GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper we introduce and study the concept of intuitionistic fuzzy g-continuous mappings in intuitionistic fuzzy topological spaces.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [16] in 1965 and fuzzy topology by chang [5] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 20 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [15], fuzzy separation axioms [4], fuzzy metric spaces [14], fuzzy continuity [9] fuzzy multifunctions [11] have been generalized for intuitionistic fuzzy topological spaces. In [13] the authores of this paper extend the concepts of fuzzy g-closed sets due to Thakur and Malviya [12] in intuitionistic fuzzy topological space. In the present paper we introduce and study the concepts of intuitionistic fuzzy g-continuous mappings in intuitionistic fuzzy topological space.

2. Preliminaries

Definition 2.1: [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$$

where the functions $\mu_A: X \to I$ and $\gamma_A: X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition 2.2: [1] Let X be a nonempty set and the intuitionistic fuzzy sets A and B be in the form $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X. Then

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(a) A \subseteq B if \forall x \in X [\mu_A(x) \le \mu_B(x) \text{ and } \gamma_A(x) \ge \gamma_B(x)];

(b) A = B if A \subseteq B and B \subseteq A;

(c) A^c = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X \};

(d) \cap A_i = \{ \langle x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) \rangle : x \in X \};

(e) \cup A_i = \{ \langle x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) \rangle : x \in X \};

(f) 0 = \{ \langle x, 0, 1 \rangle : x \in X \} and 1 = \{ \langle x, 1, 0 \rangle : x \in X \};
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Definition 2.3 [6]: Two intuitionistic fuzzy sets A and B of X said to be q-coincident (AqB for short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

Definition 2.4[6]: Let (X, \mathfrak{I}) be an intuitionistic fuzzy topological space and $A = \langle x, \mu_A(x), \gamma_A(x) \rangle$ be an intuitionistic fuzzy set in X. Then the fuzzy interior and fuzzy closure of A are defined by

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cl(A) = \bigcap \{K : K \text{ is an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K\},\ int(A) = \bigcup \{G : G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq K\}.
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Definition 2.5 [7]: Let X be a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0,1]$ and $\beta \in [0,1)$ are two real numbers such that $\alpha + \beta \le 1$ then $c(\alpha,\beta) = \langle x,c_{\alpha}, c_{1-\beta} \rangle$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of nonmembership of $c(\alpha,\beta)$.

Definition 2.6 [6]: An intuitionistic fuzzy topology on a nonempty set X is a family \mathfrak{I} of intuitionistic fuzzy sets in X satisfy the following axioms:

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(T_1) \underset{\sim}{0}, \underset{\sim}{1} \in \mathfrak{I},

(T_2) G_1 \cap G_2 \in \mathfrak{I} \text{ for any } G_1, G_2 \in \mathfrak{I},
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 $(T_3) \cup G_i \in \mathfrak{I}$ for any arbitrary family $\{G_i : i \in J\} \subseteq \mathfrak{I}$.

In this case the pair (X, \mathfrak{I}) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in \mathfrak{I} is known as an intuitionistic fuzzy open set in X.

Definition 2.7 [6]: The complement A^c of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (X, \mathfrak{I}) is called an intuitionistic fuzzy closed set in X.

Definition 2.8[6]: Let X and Y be two nonempty sets and $f: X \to Y$ be a function. Then

(a) If B = $\{\langle y, \mu_B(y), \gamma_B(y) \rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y, then the preimage of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X$

$$f^{-1}(B) = \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X$$

(b) If $A = \{\langle x, \lambda_A(x), \nu_A(x) \rangle : x \in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{ \langle y, f(\lambda_A)(y), f(\nu_A)(y) \rangle : y \in Y \}$$

where $f(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.9[9]: Let (X,\mathfrak{T}) and (Y,Φ) be two intuitionistic fuzzy topological spaces and let $f: X \rightarrow Y$ be a function. Then f is said to be intuitionistic fuzzy continuous if and only if the preimage of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy open set in X.

Definition 2.10 [8]: A family $\{G_i : i \in \land\}$ of intuitionistic fuzzy sets in X is said to be an intuitionistic fuzzy open cover of X if $\cup \{G_i : i \in \land\} = 1$ and a finite subfamily

of an intuitionistic fuzzy open cover of X which also an intuitionistic fuzzy open cover of X is called a finite subcover $\{G_i: i \in \land\}$.

Definition 2.11 [8] :An intuitionistic fuzzy topological space (X,3) is called fuzzy compact if each intuitionistic fuzzy open cover has a finite subcover.

Definition 2.12[13]: An intuitionistic fuzzy set A of a intuitionistic fuzzy topological space (X,3) is called an intuitionistic fuzzy generalized closed (intuitionistic fuzzy g-closed) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.

Definition 2.13[13]: Complement of an intutionistic fuzzy g-closed set is called intutionistic fuzzy g-open set.

Remark 2.1 [13]: Every intuitionistic fuzzy closed set (intutionistic fuzzy open set) is intuitionistic fuzzy g-closed(intutionistic fuzzy open set) but its converse may not be true.

Throughout this paper $f:(X,\mathfrak{F})\to (Y,\sigma)$ denotes a mapping from an intuitionistic fuzzy topological space (X, \mathfrak{I}) to another intuitionistic topological space (Y, σ) .

3. Intuitionistic Fuzzy g-Continuous Mapping

Definition 3.1: A mapping $f: (X, \mathfrak{I}) \to (Y, \sigma)$ is said to be intuitionistic fuzzv g-continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy g-closed in X.

Remark 3.1: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy gcontinuous, but the converse may not be true. For,

Example 3.1: Let $X = \{a, b\}, Y = (x, y)$ and the intuitionistic fuzzy set U and V are defined as follows:

 $U = \langle x, (0.5/a, 0.6/b), (0.4/a, 0.4/b) \rangle$

 $V = \langle x, (0.3/a, 0.4/b), (0.6/a, 0.6/b) \rangle$

Let $\mathfrak{I} = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be intuitionistic fuzzy topologies on X

and Y respectively. Then the mapping $f:(X,\mathfrak{F})\to (Y,\sigma)$ defined by f(a)=x and f(b)=y is intuitionistic fuzzy g-continuous but not intuitionistic fuzzy continuous.

Theorem 3.1: A mapping $f:(X, \mathfrak{I}) \to (Y, \sigma)$ is intuitionistic fuzzy g-continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy g-open in X.

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Theorem 3.2: If $f:(X, \mathfrak{I}) \to (Y, \sigma)$ is intuitionistic fuzzy g-continuous then for each intuitionistic fuzzy point $c(\alpha, \beta)$ of X and each fuzzy open set V, $f(c(\alpha, \beta)) \subseteq V$ there exist a intuitionistic fuzzy g-open set U such that $c(\alpha, \beta) \subseteq U$ and $f(U) \subseteq V$.

Proof: Let $c(\alpha, \beta)$ be a intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set such that $c(\alpha, \beta) \subseteq V$, put $U = f^{-1}(V)$ then by hypothesis U is intuitionistic fuzzy gopen set of X such that $c(\alpha, \beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 3.3: If $f:(X,\mathfrak{F})\to (Y,\sigma)$ is fuzzy g-continuous, then for each intuitionistic fuzzy point $c(\alpha,\beta)$ in X and each intuitionistic fuzzy open set V of Y such that $c(\alpha,\beta)_qV$, there exists $c(\alpha,\beta)$ in an intuitionistic g-open set U of X such that $c(\alpha,\beta)_qU$ and $f(U)\subseteq V$.

Proof: Let $c(\alpha, \beta)$ be an intuitionistic fuzzy point of X and V be an intuitionistic fuzzy open set of Y such that $f(c(\alpha, \beta))_qV$. Put $U = f^{-1}(V)$. Then by hypothesis U is an intuitionistic fuzzy g-open set of X such that $c(\alpha, \beta)_qU$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Definition 3..2: Let (X, \mathfrak{I}) be an intuitionistic fuzzy topological space. The generalized closure of a intuitionistic fuzzy set A of X denoted by gcl(A) is the intersection of all intuitionistic fuzzy g-closed sets of X which contains A.

Remark: 3. 2: It is clear that, $A \subseteq gcl(A) \subseteq cl(A)$ for any intuitionistic fuzzy set A of X.

Theorem 3. 4: If $f:(X, \mathfrak{I}) \to (Y, \sigma)$ is intuitionistic fuzzy g-continuous, then $f(gcl(A) \subseteq cl(f(A)))$ for every intuitionistic fuzzy set A of X.

Proof: Let A be an intuitionistic fuzzy set of X. Then cl(f(A)) is an intuitionistic fuzzy closed set of Y. Since f is fuzzy g-continuous $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy g-closed in X. Clearly $A \subseteq f^{-1}(cl(f(A)))$. Therefore $gcl(A) \subset gcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Hence $f(gcl(A)) \subset cl(f(A))$.

Remark 3. 3: The converse of Theorem 3. 4 may not be true. For,

Example 3.2: Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and the intuitionistic fuzzy set U and V are defined as:

$$U = \langle x, (1/a, 0/b, 0/c), (0/a, 1/b, 1/c) \rangle$$

$$V = \langle x, (1/x, 0/y, 1/z), (0/x, 1/y, 0/z) \rangle$$
 Let $\mathfrak{I} = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be intuitionistic fuzzy topologies on X

and Y respectively and $f:(X,\mathfrak{I})\to (Y,\sigma)$ be a mapping defined by f(a)=y, f(b)=x, f(c)=z. Then $f(gcl(A))\subseteq cl(f(A))$ holds for every intuitionistic fuzzy set A of X, but f is not fuzzy g-continuous.

Definition 3.3: An intuitionistic fuzzy topological space (X, \Im) is said to be intuitionistic fuzzy $T_{1/2}$ if every intuitionistic fuzzy g-closed set in X is intuitionistic fuzzy closed in X.

Theorem 3.5: A mapping f from an intuitionistic fuzzy $T_{1/2}$ -space (X, \mathfrak{I}) to an intuitionistic fuzzy topological space (Y, σ) is intuitionistic fuzzy continuous if and only if it is intuitionistic fuzzy g-continuous.

Proof: Obvious.

Remark 3.4: The composition of two intuitionistic fuzzy g-continuous mappings may not be intuitionistic fuzzy g-continuous. For,

Example 3.3: Let $X = \{a, b\}$, $Y = \{x, y\}$ and $Z = \{p, q\}$ and the fuzzy sets U, V, and W are defined as follows:

$$U = \langle x, (0.5/a, 0.7/b), (0.5/a, 0.3/b) \rangle$$

$$V = \langle x, (0.3/a, 0.2/b), (0.7/a, 0.8/b) \rangle$$

$$W = \langle x, (0.6/a, 0.4/b), (0.4/a, 0.6/b) \rangle$$
Let $\tau = \{0, U, 1\}, \sigma = \{0, V, 1\}, \text{ and } \eta = \{0, W, 1\} \text{ be intuitionistic fuzzy}$

topologies on X, Y and Z respectively. Let the mapping $f:(X, \mathfrak{I}) \to (Y, \sigma)$ be defined by f(a) = x, f(b) = y and the mapping $g:(Y, \sigma) \to (Z, \eta)$ be defined by g(x) = p and g(y) = q. Then f and g are intuitionistic fuzzy g-continuous but gof is not intuitionistic fuzzy g-continuous.

Theorem 3. 6: If $f:(X, \mathfrak{I}) \to (Y, \sigma)$ is intuitionistic fuzzy g-continuous and $g:(Y, \sigma) \to (Z, \eta)$ is intutionistic fuzzy continuous. Then gof: $(X, \mathfrak{I}) \to (Z, \eta)$ is intutionistic fuzzy g-continuous.

Proof: If A is intuitionistic fuzzy closed in Z, then f⁻¹(A) is intuitionistic fuzzy closed in Y

because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g-closed in X. Hence gof is intuitionistic fuzzy g-continuous.

Theorem 3.7: If $f:(X, \mathfrak{I}) \to (Y, \sigma)$ and $g:(Y, \sigma) \to (Z, \eta)$ are two intutionistic fuzzy g-continuous mappings and (Y, σ) is intuitionistic fuzzy $T_{1/2}$ space then $gof:(X, \mathfrak{I}) \to (Z, \eta)$ is intuitionistic fuzzy g-continuous.

Proof: Obvious.

Definition 3.4: An intuitionistic fuzzy topological space (X,\mathfrak{I}) is said to be intuitionistic fuzzy GO- compact if every intuitionistic fuzzy g-open cover of X has a finite subcover.

Theorem 3.8: Intuitionistic fuzzy g-continuous image of an intuitionistic fuzzy GO-compact space is intuitionistic fuzzy compact.

Proof: Let $f:(X, \mathfrak{I}) \to (Y, \sigma)$ be an intuitionistic fuzzy g-continuous map from an intuitionistic fuzzy GO-compact space (X, \mathfrak{I}) onto an intuitionistic fuzzy topological space (Y, σ) . Let $\{A_i: i \in \Lambda\}$ be an intuitionistic fuzzy g-open cover of Y then $\{f^{-1}(Ai): i \in \Lambda\}$ is an intuitionistic fuzzy g-open cover of X. Since X is intuitionistic fuzzy GO-compact it has finite intuitionistic fuzzy subcover say

 $\{f^{-1}(A_1),\ldots,f^{-1}(A_n)\}$. Since f is onto $\{A_1,\ldots,A_n\}$ is an intuitionistic fuzzy open cover of Y and so (Y,σ) is intuitionistic fuzzy compact.

Definition 3.5: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy GO-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed.

Remark 3.5: Every intuitionistic fuzzy GO-connected space is intuitionistic fuzzy C_5 -connected [15], but the converse may not be true. For,

Example 3. 4 Let $X = \{a,b\}$, $U = \langle x, (.5/a, .7/b), (.5/a, .3/b) \rangle$,

and $\mathfrak{I} = \{0, U, 1\}$ be an intuitionistic topology on X, then (X, \mathfrak{I}) is intutionistic fuzzy connected but not fuzzy GO-connected.

Theorem 3.9: An intuitionistic fuzzy $T_{1/2}$ -space is intuitionistic fuzzy C_5 - connected if and only if it is intuitionistic fuzzy GO-connected.

Proof: Obvious.

Theorem 3. 10: If $f:(X, \mathfrak{I}) \to (Y, \sigma)$ is an intuitionistic fuzzy g-continuous surjection and X is intuitionistic fuzzy GO-connected then Y is intuitionistic fuzzy C₅-connected.

Proof: Suppose Y is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic

fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy closed and intuitionistic fuzzy open set of X, because f intuitionistic fuzzy g-continuous surjection. Hence X is not intuitionistic fuzzy GO-connected.

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