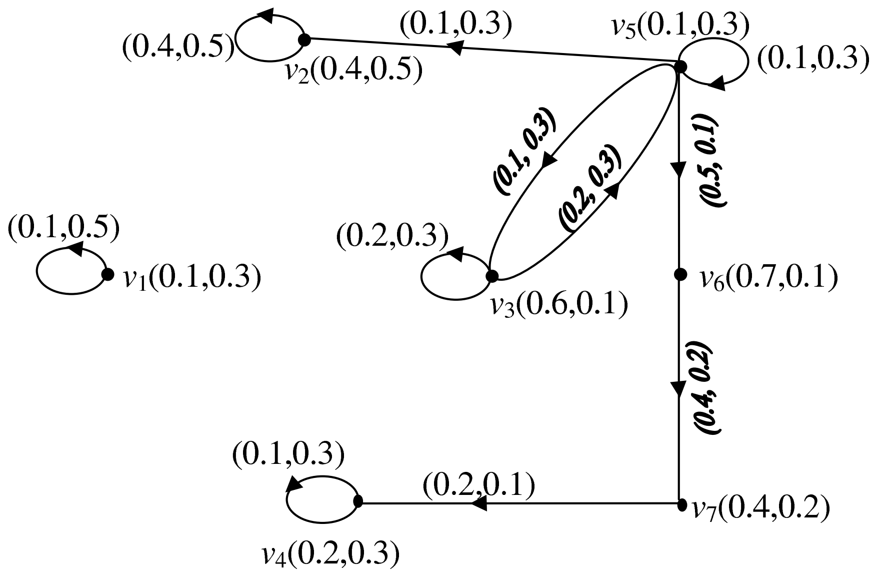


Figure 7: $G_1 \odot G_2$

## 3.3 Structural subtraction

The *structural subtraction* of two IFDGs $G_1$ and $G_2$, denoted by $G_1 \ominus G_2$, is defined as $G_1 \ominus G_2 = [V_1 - V_2, \{\langle \mu_r, \nu_r \rangle\}, \{\langle \mu_{rs}, \nu_{rs} \rangle\}]$ where $'-'$ is the set theoretic difference operation and

$$\{\langle \mu_r, \nu_r \rangle\} = \begin{cases} \langle \mu_i, \nu_i \rangle & \text{if } v_r \in V_1 \\ \langle \mu_p, \nu_p \rangle & \text{if } v_r \in V_2 \\ \langle 0, 1 \rangle & \text{otherwise.} \end{cases}$$

$\{\langle \mu_{rs}, \nu_{rs} \rangle\} = \{\langle \mu_{ij}, \nu_{ij} \rangle\}$ , for $v_r = v_i \in V_1 - V_2$ and $v_s = v_j \in V_1 - V_2$. If $V_1 - V_2 = \phi$ , then graph of $G_1 \ominus G_2$ is also empty.

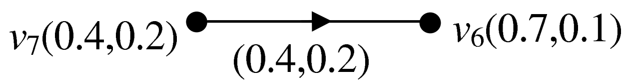


Figure 8: $G_1 \ominus G_2$

# 4 Conclusion

In this paper, a new version of IFG definition is given and operations like addition, vertexwise multiplication, multiplication, structural subtraction on index matrix representation of intuitionistic fuzzy graphs are introduced.

# References


[1] Atanassov, K., *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer Physica-Verlag, Heidelberg, 1999.

[2] Atanassov, K., Index matrix representation of the intuitionistic fuzzy graphs, *Preprint*, Sofia, 1994, 36–41.

[3] Atanassov, K., A. Shannon, Intuitionstic fuzzy graphs From $\alpha-, \beta-$ and $(\alpha, \beta)$-levels, *Notes on Intuitionistic Fuzzy Sets*, Vol. 1, 1995, No. 1, 32–35.

[4] Parvathi, R., M. G. Karunambigai, Intuitionistic fuzzy graphs, *Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence*, In: *Advances in soft computing: Computational Intelligence, Theory and Applications*, Springer-Verlag, Vol. 20, 2006, 139–150.

[5] Parvathi, R., M. G. Karunambigai, K. Atanassov, Operations on intuitionistic fuzzy graphs, *Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, August 2009, 1396–1401.

[6] Shannon, A., K. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs, *Proceedings of the 1st Workshop on Fuzzy Based Expert Systems (Lakov, D., Ed.)*, Sofia, 28–30 Sept. 1994, 59–61.

[7] Rosenfeld, A., Fuzzy graphs, *Fuzzy Sets and their Applications (Zadeh, L. A., K. S. Fu, M. Shimura, Eds.)*, Academic Press, New York, 1975, 77–95.