Notes on Intuitionistic Fuzzy Sets
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Remark on Dworniczak's intuitionistic fuzzy implications. Part 1

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Abstract: Four new intuitinistic fuzzy implications are introduced. They are modal forms of Dworniczak's intuitionistic fuzzy implication $\rightarrow_{150,\lambda}$. Their basic properties are studied.
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1 Introduction

In [7, 8, 9], Piotr Dworniczak introduce three intuitionistic fuzzy implications, that generalized the defined by the author intuitionistic fuzzy implications in [4, 5, 6]. Some of their properties are discussed in [1].

Here, following the idea from [2], we introduce four new intuitionistic fuzzy implications, modifying Dworniczak's intuitionistic fuzzy implication

$$\langle a, b\rangle \rightarrow_{150,\lambda} \langle c, d\rangle = \langle \frac{b + c + \lambda - 1}{2\lambda}, \frac{a + d + \lambda - 1}{2\lambda}\rangle,$$

where $\lambda \geq 1$.

2 Main results

Let everywhere below the variables x and y have truth values $\langle a, b\rangle$ and $\langle c, d\rangle$, where $a, b, c, d \in [0, 1], a + b \leq 1, c + d \leq 1$.

In [2, 3] the following operations and operators are defined:

$$\neg\langle a, b\rangle \equiv \neg_1\langle a, b\rangle = \langle b, a\rangle,$$

$$\langle a,b\rangle @\langle c,d\rangle = \langle \frac{a+c}{2}, \frac{b+d}{2}\rangle,$$

$$\Box\langle a,b\rangle = \langle a, 1-a\rangle,$$

$$\Diamond\langle a,b\rangle = \langle 1-b, b\rangle,$$

$$\boxplus_{\alpha,\beta,\gamma}\langle a,b\rangle = \langle \alpha a, \beta b+\gamma\rangle,$$

$$\boxtimes_{\alpha,\beta,\gamma}\langle a,b\rangle = \langle \alpha a+\gamma, \beta b\rangle,$$

where $\alpha,\beta,\gamma \in [0,1]$ and $\max(\alpha,\beta)+\gamma \leq 1$,

$$\boxdot_{\alpha,\beta,\gamma,\delta}\langle a,b\rangle = \langle \alpha a+\gamma, \beta b+\delta\rangle,$$

where $\alpha,\beta,\gamma,\delta \in [0,1]$ and $\max(\alpha,\beta)+\gamma+\delta \leq 1$.

We use the following formulas:

$$\langle a,b\rangle \to^1_{150,\lambda} \langle c,d\rangle = \Box\langle a,b\rangle \to_{150,\lambda} \Box\langle c,d\rangle,$$

$$\langle a,b\rangle \to^2_{150,\lambda} \langle c,d\rangle = \Box\langle a,b\rangle \to_{150,\lambda} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \to^3_{150,\lambda} \langle c,d\rangle = \Diamond\langle a,b\rangle \to_{150,\lambda} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \to^4_{150,\lambda} \langle c,d\rangle = \Diamond\langle a,b\rangle \to_{150,\lambda} \Box\langle c,d\rangle.$$

So, we obtain the explicit forms of the new four implications as follows:

$$\langle a,b\rangle \to^1_{150,\lambda} \langle c,d\rangle = \langle \frac{-a+c+\lambda}{2\lambda}, \frac{a-c+\lambda}{2\lambda}\rangle,$$

$$\langle a,b\rangle \to^2_{150,\lambda} \langle c,d\rangle = \langle \frac{1-a-d+\lambda}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda}\rangle,$$

$$\langle a,b\rangle \to^3_{150,\lambda} \langle c,d\rangle = \langle \frac{b+c+\lambda-1}{2\lambda}, \frac{1-b-c+\lambda}{2\lambda}\rangle,$$

$$\langle a,b\rangle \to^4_{150,\lambda} \langle c,d\rangle = \langle \frac{b-d+\lambda}{2\lambda}, \frac{-b+d+\lambda}{2\lambda}\rangle.$$

First, we check that for every $i = 1,2,3,4$ and for every $\lambda \geq 1$:

$$\langle 0,1\rangle \to^i_{150,\lambda} \langle 0,1\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,1\rangle \to^i_{150,\lambda} \langle 1,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 1,0\rangle \to^i_{150,\lambda} \langle 0,1\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 1,0\rangle \to^i_{150,\lambda} \langle 1,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle.$$

Second, we check that for every $\lambda \geq 1$:

$$\langle 0,0\rangle \to^1_{150,\lambda} \langle 0,1\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,0\rangle \to^2_{150,\lambda} \langle 0,1\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,0\rangle \to^3_{150,\lambda} \langle 0,1\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^4_{150,\lambda} \langle 0,1\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^1_{150,\lambda} \langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,0\rangle \to^2_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^3_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^4_{150,\lambda} \langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,0\rangle \to^1_{150,\lambda} \langle 1,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^2_{150,\lambda} \langle 1,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 0,0\rangle \to^3_{150,\lambda} \langle 1,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,0\rangle \to^4_{150,\lambda} \langle 1,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

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$$\langle 0,1\rangle \to^2_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 0,1\rangle \to^3_{150,\lambda} \langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 0,1\rangle \to^4_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda}\rangle,$$

$$\langle 1,0\rangle \to^1_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 1,0\rangle \to^2_{150,\lambda} \langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$

$$\langle 1,0\rangle \to^3_{150,\lambda} \langle 0,0\rangle = \langle \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda}\rangle,$$

$$\langle 1,0\rangle \to^4_{150,\lambda} \langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle.$$

Using definition from [2, 3]

$$\langle a,b\rangle \geq \langle c,d\rangle \text{ if and only if } a \geq c \text{ and } b \leq d,$$

we can prove the validity of the following theorem.

Theorem 1. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a, b\rangle \to^2_{150,\lambda} \langle c, d\rangle \geq \langle a, b\rangle \to^1_{150,\lambda} \langle c, d\rangle, \tag{1}$$

$$\langle a, b\rangle \to^4_{150,\lambda} \langle c, d\rangle \geq \langle a, b\rangle \to^3_{150,\lambda} \langle c, d\rangle, \tag{2}$$

$$\langle a, b\rangle \to^2_{150,\lambda} \langle c, d\rangle \geq \langle a, b\rangle \to^4_{150,\lambda} \langle c, d\rangle, \tag{3}$$

$$\langle a, b\rangle \to^1_{150,\lambda} \langle c, d\rangle \geq \langle a, b\rangle \to^3_{150,\lambda} \langle c, d\rangle. \tag{4}$$

Proof: For example, let us check the validity of the fourth inequality.
First, we see, that

$$-a + c + \lambda - b - c - \lambda + 1 = 1 - a - b \geq 0,$$

$$1 - b - c + \lambda - a + c - \lambda = 1 - a - b \geq 0.$$

Therefore,

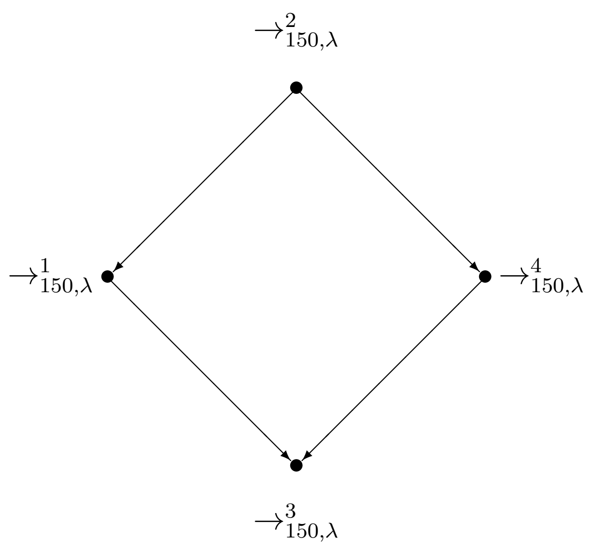
$$\langle \frac{-a + c + \lambda}{2\lambda}, \frac{a - c + \lambda}{2\lambda}\rangle \geq \langle \frac{b + c + \lambda - 1}{2\lambda}, \frac{1 - b - c + \lambda}{2\lambda}\rangle,$$

i.e.,

$$\langle a, b\rangle \to^1_{150,\lambda} \langle c, d\rangle \geq \langle a, b\rangle \to^3_{150,\lambda} \langle c, d\rangle.$$

Hence, (4) is valid. (1) – (3) are proved by analogy. □

Now, we can construct the following diagram



Now, we show that the new implications can be represented by a part of the above operators.

Theorem 2. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a, b\rangle \to^1_{150,\lambda} \langle c, d\rangle = \boxtimes_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Diamond\neg\langle a, b\rangle @ \boxplus_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Box\langle c, d\rangle,$$

$$\langle a, b\rangle \to^2_{150,\lambda} \langle c, d\rangle = \boxtimes_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Diamond\neg\langle a, b\rangle @ \boxplus_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Box\neg\langle c, d\rangle,$$

$$\langle a,b\rangle \rightarrow^1_{150,\lambda} \langle c,d\rangle = \boxplus_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Box\neg\langle a,b\rangle @ \boxtimes_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \rightarrow^3_{150,\lambda} \langle c,d\rangle = \boxtimes_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Box\neg\langle a,b\rangle @ \boxplus_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda}} \Diamond\langle c,d\rangle.$$

The proofs of these assertions are similar to the above one. □

Corollary. For every $a, b, c, d \in [0,1]$, so that $a+b \leq 1$ and $c+d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a,b\rangle \rightarrow^1_{150,\lambda} \langle c,d\rangle = \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda},0} \Diamond\neg\langle a,b\rangle @ \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},0,\frac{\lambda-1}{\lambda}} \Box\langle c,d\rangle,$$

$$\langle a,b\rangle \rightarrow^2_{150,\lambda} \langle c,d\rangle = \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda},0} \Diamond\neg\langle a,b\rangle @ \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},0,\frac{\lambda-1}{\lambda}} \Box\neg\langle c,d\rangle,$$

$$\langle a,b\rangle \rightarrow^1_{150,\lambda} \langle c,d\rangle = \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},0,\frac{\lambda-1}{\lambda}} \Box\neg\langle a,b\rangle @ \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda},0} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \rightarrow^3_{150,\lambda} \langle c,d\rangle = \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},0,\frac{\lambda-1}{\lambda}} \Box\neg\langle a,b\rangle @ \boxdot_{\frac{1}{\lambda},\frac{1}{\lambda},\frac{\lambda-1}{\lambda},0} \Diamond\langle c,d\rangle.$$

3 Conclusion

In the next parts of the research, we will study from one side other properties of the new implications and from another – the modifications of the two other Dworniczak's implications and their properties.

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