

# Generalized Net Model of a Part of an Overall Telecommunication System with a Queue in the Switching Stage

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**Abstract:** A new model of an overall telecommunication system is proposed. It is based on the model described in [7] and includes a queue in the Switching stage. Two conceptual models of the Switching stage are proposed – one using the apparatus of the Service Systems Theory and one using the apparatus of the Generalized nets theory.

**Keywords:** Generalized nets, Overall telecommunication system, Queuing in the switching stage.

**AMS Classification:** 68Q85.

## 1 Introduction and related work

In [7] a detailed conceptual traffic model of an overall virtual circuit switching telecommunication system including users' behaviour is proposed. On the basis of this conceptual model, an analytical macro-state model in stationary state with BPP (Bernoulli–Poisson–Pascal) input flow, repeated calls, limited number of homogeneous terminals, losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned communication is developed and studied (see [8]). The model is used to solve the important teletraffic problems of dimensioning and redimensioning of a telecommunication network with a preassigned value of the Quality of Service (QoS) parameters. The

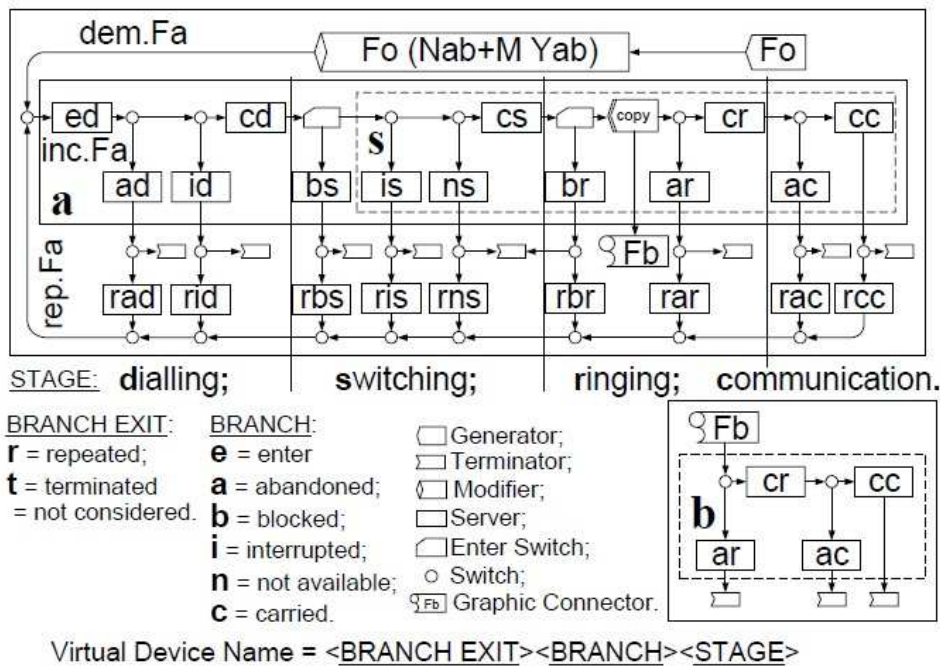


Figure 1. Classical conceptual model of an overall telecommunication system.

described approach is applicable directly to every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of the core and access networks traffic modelling. For packet switching systems, like Internet, the proposed approach may be used as a basis for comparison.

In the present paper, as opposed to [7, 8], a model with a queue in the Switching stage of the system is proposed. First, we describe the new model in terms of the Service Systems Theory and then a Generalized net model of a part of the Switching stage with a queue.

## 2 Classical conceptual model of an overall telecommunication system

The conceptual model described in [7, 8] considers user's behaviour, limited number of homogeneous terminals, losses due to abandoned and interrupted dialling, blocked and interrupted switching, unavailable intent terminal, blocked and abandoned ringing and abandoned communication. The network traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately but in their interrelation. Two types of virtual devices are used: base and comprising base devices. The base virtual devices, their names, types and graphic representation are shown in Fig. 1 (see [8]). There are six different types of base virtual devices: Generator, Terminator, Modifier, Server, Enter Switch, Switch and Graphic connector. Every base virtual device, with the exception of the Switch, has no more than one entrance and/or one exit. Switches have one or two entrances and one or two exits.

Every base virtual device has the following parameters:  $F$  - intensity (frequency rate) of the flow [calls/second];  $Y$  - intensity of the device traffic [Erlangs];  $P$  - probability of directing the

external flow to the device considered;  $T$  - service time of a call in the device [seconds];  $N$  - number of service places (lines, servers) in the virtual device (capacity of the device). For the purpose of more detailed description of the intensity of the flow, a special notation including qualifiers (see [5]) is used: *inc.F* stands for incoming flow; *dem.F* for demand flow; *ofr.F* for offered flow; *rep.F* for repeated flow (see [5]). The same notation is used to characterize the traffic intensity ( $Y$ ).

The following comprising virtual devices are considered in the model denoted by **a**, **b**, **s** (see Fig. 1) and **ab** (not shown in Fig. 1).

- **a** denotes the virtual device that comprises all A-terminals (calling) in the system. It is shown with a continuous line box in Fig. 1. The devices outside the **a**-device belong to the network environment. The calls in the environment do not occupy network devices. They contribute to the incoming flows to the network;
- **b** denotes the virtual device that comprises all B-terminals (called) in the system, the paths of the calls occupying B-terminal and corresponding base virtual devices. It is shown in a dashed line box in Fig. 1;
- **ab** denotes the virtual device that comprises all the terminals (calling and called) in the system. It is not shown in Fig. 1;
- **s** denotes the virtual device corresponding to the switching system. It is shown with a dashed line box into the **a**-device in Fig.1.

For better understanding of the conceptual model in Fig. 1, some remarks have to be made. The flow of calls (B-calls) occupying the B-terminals is coming from the Copy device. The justification for this modeling approach is, in the first place, the fact that at the beginning of the ringing a second (B) terminal in the system becomes busy. In the second place, the paths of the A and B-calls are different in the telecommunication system's environment after releasing the terminals. This becomes clear when one compares the environments of the **a** and **b** - devices in Fig. 1.

Another remark that must be made is that there are two virtual devices of the type Enter Switch – before the Blocked Switching (**bs**) and Blocked Ringing (**br**) devices. When there is no free line in the switching system or the intent B-terminal is busy, these devices deflect calls. The probabilities of the call attempts entering these devices depend on the macrostate of the system denoted by  $Y_{ab}$ . The macrostate of a (virtual) device or of the overall network, considered as a device, is defined as the mean number of simultaneously served calls in this device in the observed time interval. Its analogue is the “mean traffic intensity” in [5].

In the conceptual model, there are at least 35 important virtual devices – 31 base virtual devices and 4 comprising virtual devices (**a**, **b**, **ab**, **s**). Their importance is explained by the fact that the values of their parameters determine the characteristics and the state of the telecommunication system. Every virtual device has 5 parameters and therefore the total number of parameters of the system is 175. In [8] some assumptions for the telecommunication system are stated which allow for a system of 10 equations to be obtained for the 11 base dynamic parameters

$Fo, Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fs, Ts, crr.Ys, ofr.Ys$ . This system is used for solving different teletraffic problems such as dimensioning and redimensioning of a telecommunication network.

Our goal is to extend this conceptual model with the inclusion of a queue in the Switching system. This new conceptual model is a basis for derivation of a new analytical system of equations describing the telecommunication system. The new analytical model can be used for solving various teletraffic tasks such as system state task, technical characteristics task, human behaviour tasks etc.

### 3 Conceptual model of the Switching stage of an overall telecommunication system with a queue

We consider an overall telecommunication network as the one described in the previous section but now the call attempts are serviced by the Switching system with a queue.

The new conceptual model in terms of Service Systems Theory is shown in Fig. 2. In comparison to the conceptual model in Fig. 1, the branch **bs** is removed because the blocked call attempts from the Enter Switch remain in the queue and they are not redirected to other virtual devices. New virtual devices in the model are a device of type Queue denoted by **q**, the Enter Switch before it and all devices of the **bq** branch. The switching system is denoted by **s**. The Enter Switch device before the **q** device redirects the call attempts when the queue is full. The base device **q** has the same parameters as the other base devices:  $Fq, Yq, Tq, Pq, Nq$ . The capacity of the Queue device  $Nq$  is actually the length of the queue. The queue discipline considered in the model is FIFO. The Enter switch device between the Queue device and the **s**-device has one important parameter – the probability of blocked switching ( $Pbs$ ). The **s**-device has the same parameters as the other server devices.

In order to compactly describe single queueing stations in an unambiguous way, the so called Kendall notation is often used (see [4]). It consists of 6 identifiers, separated by vertical bars, as follows:

$$Arrivals | Services | Servers | Buffersize | Population | Scheduling$$

where “Arrivals” characterises the arrival process (arrival distribution), “Service” characterizes the service process (service distribution), “Servers” – the number of servers, “Buffersize” – the total capacity, which includes the customers possibly in the server (infinite if not specified), “Population” – the size of the customer population (infinite if not specified) and “Scheduling” – the employed service discipline.

The queuing process in Switching stage of the telecommunication network in Kendall notation is  $M|M|Nq|Nq + Ns|Nab|FIFO$ , where  $M$  stands for exponential distribution,  $Ns$  is the capacity of the Switching system (number of equivalent internal switching lines) and  $Nab$  is the total number of active terminals (calling and called). This is related to the derivation of the analytical model of the system, and falls out of the scope of the present paper.

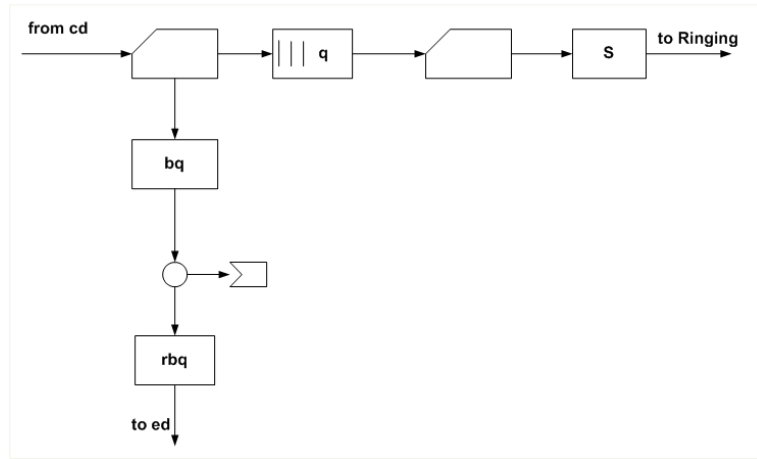


Figure 2. Conceptual model of a part of the Switching stage of an overall telecommunication system with a queue.

The important parameters of the devices in Fig. 2 can be divided into two groups. The first group consists of known parameters whose values can be obtained from the model [7, 8]. These parameters are:

- $T_s, N_s, Pbs, Y_s$ ;
- $ofr.Fq, Nq$ .

The second group consists of the unknown parameters related to the queuing process:

- $Fq, Pbq, Tq, Yq, Tsq$ .

The parameter  $Tsq$  represents the service time in the Switching system when it has reached its capacity, i.e., when  $Y_s = N_s$ . In order to describe the queuing process in detail, we consider the following cases separately depending on the value of  $Y_s$ .

Case 1. If the Switching system has reached its capacity, i.e.  $Y_s = N_s$ , and there are call attempts waiting to be serviced in the Queue device, i.e.  $Yq > 0$ , then  $Pbs > 0$ . In this case the intensity of the flow carried by the Queue device is equal to the intensity of the flow leaving the Switching system, i.e.  $crr.Fq = out.Fs$  where the qualifier “out” is abbreviation of outgoing. For the outgoing flow from the Switching system we have

$$out.Fs = \frac{Y_s}{T_s},$$

which is a restatement of the Little’s theorem for this special case. Since  $Y_s = N_s$  we also have

$$out.Fs = \frac{N_s}{T_s} = crr.Fq.$$

Case 2. If the Switching system has not reached its capacity but there are call attempts being serviced, i.e.  $0 < Y_s < N_s$ , then  $Yq = 0$  and  $Pbs = 0$ . In this case there are no call attempts waiting to be serviced in the Queue and all call attempts are passed through the device Enter switch between the **q** device and the **s** device (see Fig. 2). The equality  $Fq = Fs$  holds.

Case 3. Finally, if there are no call attempts being serviced by the Switching system, i.e.  $Y_s = 0$ , then  $Fq = Fs = 0$  and  $Yq = 0$ .

## 4 Generalized net model of the Switching stage with queue

The apparatus of Generalized Nets (GNs) has been used to represent the functioning and performance results of various types of systems in the sense of the Abstract Systems Theory of M. Mesarovich and Y. Takahara (see [2,3]). In [6] GNs are used as an alternative approach to the conceptual modeling of an overall telecommunication system. Two GN models are proposed – one using standard GNs and one using Generalized nets with characteristics of the places (see [1]). In the present paper we propose a GN model of the queuing process in an overall telecommunication system. It corresponds to the Service Systems Theory model from the previous section. The graphical representation of the net is shown in Fig. 3.

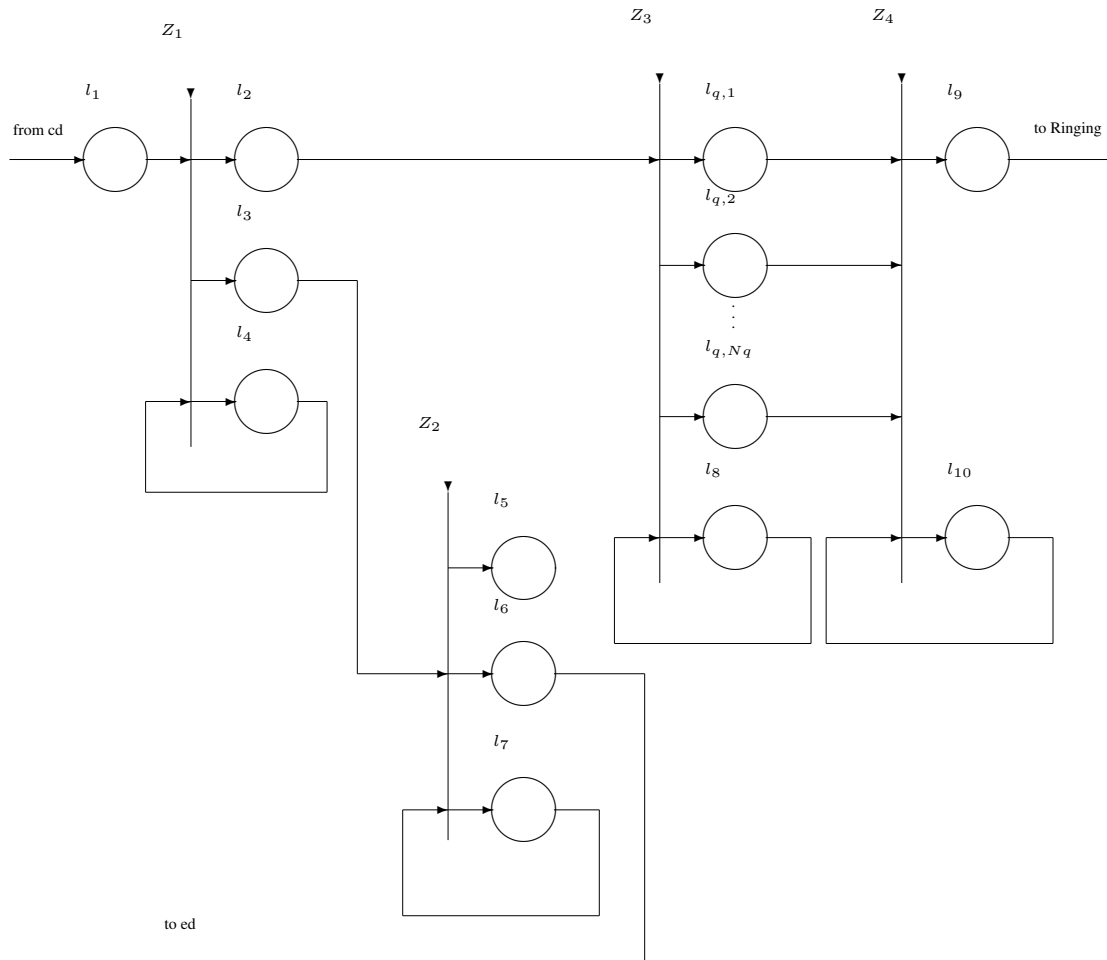


Figure 3. Generalized net model of the Switching stage of an overall telecommunication system with a queue.

The places of the GN correspond to base virtual devices as follows:

- $l_1$  represents the **cd** device from Fig. 1. It has no analogue in Fig. 2;
- $l_3$  and  $l_4$  represent the base virtual device **bq**;
- $l_6$  and  $l_7$  represent the base virtual device **rbq**;
- $l_5$  corresponds to the Terminator after **bq**;

- $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$  and  $l_8$  represent the base virtual device **q**;
- $l_9$  and  $l_{10}$  correspond to the **s** device (comprising device of the Switching system);

Each of the four transitions has the following meaning:

- $Z_1$  represents the function of the Enter Switch device before the **q** device;
- $Z_2$  represents the Switch device below the **bq** device;
- $Z_3$  represents the service of call attempts in the **q** device;
- $Z_4$  represents the service of call attempts in the Switching system (**s** device in Fig. 2).

Five different types of tokens are used in the model.

- Tokens of type  $\alpha$  represent the call attempts. They enter the net in place  $l_1$  with characteristic “*volume, duration of service, call destination*”.
- A token of type  $\beta$  stays in place  $l_4$  in the initial moment. It is used to accumulate data about the **bq** device.
- A token of type  $\gamma$  stays in place  $l_7$  in the initial moment. It is used to accumulate data about the **rbq** device.
- A token of type  $\delta$  stays in place  $l_8$  in the initial moment. It is used to accumulate data about the **q** device.
- A token of type  $\epsilon$  stays in place  $l_{10}$  in the initial moment. It is used to accumulate data about the **s** device.

All tokens except the  $\alpha$ -tokens have initial characteristic: “*initial values of  $Y_{dn}, P_{dn}, F_{dn}, T_{dn}$* ”, where  $dn$  should be replaced by the corresponding device name.

Detailed description of each of the transitions follows. The GN is a reduced one and every transition has the form  $Z = \langle L', L'', r \rangle$ .

$$Z_1 = \langle \{l_1, l_4\}, \{l_2, l_3, l_4\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|ccc} & l_2 & l_3 & l_4 \\ \hline l_1 & W_{1,2} & W_{1,3} & W_{1,4} \\ l_4 & false & false & true \end{array}$$

and

- $W_{1,2} = “Y_q < N_q”$ ;
- $W_{1,3} = \neg W_{1,2}$ ;
- $W_{1,4} = W_{1,3}$ .

When the truth value of the predicate  $W_{1,2}$  is “true”, the token  $\alpha$  enters place  $l_2$  without obtaining any new characteristic. When the truth value of the predicate  $W_{1,3}$  is “true” the token  $\alpha$  splits into two identical tokens one of which enters place  $l_3$  and the other one merges with the  $\beta$  token in place  $l_4$ . In place  $l_3$  the tokens do not obtain new characteristics. Token  $\beta$  in place  $l_4$  obtains the characteristic “*current value of  $Y_{bq}$* ”.

$$Z_2 = \langle \{l_3, l_7\}, \{l_5, l_6, l_7\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|ccc} & l_5 & l_6 & l_7 \\ \hline l_3 & W_{3,5} & W_{3,6} & W_{3,7} \\ l_7 & false & false & true \end{array}$$

and

- $W_{3,5}$  = “the current call is terminated (with a given probability  $1 - Prbq$ )”;
- $W_{3,6}$  = “the current call is repeated (with a given probability  $Prbq$ )”;
- $W_{3,7} = W_{3,6}$ .

When the truth value of the predicate  $W_{3,5}$  is “true” the current token  $\alpha$  enters place  $l_5$  without obtaining any new characteristic. When the truth value of the predicate  $W_{3,6}$  is “true” the current token  $\alpha$  splits into two identical tokens one of which enters place  $l_6$  and the other one enters place  $l_7$  where it merges with the  $\gamma$  token. In place  $l_6$  the tokens do not obtain any new characteristics. Token  $\gamma$  in place  $l_7$  obtains the characteristic “*current value of  $Y_{rbq}$* ”.

$$Z_3 = \langle \{l_2, l_8\}, \{l_{q,1}, l_{q,2}, \dots, l_{q,Nq}, l_8\}, r_3 \rangle,$$

where

$$r_3 = \begin{array}{c|ccccc} & l_{q,1} & l_{q,2} & \dots & l_{q,Nq} & l_8 \\ \hline l_2 & W_{2,q1} & W_{2,q2} & \dots & W_{2,qNq} & W_{2,8} \\ l_8 & false & false & \dots & false & true \end{array}$$

and

- $W_{2,qi}$  = “The output  $l_{q,i}$  is the highest priority empty place among places  $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$ ” for  $i = 1, 2, \dots, Nq$ ;
- $W_{2,8} = W_{2,q1} \vee W_{2,q2} \vee \dots \vee W_{2,qNq}$ .

In order to employ FIFO queuing discipline, each of the output places  $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$  should have capacity 1 and their priorities can be such that  $\pi_L(l_{q,1}) > \pi_L(l_{q,2}) > \dots > \pi_L(l_{q,Nq})$ . When at least one of the predicates  $W_{2,qi}$  for  $i = 1, 2, \dots, Nq$  is true, the  $\alpha$  token in place  $l_2$  splits into two identical tokens, the first of which enters the corresponding output place among places  $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$  without obtaining new characteristic. The other one enters place  $l_8$  where it merges with the  $\delta$  token. Token  $\delta$  obtains the characteristic “*current value of  $Y_q, T_q, F_q$ , list of all tokens in the output places  $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$  and the duration of their stay in the place*”.



$$Z_4 = \langle \{l_{q,1}, l_{q,2}, \dots, l_{q,Nq}, l_{10}\}, \{l_9, l_{10}\}, r_4 \rangle,$$

where

$$r_4 = \begin{array}{c|cc} & l_9 & l_{10} \\ \hline l_{q,1} & W_{q1,9} & W_{q1,10} \\ l_{q,2} & W_{q2,9} & W_{q2,10} \\ \vdots & \vdots & \vdots \\ l_{q,Nq} & W_{qNq,9} & W_{qNq,10} \\ l_{10} & false & true \end{array}$$

and

- $W_{qi,9}$  = “The current token in place  $l_{q,i}$  has stayed more time in the place than the tokens in all places  $l_{q,1}, l_{q,2}, \dots, l_{q,Nq}$  and  $Y_s < Ns$ ”, for  $i = 1, 2, \dots, Nq$ ;
- $W_{qi,10} = W_{qi,9}$  for  $i = 1, 2, \dots, Nq$ .

When at least one of the predicates  $W_{qi,9}$  for  $i = 1, 2, \dots, Nq$  is “true” the  $\alpha$  token in the corresponding input place splits into two identical tokens the first of which enters place  $l_9$  without obtaining new characteristic. The other one enters place  $l_{10}$  where it merges with the  $\epsilon$  token. Token  $\epsilon$  obtains the characteristic “*current value of  $Y_s, T_s, F_s$* ”.

## 5 Conclusions and future work

The Classical conceptual model of an overall telecommunication system with a Switching stage with queue as described in the present paper is a new conceptual model of an overall telecommunication system. This new model is a first step towards the construction of analytical model of the system which can be used to solve important teletraffic problems. The GN model of the Switching stage with queue can also be used as a base for derivation of analytical dependencies between the parameters of the system. Its graphical representation uses only three types of objects: arcs, places and transitions. The conceptual model in terms of Service Systems Theory uses more objects from different types (the virtual devices), each of which has a specific function. In the GN model, the function of each of the virtual devices is described in terms of the GN theory in the description of the transitions.

In our future work, we plan to construct GN models of the other stages: Dialling, Ringing, Communication, and a GN model of the telecommunication system as a whole. We consider using some GNs extensions such as Generalized nets with characteristics of the places, Generalized nets with volumetric tokens, etc. These models will be compared to models in which ordinary GNs are used.

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