Notes on Intuitionistic Fuzzy Sets
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Sustainable material selection using complex type 2 intuitionistic fuzzy analytic hierarchy process

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Abstract: Sustainable material selection has become a crucial challenge in the construction industry due to increasing environmental concerns, resource limitations, and the demand for long-term ecological balance. Traditional evaluation approaches often struggle to adequately

address the complexity and uncertainty inherent in expert judgments. To overcome these limitations, this study proposes a novel decision-making framework based on the Complex Intuitionistic Type-2 Fuzzy Analytic Hierarchy Process (CIT2F-AHP). The method integrates the strengths of fuzzy set theory and analytic hierarchy process to capture both magnitude and phase information, enabling a more robust representation of expert opinions under uncertainty. A comprehensive set of evaluation criteria covering environmental, technical, economic, social, and regulatory aspects is adopted for assessing sustainable alternatives. The proposed approach is applied to compare low-carbon cement and concrete options, including carbon-cured concrete, geopolymer concrete, alkali-activated binders, and limestone calcined clay cement. The findings highlight the effectiveness of CIT2F-AHP in ranking sustainable materials and demonstrate its potential as a reliable decision-support tool for practitioners and policymakers in advancing green construction practices.
Keywords: Sustainable materials, Construction industry, Complex fuzzy sets, Intuitionistic fuzzy sets, Analytic hierarchy process, Type 2 fuzzy sets.

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1 Introduction

The construction industry is one of the largest contributors to global carbon emissions and resource consumption, making the transition toward sustainable materials a pressing priority. Growing concerns over climate change, environmental degradation, and resource scarcity have accelerated the demand for innovative solutions that minimize ecological impact while maintaining technical and economic feasibility. Sustainable material selection has therefore emerged as a critical challenge, requiring decision-makers to balance environmental, technical, economic, social, and regulatory considerations.

Traditional decision-making approaches often fall short when addressing the complexity and uncertainty involved in sustainable material evaluation. Expert judgments in this domain are inherently imprecise, and conventional models are limited in capturing the multidimensional uncertainty of such assessments. Fuzzy set theory [10] has long been recognized as a powerful tool for handling vagueness and subjectivity in decision-making; however, classical fuzzy models do not fully account for the richness of expert opinions. A lot of new extensions of ordinary fuzzy sets have been introduced in the literature such as intuitionistic fuzzy sets [3] and picture fuzzy sets [4]. Recent advances such as complex fuzzy sets and intuitionistic fuzzy sets have introduced greater flexibility by incorporating both membership hesitancy and phase information, thus enabling a more nuanced representation of uncertainty. Complex Fuzzy Sets (CFS) were developed by Ramot and colleagues, allowing membership functions to take values on the unit circle in the complex plane rather than being restricted to the interval [0, 1] (see [8]).

Within this context, the Analytic Hierarchy Process (AHP) remains one of the most widely used multi-criteria decision-making (MCDM) techniques for prioritizing alternatives under structured hierarchies of criteria [9]. Nevertheless, the conventional AHP approach is limited when judgments are uncertain or ambiguous. To address these shortcomings, this study employs the Complex Type-2 Intuitionistic Fuzzy Analytic Hierarchy Process (CIT2F-AHP), which integrates the strengths of complex fuzzy theory with AHP. By capturing both magnitude and phase in membership functions and extending them to type-2 intuitionistic sets, CIT2F-AHP provides a more robust mechanism for modeling hesitation, ambiguity, and uncertainty in expert evaluations.

In this study, a comprehensive decision framework based on CIT2F-AHP is developed to assess sustainable material alternatives in the construction industry. The method is applied to evaluate four promising low-carbon cement and concrete options: carbon-cured concrete, geopolymer concrete, alkali-activated binders, and limestone calcined clay cement. The evaluation is carried out across a broad spectrum of criteria encompassing environmental performance, technical feasibility, economic viability, social impact, and regulatory compliance.

The main contribution of this paper is twofold: first, it advances methodological research by demonstrating the applicability of the CIT2F-AHP in a sustainability-oriented material selection context; second, it provides practical insights into the ranking and prioritization of low-carbon materials for the construction industry. The results highlight the potential of this approach as a reliable decision-support tool for practitioners, researchers, and policymakers striving to promote greener construction practices.

2 Sustainable materials

Sustainable materials are those that are produced, used, and disposed of in ways that minimize negative impacts on the environment, conserve natural resources, and promote long-term ecological balance. Unlike conventional materials that may rely heavily on nonrenewable resources or energy-intensive manufacturing processes, sustainable materials are chosen or engineered with an emphasis on renewability, recyclability, and lower carbon footprints. They are central to addressing global challenges such as climate change, resource depletion, and pollution. Figure 1 shows some examples of sustainable materials.

a) Bio bricks b) Geopolymer concrete c) Permeable pavement

Figure 1. Some sustainable materials

One of the key aspects of sustainable materials is their life cycle impact—from raw material extraction to production, use, and eventual disposal or reuse. For instance, natural fibers like bamboo, hemp, and jute are increasingly used as alternatives to synthetic fibers because they grow quickly, require fewer chemicals, and are biodegradable. Similarly, recycled metals, plastics, and glass reduce the need for virgin resource extraction, thereby cutting down energy consumption and greenhouse gas emissions. Innovations in material science, such as bioplastics made from plant starches or composites derived from agricultural waste, further highlight the potential of sustainable alternatives.

Sustainable materials are not limited to construction or manufacturing; they play a significant role across industries. In architecture and engineering, for example, green building materials such as reclaimed wood, low-carbon cement, and recycled steel are transforming the way infrastructure is developed. In fashion, the shift toward organic cotton, recycled fabrics, and closed-loop textile systems helps reduce water use and landfill waste. Even in technology, companies are exploring biodegradable electronics and modular designs that extend product lifespans.

Ultimately, adopting sustainable materials is both an environmental necessity and an economic opportunity. While the transition may involve higher upfront costs or challenges in scaling production, the long-term benefits include reduced waste, enhanced resource efficiency, and alignment with global sustainability goals. As industries, governments, and consumers increasingly prioritize eco-conscious choices, sustainable materials will continue to shape innovation and drive a more resilient and responsible future.

3 Complex fuzzy sets: A literature review

Complex fuzzy sets (CFSs) are an extension of classical fuzzy sets that incorporate complex numbers into the membership functions [8]. In traditional fuzzy sets, the membership degree of an element lies within the real unit interval [0, 1], representing the extent to which an element belongs to a set. Complex fuzzy sets, however, assign a complex number with both magnitude and phase to represent membership. The magnitude typically indicates the degree of belonging, while the phase angle provides additional interpretive power, such as encoding periodicity, hesitation, or dual perspectives in uncertain information. This dual representation allows complex fuzzy sets to model more nuanced and multidimensional forms of uncertainty than classical fuzzy theory.

The key motivation for introducing complex fuzzy sets stems from the need to handle problems where real-valued membership grades are insufficient. For example, in domains such as signal processing, decision-making under uncertainty, and control systems, the phase component can capture relationships that real-valued fuzzy sets cannot express. The phase term may encode hidden patterns, cyclic behaviors, or even subjective hesitation in expert judgments. As such, CFSs offer a richer mathematical framework that is better aligned with the complexities of real-world problems.

Over the years, researchers have developed various operations and extensions for complex fuzzy sets, including union, intersection, complement, and aggregation operators. These operations are generalized from classical fuzzy logic but adapted to handle complex-valued membership grades. Moreover, applications of CFSs have emerged in fields like pattern recognition, medical diagnosis, and decision support systems. In multi-criteria decision-making, for instance, complex fuzzy information can represent both the strength of preference and the underlying uncertainty of experts' opinions.

In summary, complex fuzzy sets represent a powerful extension of fuzzy theory, broadening the scope of uncertainty modeling. By incorporating both magnitude and phase into membership functions, they enable a more comprehensive representation of complex, imprecise, and multidimensional information. As research in this area continues, CFSs are expected to play an increasingly important role in advanced decision-making, data analysis, and intelligent system design.

In Scopus, we searched for "complex fuzzy sets" and "complex fuzzy numbers" in titles, abstracts, and keywords. In total, 379 studies were found in the Scopus database based on a search conducted in October 2025. To capture different aspects of the field, we did not restrict the results to journal articles; conference papers and proceedings were also included. Figure 1 shows the total number of studies published in this area. It is clear that, especially after 2017, there has been a dramatic increase in the use of these sets. VOSviewer was used to analyze the characteristics of these studies.

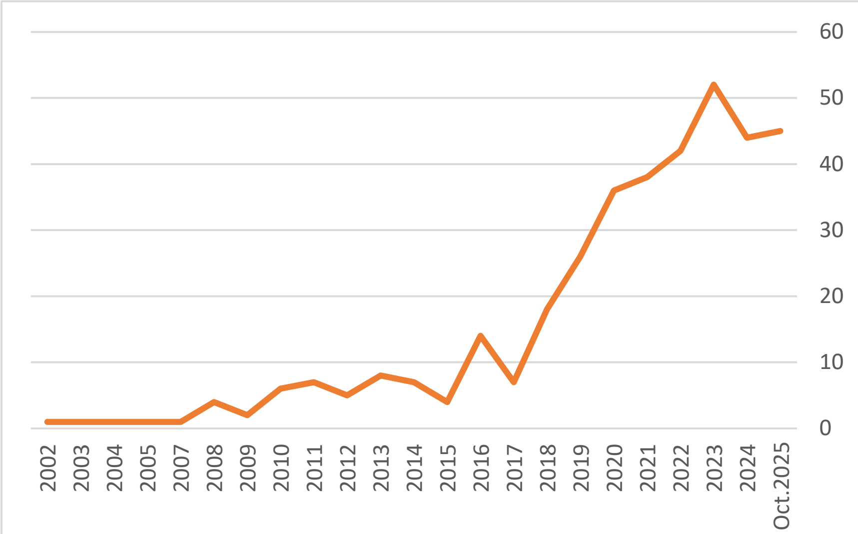

Figure 1. Number of studies per year

Figure 2 shows the distribution of publication venues. Because most studies were published after 2017, research on complex fuzzy sets can be considered emerging. Surprisingly, however, the majority of outputs are journal articles rather than papers in conference proceedings, indicating a degree of maturity in the field.

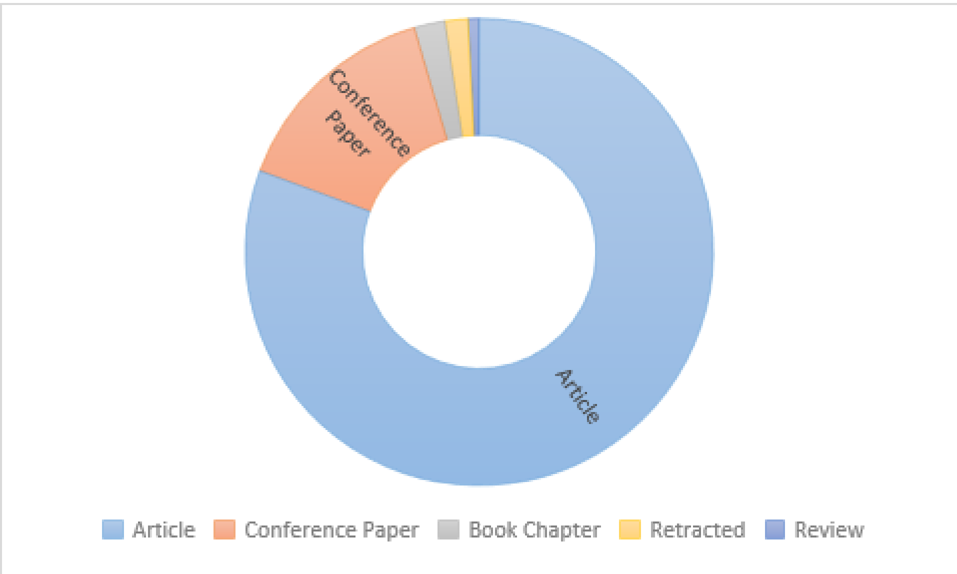

Figure 2. Sources of the studies

To identify the different research groups in the field, we conducted a VOSviewer co-authorship analysis. All authors were included, even those with a single publication. The results are presented in Figure 3.

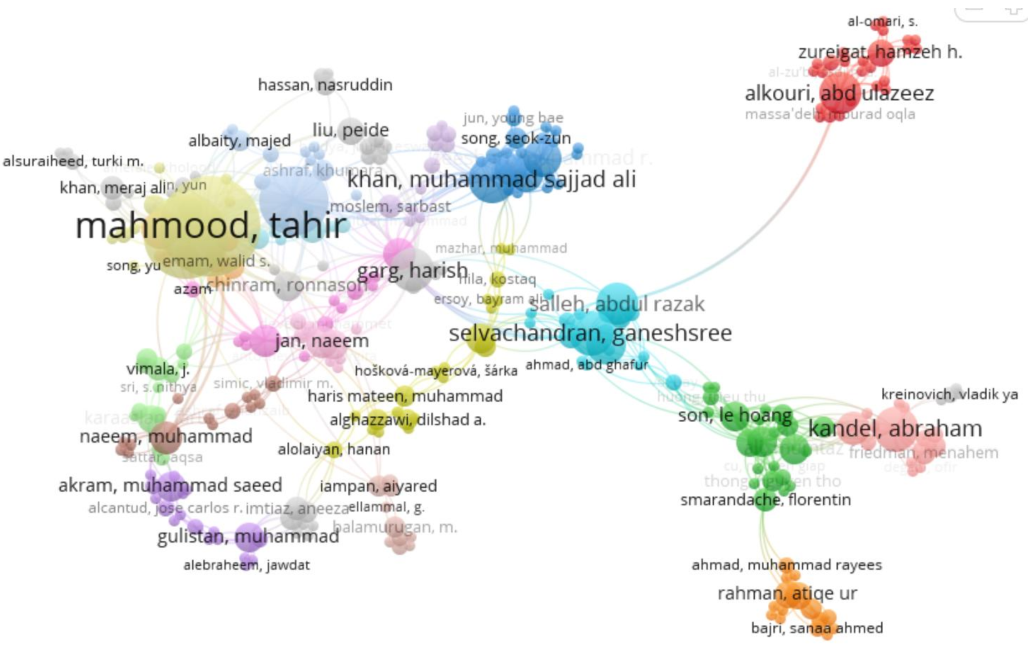

Figure 3. The co-authourship map (VosViewer)

The results clearly show several distinct research groups publishing in this field, and this segmented structure is also evident across application domains. While some papers are purely conceptual, developing mathematical models, others use complex fuzzy numbers in a range of application areas. A keyword analysis was conducted, and keyword densities were visualized using VOSviewer. The minimum occurrence threshold for keywords was set to 3. The results are presented in Figure 4.

Figure 4. Density Visualization of Keywords (VosViewer)

The density visualization highlights a strong link between decision-making and complex fuzzy sets, with mathematical operations, such as aggregation operators, still being actively studied. It also reveals connections to intelligent systems, including inference and learning approaches. As for applications, signal processing and receiver design emerge as particularly important areas.

4 Complex intuitionistic type 2 fuzzy sets

Let an CIT2FS be defined as in Equation (1), [1].

$$\tilde{T} = (x; (\mu, v))_{IT2} = \left(x; \left(\omega e^{i2\pi\beta}, \tau e^{i2\pi\delta}\right)\right)_{IT2}, \quad (1)$$

where

$$0 \le \mu \le 1,\ 0 \le v \le 1,\ 0 \le \omega \le 1,\ 0 \le \tau \le 1,\ 0 \le \beta \le 1,\ 0 \le \delta \le 1;$$

$$0 \le {\omega_{IT2}}^2 + {\tau_{IT2}}^2 \le 1;\ 0 \le {\beta_{IT2}}^2 + {\delta_{IT2}}^2 \le 1 .$$

The hesitancy degree is given as

$$h_{Py} = \sqrt{\left(1 - (\omega_{IT2}^2 + \tau_{IT2}^2)\right)} e^{i2\pi\sqrt{1-(\beta_{IT2}^2+\delta_{IT2}^2)}} . \quad (2)$$

The score function of a CIT2FS is given by Equation (3):

$$S_{\tilde{T}} = \frac{\omega_{IT2}^2 - \tau_{IT2}^2 + \beta_{IT2}^2 - \delta_{IT2}^2}{2} . \quad (3)$$

The accuracy function of a CIT2FS is given by Equation (4):

$$A_{\tilde{T}} = \frac{\omega_{IT2}^2 + \tau_{IT2}^2 + \beta_{IT2}^2 + \delta_{IT2}^2}{2} , \quad (4)$$

where $S_{\tilde{T}} \in [-1, 1]$ and $A_{\tilde{T}} \in [0, 1]$.

Let $\tilde{A} = \left(\omega_{\tilde{A}} e^{i2\pi\beta_{\tilde{A}}}, \tau_{\tilde{A}} e^{i2\pi\delta_{\tilde{A}}}\right)_{IF2}$ and $\tilde{B} = \left(\omega_{\tilde{B}} e^{i2\pi\beta_{\tilde{B}}}, \tau_{\tilde{B}} e^{i2\pi\delta_{\tilde{B}}}\right)_{IF2}$ be two CIT2FSs and let λ be a non-negative scalar. Some arithmetic operations to these CIT2FSs can be given as follows:

$$\lambda\tilde{A} = \left(\sqrt{1 - (1 - {\omega_{\tilde{A}}}^2)^{\lambda}} e^{i2\pi\sqrt{1-\left(1-\left(\beta_{\tilde{A}}/(2\pi)\right)^2\right)^{\lambda}}}, {\tau_{\tilde{A}}}^{\lambda} e^{i2\pi\left(\delta_{\tilde{A}}/(2\pi)\right)^2}\right), \quad (5)$$

$$\tilde{A}^{\lambda} = \left({\omega_{\tilde{A}}}^{\lambda} e^{i2\pi\left(\beta_{\tilde{A}}/(2\pi)\right)^2}, \sqrt{1 - (1 - {\tau_{\tilde{A}}}^2)^{\lambda}} e^{i2\pi\sqrt{1-\left(1-\left(\delta_{\tilde{A}}/(2\pi)\right)^2\right)^{\lambda}}}\right), \quad (6)$$

$$\tilde{A} \oplus \tilde{B} = \left(\sqrt{{\omega_{\tilde{A}}}^2 + {\omega_{\tilde{B}}}^2 - {\omega_{\tilde{A}}}^2 {\omega_{\tilde{B}}}^2} e^{i2\pi\sqrt{\left(\beta_{\tilde{A}}/2\pi\right)^2 + \left(\beta_{\tilde{B}}/2\pi\right)^2 - \left(\beta_{\tilde{A}}/2\pi\right)^2 \left(\beta_{\tilde{B}}/2\pi\right)^2}}, \tau_{\tilde{A}} \tau_{\tilde{B}} e^{i2\pi\left(\delta_{\tilde{A}}/2\pi\right)\left(\delta_{\tilde{B}}/2\pi\right)}\right), \quad (7)$$

$$\tilde{A} \otimes \tilde{B} = \left(\omega_{\tilde{A}} \omega_{\tilde{B}} e^{i2\pi\left(\beta_{\tilde{A}}/2\pi\right)\left(\beta_{\tilde{B}}/2\pi\right)}, \sqrt{{\tau_{\tilde{A}}}^2 + {\tau_{\tilde{B}}}^2 - {\tau_{\tilde{A}}}^2 {\tau_{\tilde{B}}}^2} e^{i2\pi\sqrt{\left(\delta_{\tilde{A}}/2\pi\right)^2 + \left(\delta_{\tilde{B}}/2\pi\right)^2 - \left(\delta_{\tilde{A}}/2\pi\right)^2 \left(\delta_{\tilde{B}}/2\pi\right)^2}}\right). \quad (8)$$

5 CIT2F-AHP method

The weights vector $\{w_1, w_2, \dots, w_i, \dots w_n\}$, $i = 1, 2, \dots, n$ is obtained by the following CIT2F-AHP method. Consider the following hierarchy in Figure 4. The main criteria are first pairwise-compared with respect to the goal and then the alternatives are pairwise-compared with respect to each main-criterion by Ξ experts, $\xi = 1,2, \dots, \Xi$. The associated weight vector of the experts is represented by $\varrho_\xi = (\varrho_1, \varrho_2, \varrho_3, \dots, \varrho_\Xi)$, $\xi = 1,2, \dots, \Xi$.

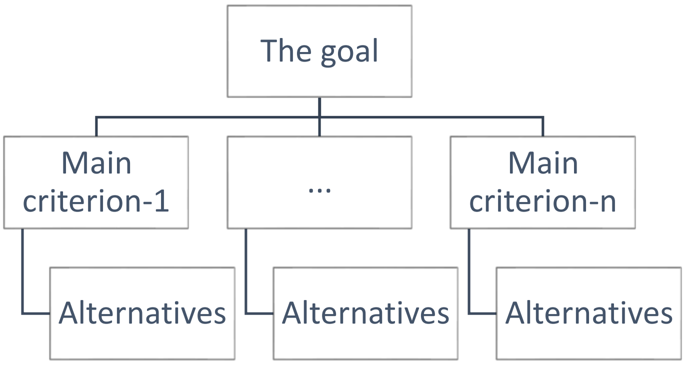

Figure 4. Goal / criteria / alternatives hierarchy

The linguistic scale in Table 1 is used for the pairwise comparisons.

Table 1. Linguistic scale

Linguistic terms for criteria evaluation	**Linguistic terms for alternative ratings**	**CIT2FS values ($\tilde{x}_{ij}$)**
Absolutely Less Important (ALI)	Certainly Low Performance (CLP)	$\langle(0.25, 0.30), (0.85, 0.95)\rangle$
Much Less Important (MLI)	Very Low Performance (VLP)	$\langle(0.30, 0.35), (0.80,0.90)\rangle$
Less Important (LI)	Less Performance (LP)	$\langle(0.35, 0.40), (0.70, 0.80)\rangle$
Slightly Less Important (SLI)	Slightly Less Performance (BAP)	$\langle(0.40, 0.45), (0.50, 0.60)\rangle$
Equally Important (EI)	Equal Performance (AP)	$\langle(0.50, 0.50), (0.50,0.50)\rangle$
Slightly More Important (SMI)	Above Average Performance (AAP)	$\langle(0.50, 0.60), (0.40, 0.45)\rangle$
More Important (MI)	High Performance (HP)	$\langle(0.70, 0.80), (0.35, 0.40)\rangle$
Much More Important (MMI)	Very High Performance (VHP)	$\langle(0.80,0.90), (0.35, 0.40)\rangle$
Absolutely More Important (AMI)	Certainly High Performance (CHP)	$\langle(0.85, 0.95), (0.25, 0.30)\rangle$

Step 1. Pairwise-compare the main-criteria with respect to the goal. Each expert fills in the matrix as in Table 2.

Table 2. Linguistic pairwise-comparison matrix with respect to the goal

	C_1	C_2	$\cdots$	C_n
C_1	l_{11}	l_{12}	$\cdots$	l_{1n}
C_2	l_{21}	l_{22}	$\cdots$	l_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	l_{n1}	l_{n2}	$\cdots$	l_{nn}

Linguistic pairwise-comparison matrices are converted to CIT2F pairwise-comparison matrices by substituting the corresponding CIT2F values ($\tilde{x}_{ij}$) in the linguistic scale as in Table 3.

Table 3. Pairwise-comparisons of the criteria with respect to the goal

	C_1	C_2	$\cdots$	C_n
C_1	$\langle(0.5,0.5),(0.5,0.5)\rangle$	$\langle\tilde{x}_{12}\rangle$	$\cdots$	$\langle\tilde{x}_{1n}\rangle$
C_2	$\langle\tilde{x}_{21}\rangle$	$\langle(0.5,0.5),(0.5,0.5)\rangle$	$\cdots$	$\langle\tilde{x}_{2n}\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\langle\tilde{x}_{n1}\rangle$	$\langle\tilde{x}_{n2}\rangle$	$\cdots$	$\langle(0.5,0.5),(0.5,0.5)\rangle$

Step 2. Measure the consistency ratio of each matrix by defuzzifying the CIT2F numbers by Equation (9), [5].

$$S_{IT2}=\begin{cases}1, & i=j;\ i=1,2,\ldots,n;\ j=1,2,\ldots,n;\\ 10\times\sqrt{\left(\omega_{ij}-\tau_{ij}\right)^2+\left(\beta_{ij}-\delta_{ij}\right)^2} & \text{if } \omega_{ij}>\tau_{ij};\beta_{ij}>\delta_{ij};\ i\neq j;\\ 1/\left(10\times\sqrt{\left(\omega_{ij}-\tau_{ij}\right)^2+\left(\beta_{ij}-\delta_{ij}\right)^2}\right), & \text{reciprocal values.}\end{cases} \quad (9)$$

The *Consistency Ratio* (CR) is equal to Consistency Index divided by Random Index (CI/RI) and should be at most equal to 0.10 to be a consistent matrix. *Random Index* (RI) is determined from a readily given table. The *Consistency Index* (CI) is computed by Equation (10):

$$CI=\frac{\lambda_{\max}-n}{n-1}. \quad (10)$$

Step 3. Aggregate the pairwise-comparison matrices filled by the experts by using the complex intuitionistic type-2 fuzzy Aczel–Alsina weighted averaging (CIT2FAAWA) operator given by Equation (11). Ω can be taken as 2, [7].

$$\text{CIT2FAAWA}(\zeta_1,\zeta_2,\zeta_3,\ldots,\zeta_\Xi)=\left(\left(\omega^{agg}e^{i2\pi\beta^{agg}},\tau^{agg}e^{i2\pi\delta^{agg}}\right)\right)$$

$$=\left(\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\varrho_\xi\left(-ln\left(1-\omega_\xi{}^2\right)\right)^\Omega\right)\right)^{1/\Omega}}}e^{i2\pi\sqrt{1-e^{-\left(\sum_{\xi=1}^{\Xi}\left(\varrho_\xi\left(-ln\left(1-\beta_\xi{}^2\right)\right)^\Omega\right)\right)^{1/\Omega}}}},\right.$$

$$\left.e^{-\left(\sum_{\xi=1}^{\Xi}\left(\varrho_\xi\left(-ln(\tau_\xi)\right)^\Omega\right)\right)^{1/\Omega}}e^{i2\pi\left(e^{-\left(\sum_{\xi=1}^{\Xi}\left(\varrho_\xi\left(-(\delta_\xi)\right)^\Omega\right)\right)^{1/\Omega}}\right)}\right) \tag{11}$$

Step 4. Defuzzify the aggregated values in the CIT2F pairwise comparison matrix by using Equation (9).

Step 5. Compute the weights of criteria and the priorities of alternatives from the defuzzified matrix obtained in Step 4. The crisp pairwise comparison matrix is given by Table 4.

Table 4. Defuzzified pairwise comparison matrix
with respect to the goal or with respect to the criteria

	C_1 or A_1	C_2 or A_2	$\cdots$	C_n or A_n
C_1 or A_1	1	$(S_{IT2})_{12}$	$\cdots$	$(S_{IT2})_{1n}$
C_2 or A_2	$(S_{IT2})_{21}$	1	$\cdots$	$(S_{IT2})_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n or A_n	$(S_{IT2})_{n1}$	$(S_{IT2})_{n2}$	$\cdots$	1

Step 6. Follow the same procedure for computing weights or priorities as in the classical AHP.

6 Application

Low-carbon cement & concrete alternatives in construction industry are as follows: A1. Carbon-cured concrete**:** Traps CO_2 during curing, improving strength and sustainability; A2. Geopolymer concrete: Uses industrial by-products like fly ash and slag instead of Portland cement; A3. Alkali-activated binders: Lower CO_2 footprint than traditional cement; A4. Limestone calcined clay cement (LC^3): Cuts CO_2 emissions by up to 40%.

When evaluating alternative sustainable materials in the construction industry**,** researchers and practitioners usually rely on a set of criteria that capture environmental, technical, economic, and social aspects. The list of the most common evaluation criteria are:

- **C1: Environmental Criteria** (Carbon footprint / CO_2 emissions (during production, transport, and lifecycle); Embodied energy (energy consumed in extraction, processing, manufacturing, transport); Resource efficiency (use of renewable, abundant, or recycled raw materials); Recyclability and reusability; Biodegradability; Waste reduction potential; and Impact on biodiversity and ecosystems);

- **C2: Technical & Performance Criteria** (Mechanical strength (compressive, tensile, flexural strength); Durability and longevity (resistance to weathering, corrosion, decay, fire, and pests); Thermal performance (insulation, heat transfer, thermal mass); Acoustic performance (sound insulation, absorption); Moisture resistance and permeability; Compatibility with conventional construction methods; and Ease of maintenance);
- **C3: Economic Criteria** (Initial cost (material cost, production, transportation, installation); Life-cycle cost (maintenance, replacement, operation expenses); Local availability (reducing transport cost and supporting local economy); Market readiness / scalability; Return on investment);
- **C4: Social & Health Criteria** (Worker and occupant health impact (toxicity, off-gassing, VOC emissions); Indoor air quality contribution; Aesthetic and cultural acceptance; Job creation and social benefits (local sourcing, community involvement); User comfort and safety**);**
- **C5: Regulatory & Certification Criteria** (Compliance with building codes and standards; Certifications (LEED, BREEAM, WELL, ISO standards, etc.); Fire safety and hazard regulations).

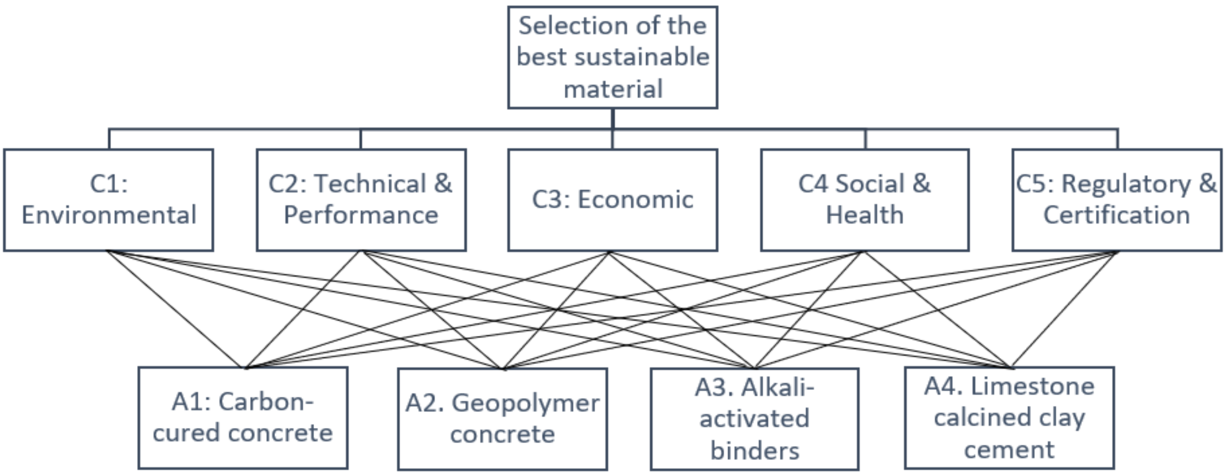

Figure 5. Hierarchy of the problem

The evaluations were gathered from one materials scientist and two civil engineers with substantial expertise in the construction industry. One of the experts has over five years of professional experience, while the other two each possess more than a decade of industry practice.

Table 1. Linguistic pairwise comparison matrix of main criteria by experts

	E_1					E_2					E_3				
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5
C_1	EI	MI	MMI	SMI	AMI	EI	SMI	LI	MI	SLI	EI	LI	SLI	ALI	MLI
C_2	LI	EI	SMI	SLI	MI	SLI	EI	MLI	SMI	LI	MI	EI	SMI	LI	SLI
C_3	MLI	SLI	EI	LI	SMI	MI	MMI	EI	AMI	SMI	SMI	SLI	EI	MLI	SLI
C_4	SLI	SMI	MI	EI	MI	LI	SLI	ALI	EI	LI	AMI	MI	MMI	EI	SMI
C_5	ALI	LI	SLI	LI	EI	SMI	MI	SLI	MI	EI	MMI	SMI	SMI	SLI	EI

Table 7. Linguistic pairwise comparison matrix of alternatives by experts

		E_1				E_2				E_3			
		A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
C_1	A_1	EP	ALP	SMP	LP	EP	SLP	SMP	MLP	EP	SLP	MLP	SMP
	A_2	AMP	EP	MMP	SMP	SMP	EP	MP	SLP	SMP	EP	LP	MP
	A_3	SLP	MLP	EP	SLP	SLP	LP	EP	LP	MMP	MP	EP	MMP
	A_4	MP	SLP	SMP	EP	MMP	SMP	MP	EP	SLP	LP	MLP	EP
C_2	A_1	EP	MP	SMP	AMP	EP	SMP	LP	ALP	EP	MMP	MP	AMP
	A_2	LP	EP	LP	MP	SLP	EP	SLP	MLP	MLP	EP	LP	SMP
	A_3	SLP	MP	EP	MMP	MP	SMP	EP	SLP	LP	MP	EP	MP
	A_4	ALP	LP	MLP	EP	AMP	MMP	SMP	EP	ALP	SLP	LP	EP
C_3	A_1	EP	SLP	MP	ALP	EP	MP	SMP	MMP	EP	SMP	MP	MMP
	A_2	SMP	EP	MMP	SLP	LP	EP	SLP	SMP	SLP	EP	MP	MMP
	A_3	LP	MLP	EP	ALP	SLP	SMP	EP	AMP	LP	LP	EP	SMP
	A_4	AMP	SMP	AMP	EP	MLP	SLP	ALP	EP	MLP	MLP	SLP	EP
C_4	A_1	EP	AMP	MMP	SMP	EP	SLP	SMP	LP	EP	MP	SMP	MMP
	A_2	ALP	EP	SLP	MLP	SMP	EP	MP	SLP	LP	EP	SLP	MP
	A_3	MLP	SMP	EP	SLP	SLP	LP	EP	ALP	SLP	SMP	EP	MMP
	A_4	SLP	MMP	SMP	EP	MP	SMP	AMP	EP	MLP	LP	MLP	EP
C_5	A_1	EP	MLP	SLP	SMP	EP	MLP	SLP	SMP	EP	LP	SMP	SLP
	A_2	MMP	EP	MMP	AMP	MMP	EP	MMP	AMP	MP	EP	AMP	MP
	A_3	SMP	MLP	EP	MP	SMP	MLP	EP	MMP	SLP	ALP	EP	MLP
	A_4	SLP	ALP	LP	EP	SLP	ALP	MLP	EP	SMP	LP	MMP	EP

The initial linguistic pairwise-comparison matrices on the criteria importance and alternative ratings are collected from the three experts and converted to CIT2F pairwise-comparison matrices using the linguistic scale as in Table 1 to proceed with the consistency check on each matrix. After defuzzifying the CIT2F pairwise-comparison matrices using Equation 9, *CR* is computed for each matrix. For the matrices where the $CR > 0.1$, the expert is asked to revise his/her evaluation until $CR \leq 0.1$. Table 6 shows the final linguistic pairwise comparison matrix of main criteria with respect to goal evaluated by each expert and Table 7 shows the linguistic pairwise comparison matrix of the alternatives with respect to main criteria, after the consistency is obtained for each evaluation.

To illustrate the consistency calculation, Table 8 presents the defuzzified pairwise comparison matrix on the criteria evaluation given by Expert 1.

Table 8. Defuzzified pairwise comparison of the main criteria evaluated by Expert 1.

	E_1				
	C_1	C_2	C_3	C_4	C_5
C_1	1	5.315	6.727	1.803	8.846
C_2	0.188	1	1.803	0.555	5.315
C_3	0.149	0.555	1	0.188	1.803
C_4	0.555	1.803	5.315	1	5.315
C_5	0.113	0.188	0.555	0.188	1
CI	0.032				
RI	1.11				
CR	0.028				

Then, the pairwise-comparison matrices on the main-criteria and alternatives are aggregated using the CIT2AAWA operator given in Equation 11. Table 9 and Table 10 present the aggregated CIT2F matrix on the criteria importance and on the alternative ratings, respectively.

Table 9. Aggregated CIT2F matrix for criteria evaluation

	C_1	C_2	C_3	C_4	C_5
C_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.591e^{i2\pi(0.692)}, 0.436e^{i2\pi(0.490)})$	$(0.673e^{i2\pi(0.789)}, 0.470e^{i2\pi(0.538)})$	$(0.588e^{i2\pi(0.690)}, 0.445e^{i2\pi(0.495)})$	$(0.726e^{i2\pi(0.861)}, 0.405e^{i2\pi(0.469)})$
C_2	$(0.436e^{i2\pi(0.490)}, 0.591e^{i2\pi(0.692)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.463e^{i2\pi(0.557)}, 0.468e^{i2\pi(0.520)})$	$(0.434e^{i2\pi(0.513)}, 0.499e^{i2\pi(0.570)})$	$(0.579e^{i2\pi(0.677)}, 0.470e^{i2\pi(0.538)})$
C_3	$(0.470e^{i2\pi(0.538)}, 0.673e^{i2\pi(0.789)})$	$(0.468e^{i2\pi(0.520)}, 0.463e^{i2\pi(0.557)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.725e^{i2\pi(0.861)}, 0.433e^{i2\pi(0.492)})$	$(0.475e^{i2\pi(0.567)}, 0.428e^{i2\pi(0.489)})$
C_4	$(0.445e^{i2\pi(0.495)}, 0.588e^{i2\pi(0.690)})$	$(0.499e^{i2\pi(0.570)}, 0.434e^{i2\pi(0.513)})$	$(0.433e^{i2\pi(0.492)}, 0.725e^{i2\pi(0.861)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.591e^{i2\pi(0.692)}, 0.436e^{i2\pi(0.490)})$
C_5	$(0.405e^{i2\pi(0.469)}, 0.726e^{i2\pi(0.861)})$	$(0.470e^{i2\pi(0.538)}, 0.579e^{i2\pi(0.677)})$	$(0.428e^{i2\pi(0.489)}, 0.475e^{i2\pi(0.567)})$	$(0.436e^{i2\pi(0.490)}, 0.591e^{i2\pi(0.692)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$

Table 10. Aggregated CIT2F matrix for alternatives evaluation

		A_1	A_2	A_3	A_4
C_1	A_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.370e^{i2\pi(0.419)}, 0.563e^{i2\pi(0.658)})$	$(0.463e^{i2\pi(0.557)}, 0.468e^{i2\pi(0.520)})$	$(0.416e^{i2\pi(0.499)}, 0.559e^{i2\pi(0.617)})$
	A_2	$(0.563e^{i2\pi(0.658)}, 0.370e^{i2\pi(0.419)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.713e^{i2\pi(0.823)}, 0.414e^{i2\pi(0.468)})$	$(0.594e^{i2\pi(0.694)}, 0.407e^{i2\pi(0.467)})$
	A_3	$(0.468e^{i2\pi(0.520)}, 0.463e^{i2\pi(0.557)})$	$(0.414e^{i2\pi(0.468)}, 0.713e^{i2\pi(0.823)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.673e^{i2\pi(0.789)}, 0.470e^{i2\pi(0.538)})$
	A_4	$(0.559e^{i2\pi(0.617)}, 0.416e^{i2\pi(0.499)})$	$(0.407e^{i2\pi(0.467)}, 0.594e^{i2\pi(0.694)})$	$(0.470e^{i2\pi(0.538)}, 0.673e^{i2\pi(0.789)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$
C_2	A_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.718e^{i2\pi(0.828)}, 0.365e^{i2\pi(0.415)})$	$(0.591e^{i2\pi(0.692)}, 0.436e^{i2\pi(0.490)})$	$(0.806e^{i2\pi(0.922)}, 0.321e^{i2\pi(0.374)})$
	A_2	$(0.365e^{i2\pi(0.415)}, 0.718e^{i2\pi(0.828)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.369e^{i2\pi(0.419)}, 0.610e^{i2\pi(0.707)})$	$(0.589e^{i2\pi(0.691)}, 0.443e^{i2\pi(0.494)})$
	A_3	$(0.436e^{i2\pi(0.490)}, 0.591e^{i2\pi(0.692)})$	$(0.610e^{i2\pi(0.707)}, 0.369e^{i2\pi(0.419)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.714e^{i2\pi(0.823)}, 0.388e^{i2\pi(0.447)})$
	A_4	$(0.321e^{i2\pi(0.374)}, 0.806e^{i2\pi(0.922)})$	$(0.443e^{i2\pi(0.494)}, 0.589e^{i2\pi(0.691)})$	$(0.388e^{i2\pi(0.447)}, 0.714e^{i2\pi(0.823)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$
C_3	A_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.594e^{i2\pi(0.694)}, 0.407e^{i2\pi(0.467)})$	$(0.661e^{i2\pi(0.763)}, 0.365e^{i2\pi(0.415)})$	$(0.752e^{i2\pi(0.862)}, 0.422e^{i2\pi(0.473)})$
	A_2	$(0.407e^{i2\pi(0.467)}, 0.594e^{i2\pi(0.694)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.714e^{i2\pi(0.823)}, 0.388e^{i2\pi(0.447)})$	$(0.680e^{i2\pi(0.795)}, 0.407e^{i2\pi(0.467)})$
	A_3	$(0.365e^{i2\pi(0.415)}, 0.661e^{i2\pi(0.763)})$	$(0.388e^{i2\pi(0.447)}, 0.714e^{i2\pi(0.823)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.729e^{i2\pi(0.864)}, 0.381e^{i2\pi(0.434)})$
	A_4	$(0.422e^{i2\pi(0.473)}, 0.752e^{i2\pi(0.862)})$	$(0.407e^{i2\pi(0.467)}, 0.680e^{i2\pi(0.795)})$	$(0.381e^{i2\pi(0.434)}, 0.729e^{i2\pi(0.864)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$
C_4	A_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.754e^{i2\pi(0.878)}, 0.339e^{i2\pi(0.398)})$	$(0.686e^{i2\pi(0.801)}, 0.382e^{i2\pi(0.432)})$	$(0.679e^{i2\pi(0.795)}, 0.436e^{i2\pi(0.490)})$
	A_2	$(0.339e^{i2\pi(0.398)}, 0.754e^{i2\pi(0.878)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.582e^{i2\pi(0.679)}, 0.436e^{i2\pi(0.510)})$	$(0.577e^{i2\pi(0.675)}, 0.478e^{i2\pi(0.544)})$
	A_3	$(0.382e^{i2\pi(0.432)}, 0.686e^{i2\pi(0.801)})$	$(0.436e^{i2\pi(0.510)}, 0.582e^{i2\pi(0.679)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.672e^{i2\pi(0.788)}, 0.481e^{i2\pi(0.545)})$
	A_4	$(0.436e^{i2\pi(0.490)}, 0.679e^{i2\pi(0.795)})$	$(0.478e^{i2\pi(0.544)}, 0.577e^{i2\pi(0.675)})$	$(0.481e^{i2\pi(0.545)}, 0.672e^{i2\pi(0.788)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$
C_5	A_1	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.320e^{i2\pi(0.369)}, 0.760e^{i2\pi(0.856)})$	$(0.443e^{i2\pi(0.521)}, 0.461e^{i2\pi(0.537)})$	$(0.475e^{i2\pi(0.567)}, 0.428e^{i2\pi(0.489)})$
	A_2	$(0.760e^{i2\pi(0.856)}, 0.320e^{i2\pi(0.369)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.820e^{i2\pi(0.923)}, 0.310e^{i2\pi(0.360)})$	$(0.820e^{i2\pi(0.929)}, 0.277e^{i2\pi(0.327)})$
	A_3	$(0.461e^{i2\pi(0.537)}, 0.443e^{i2\pi(0.521)})$	$(0.310e^{i2\pi(0.360)}, 0.820e^{i2\pi(0.923)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$	$(0.712e^{i2\pi(0.822)}, 0.420e^{i2\pi(0.472)})$
	A_4	$(0.428e^{i2\pi(0.489)}, 0.475e^{i2\pi(0.567)})$	$(0.277e^{i2\pi(0.327)}, 0.820e^{i2\pi(0.929)})$	$(0.420e^{i2\pi(0.472)}, 0.712e^{i2\pi(0.822)})$	$(0.500e^{i2\pi(0.500)}, 0.500e^{i2\pi(0.500)})$

The aggregated CIT2F matrices are then defuzzified using Equation 9 to obtain crisp pairwise comparison matrix for the main-criteria as given in Table 11 and for alternatives evaluation as in Table 12. Following the computation in classical AHP, the criteria weights are computed from the defuzzified comparison matrix and presented in Table 11.

Table 11. Defuzzified pairwise comparison matrix for criteria evaluation

	C_1	C_2	C_3	C_4	C_5	**Criteria weights**
C_1	1.000	2.546	3.222	2.414	5.069	0.433
C_2	0.393	1.000	2.615	1.155	1.762	0.199
C_3	0.310	0.382	1.000	4.703	0.907	0.150
C_4	0.414	0.866	0.213	1.000	2.546	0.124
C_5	0.197	0.567	1.103	0.393	1.000	0.094

For the alternative ratings, the defuzzified comparison matrix in Table 12 presents the pairwise comparison of the alternatives with respect to each main criterion, separately. With respect to each criterion, the crisp alternative ratings are obtained from the defuzzified pairwise comparison matrices as shown in the last column of Table 12.

Table 12. Defuzzified pairwise comparison matrices for alternative evaluation

		A_1	A_2	A_3	A_4	Performance ratings
C_1	A_1	1.000	0.325	2.615	0.541	0.173
	A_2	3.081	1.000	4.640	2.936	0.536
	A_3	0.382	0.216	1.000	3.222	0.151
	A_4	1.849	0.341	0.310	1.000	0.140
C_2	A_1	1.000	5.429	2.546	7.317	0.553
	A_2	0.184	1.000	0.267	2.448	0.103
	A_3	0.393	3.750	1.000	4.976	0.287
	A_4	0.137	0.409	0.201	1.000	0.057
C_3	A_1	1.000	2.936	4.566	5.101	0.521
	A_2	0.341	1.000	4.976	4.269	0.297
	A_3	0.219	0.201	1.000	5.529	0.127
	A_4	0.196	0.234	0.181	1.000	0.055
C_4	A_1	1.000	6.348	4.780	3.896	0.613
	A_2	0.158	1.000	2.234	1.644	0.162
	A_3	0.209	0.448	1.000	3.085	0.136
	A_4	0.257	0.608	0.324	1.000	0.088
C_5	A_1	1.000	0.152	4.233	0.907	0.137
	A_2	6.564	1.000	7.597	8.106	0.702
	A_3	0.236	0.132	1.000	4.562	0.096
	A_4	1.103	0.123	0.219	1.000	0.065

In the last step, the final performance ratings of the alternatives are computed by weighting the performance ratings (Table 12) by the criteria weights (Table 11). Table 13 shows the final performance rating of each alternative.

Table 13. Final performance ratings of the alternatives

Criteria weights	C_1	C_2	C_3	C_4	C_5	Final performance ratings
	0.433	0.199	0.150	0.124	0.094	
A_1	0.173	0.553	0.521	0.613	0.137	0.352
A_2	0.536	0.103	0.297	0.162	0.702	0.383
A_3	0.151	0.287	0.127	0.136	0.096	0.168
A_4	0.140	0.057	0.055	0.088	0.065	0.097

The most importance criterion is obtained as C_1 "Environmental criteria", which is aligned with the expectation under selecting the most sustainable concrete material, followed by prioritization of C_2 "Technical and Performance criteria". The ranking of criteria is derived as $C_1 > C_2 > C_3 > C_4 > C_5$. The alternative ratings are found to be $A_2 > A_1 > A_3 > A_4$. Thus, the most sustainable concrete material in the construction industry is determined as Geopolymer concrete (A_2). The remaining alternatives are ranked from the best to the worst as follows: Carbon-cured concrete (A_1), Alkali-activated binders (A_3), and Limestone calcined clay cement (A_4).

7 Comparative analysis

In this section, the results derived from the applied CIT2F-AHP method utilizing the Aczel–Alsina weighted averaging (WA) operator (Equation (11)) (Jin 2023) are compared using different weighting averaging aggregation operators Dombi [2] and Einstein [6]. Additionally, the variable parameter Ω used in Aczel–Alsina weighting averaging operator is selected to be 3 in order to observe its effect on the criteria and alternative rankings.

Table 14. Criteria weights obtained by applying different WA operators

	Aczel–Alsina $\Omega = 2$ [7]	**Aczel–Alsina $\Omega = 3$ [7]**	**Dombi [2]**	**Einstein [6]**
C_1	0.433	0.481	0.487	0.212
C_2	0.199	0.169	0.158	0.124
C_3	0.150	0.183	0.187	0.175
C_4	0.124	0.095	0.097	0.250
C_5	0.094	0.072	0.071	0.239

Table 15. Final performance ratings of alternatives obtained by applying different WA operators

	Aczel–Alsina $\Omega = 2$ [7]	**Aczel–Alsina $\Omega = 3$ [7]**	**Dombi [2]**	**Einstein [6]**
A_1	0.352	0.340	0.331	0.326
A_2	0.383	0.396	0.402	0.385
A_3	0.168	0.185	0.187	0.153
A_4	0.097	0.079	0.080	0.136

The results illustrate that all four approaches yield the same ranking with only minor differences in the final performance ratings. The results obtained from Einstein WA operator deviate slightly from the rest of the approaches as it yielded a greater value for the least-preferred alternative and lower value for the most-preferred one.

8 Conclusion

This study proposed an advanced decision-making framework, the Complex Type-2 Intuitionistic Fuzzy Analytic Hierarchy Process (CIT2F-AHP), to address the challenge of sustainable material selection in the construction industry under uncertainty. By integrating the analytic hierarchy process with complex type-2 intuitionistic fuzzy logic, the model effectively captured both the magnitude and phase information of expert judgments, offering a richer representation of hesitation, ambiguity, and multidimensional uncertainty.

The proposed CIT2F-AHP method was applied to evaluate four low-carbon cement and concrete alternatives—carbon-cured concrete, geopolymer concrete, alkali-activated binders, and limestone calcined clay cement—across comprehensive environmental, technical, economic, social, and regulatory criteria. The findings revealed that environmental and technical factors are the most influential in determining sustainability performance, while geopolymer concrete emerged as the most sustainable alternative among those analyzed. This aligns with current trends emphasizing reduced carbon emissions and the use of industrial by-products in construction materials.

Furthermore, the comparative analysis using different weighted averaging aggregation operators (Aczel–Alsina, Dombi, and Einstein) confirmed the robustness and consistency of the proposed approach, as all operators produced the same ranking order with minor quantitative variations. This demonstrates the stability and flexibility of the CIT2F-AHP framework across different aggregation settings.

Overall, the CIT2F-AHP method provides both methodological and practical contributions. Methodologically, it extends the capability of fuzzy-based MCDM models by incorporating complex type-2 intuitionistic information. Practically, it offers a powerful and reliable decision-support tool for engineers, researchers, and policymakers engaged in sustainable construction material selection. Future studies could extend this framework by integrating dynamic weighting mechanisms, real-time data from digital construction environments, or coupling it with life cycle assessment and circular economy indicators to further enhance decision quality in sustainability-focused applications.

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