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A note on Properties of Temporal Intuitionistic Fuzzy Sets

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Abstract

Fuzzy Sets (FSs) are generalized from the notion of classical (crisp) sets (Zadeh 1965) [8]. A fuzzy subset A of E can be characterized with a membership function $\mu_A : E \to [0, 1]$. Krassimir T. Atanassov further generalized fuzzy sets into Intuitionistic Fuzzy Sets (IFSs) in which non-membership function $\nu_A : E \to [0, 1]$ is also considered [7]. Further he extended IFSs into Temporal Intuitionistic Fuzzy Sets (TIFSs) in which time-moments are also taken into consideration. TIFSs are useful in time-based mathematical modeling. In this paper, some properties of TIFSs are discussed.

Keywords: IFS, TIFS, closure, interior, tautological sets, max-min implications, level sets, $G^*_{\alpha,\beta}$, $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$.
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1 Introduction

Time is an important feature in our real world. In many application domains, such as medicine, history, traffic, cadaster, weather, criminal and financial, time is needed to record that when the object exists or changes and when the event happens. There are numerous works to show time's importance in the area of temporal geographical information system [1], spatio-temporal data bases [2], temporal data bases [3,4] and temporal reasoning [5]. It is well-known that time is monotone and time is a fundamental issue for modeling dynamic information. Time has two sorts of forms: time instant and time interval. Instant represents a point in time and interval represents the range between two points in time. Usually, a temporal model can be based on either or on both of them. Consider time instant as time interval if the granularity of time dimension is sufficiently increased [6] or the closed interval's starting point is equal to the ending point. And now, the vast majority of related work on modeling time concentrates on modeling crisp and definite temporal information [6]. However, in fact temporal information is not crisp, but is uncertain and vague. For example, when the rain begins is not definite, because we do not evaluate the concrete time. Time information in the process of rain is often uncertain. It is only approximate. So, fuzzy sets are used to represent this time information.

The advent of the concept of "Fuzzy Set" introduced by Lotfi A. Zadeh in 1965 is one of the most important events in Mathematics. It is not only an abstract mathematical object, extending J. Lukasiewicz's idea for many-valued logic, but also during the last thirty years, one of the most used mathematical concepts in practice. For these reasons, fuzzy set is an object of different extensions and modifications. One of them is the concept of IFSs.

The IFS theory is based on the extensions of corresponding definitions of fuzzy set objects into definitions of new objects and their properties.

Definition 1.1 *An IFS* A *in* E *is defined as an object of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in E\}$ *where the functions* $\mu_A : E \to [0,1]$ *and* $\nu_A : E \to [0,1]$ *define the degree of membership and the degree of non-membership of the element* $x \in E$*, respectively, and for every* $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Obviously, each fuzzy set may be written as $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in E\}$.

Definition 1.2 *The closure and interior over the IFS A, denoted as* $C(A)$ *and* $I(A)$*, are defined by* $C(A) = \{\langle x, \max \mu_A(x), \min \nu_A(x)\rangle : x \in E\}, I(A) = \{\langle x, \min \mu_A(x), \max \nu_A(x)\rangle : x \in E\}$.

Definition 1.3 *For every IFS A, the necessity operator and possibility operator are defined as* $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in E\}$, $A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle : x \in E\}$.

The purpose of generalizing these sets is to deal with imprecise, uncertain, vague information. Time modeling is one such situation in many application domains. IFSs are further classified into many types based on the domains of membership and non-membership functions. One of these types is TIFS where the "time-moments" play significant role. However, the information is often not crisp but uncertain, subjective and vague. In this paper, some properties of TIFSs are defined and analyzed. This paper is organized as follows. Section 2 provides basic concepts concerned with TIFS. Section 3 deals with some properties of TIFS. Also operators like max-min implication operator, level operators, $G^*_{\alpha,\beta}$, $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ are defined. Section 4 concludes the paper and provides some outlook about future work.

2 Temporal Intuitionistic Fuzzy Sets

Let E be an universe and T be a non-empty set. The elements of T are called "time-moments".

Definition 2.1 *A temporal intuitionistic fuzzy set is an object of the form* $A(T) = \{\langle (x,t), \mu_A(x,t), \nu_A(x,t)\rangle : (x,t) \in E \times T\}$ *where*

(a) $A \subset E$ *is a fixed set.*

(b) $0 \le \mu_A(x,t) + \nu_A(x,t) \le 1$ *for every* $(x,t) \in E \times T$.
(c) $\mu_A(x,t)$ *and* $\nu_A(x,t)$ *are the degrees of membership and non-membership, respectively, of the element* $x \in E$ *at the time-moment* $t \in T$.

Note 2.1 *1.Obviously, every ordinary IFS can be regarded as TIFS for which* T *is a singleton set.*
2.All operations and operators on the IFSs can be defined for the TIFSs.

2.1 Notations

Throughout this paper, the following notations are used.

$\mu_A(x,t) = \mu_A$; $\nu_A(x,t) = \nu_A$; $\overline{\mu}_A(x,t) = \overline{\mu}_A$; $\overline{\nu}_A(x,t) = \overline{\nu}_A$;
$\mu_{A(T')}(x,t) = \mu_{A(T')}$; $\nu_{A(T')}(x,t) = \nu_{A(T')}$; $\mu_{A(T'')}(x,t) = \mu_{A(T'')}$;
$\nu_{A(T'')}(x,t) = \nu_{A(T'')}$; $\overline{\mu}_{A(T')}(x,t) = \overline{\mu}_{A(T')}$; $\overline{\nu}_{A(T')}(x,t) = \overline{\nu}_{A(T')}$;
$\overline{\mu}_{A(T'')}(x,t) = \overline{\mu}_{A(T'')}$; $\overline{\nu}_{A(T'')}(x,t) = \overline{\nu}_{A(T'')}$.

2.2 Basic operators on TIFSs

Consider two TIFSs, $A(T') = \{\langle (x,t), \mu_A, \nu_A \rangle : (x,t) \in E \times T'\}$ and $B(T'') = \{\langle (x,t), \mu_A, \nu_A \rangle : (x,t) \in E \times T''\}$.

Definition 2.2 *Consider two TIFSs,* $A(T')$ *and* $B(T'')$*.The basic operations namely intersection,union and complement are defined as follows:*

$A(T') \cap B(T'') = \{\langle (x,t), \min(\overline{\mu}_A, \overline{\mu}_B), \max(\overline{\nu}_A, \overline{\nu}_B) \rangle : (x,t) \in E \times (T' \cup T'')\}$
$A(T') \cup B(T'') = \{\langle (x,t), \max(\overline{\mu}_A, \overline{\mu}_B), \min(\overline{\nu}_A, \overline{\nu}_B) \rangle : (x,t) \in E \times (T' \cup T'')\}$
$Comp\,(A(T')) = \{\langle (x,t), \nu_A, \mu_A \rangle : (x,t) \in E \times T'\}$
where

$$\overline{\mu}_A = \begin{cases} \mu_A & if \quad t \in T' \\ 0 & if \quad t \in T'' - T' \end{cases} \qquad \overline{\nu}_A = \begin{cases} \nu_A & if \quad t \in T' \\ 1 & if \quad t \in T'' - T' \end{cases}$$

and

$$\overline{\mu}_B = \begin{cases} \mu_B & if \quad t \in T'' \\ 0 & if \quad t \in T' - T'' \end{cases} \qquad \overline{\nu}_B = \begin{cases} \nu_B & if \quad t \in T'' \\ 1 & if \quad t \in T' - T'' \end{cases}$$

Definition 2.3 *The "closure" and "interior" operators* C^* *and* I^* *respectively, over a TIFS* $A(T)$ *are defined as*

$$C^*(A(T)) = \{\langle (x,t), \max_{t\in T} \mu_{A(T)}, \min_{t\in T} \nu_{A(T)} \rangle : (x,t) \in E \times T\}$$
$$I^*(A(T)) = \{\langle (x,t), \min_{t\in T} \mu_{A(T)}, \max_{t\in T} \nu_{A(T)} \rangle : (x,t) \in E \times T\}$$

Theorem 2.1 *For every two TIFSs* $A(T')$ *and* $B(T'')$*, the following results are true, if* $T' \cap T'' = \phi$.

$$C^*(A(T') \cap B(T'')) = C^*(A(T')) \cap C^*(B(T'') \tag{2.1}$$
$$C^*(A(T') \cup B(T'')) = C^*(A(T')) \cup C^*(B(T'') \tag{2.2}$$

Proof :

Given $T' \cap T'' = \phi$. Let $T = T' \cup T''$.

$$
\begin{aligned}
\text{Consider,}\quad & C^*(A(T') \cap B(T'')) \\
&= C^*(\{\langle (x,t), \min(\mu_A, 0), \max(\nu_A, 1)\rangle : (x,t) \in E \times T\}) \text{ if } t \in T' \\
&= \{\langle (x,t), 0, 1\rangle\rangle : (x,t) \in E \times T\} \text{ if } t \in T' \\
\text{and}\quad & C^*(A(T')) \cap C^*(B(T'')) \\
&= \{\langle (x,t), \min(\max_{t\in T'} \overline{\mu}_{A(T')}, 0), \max(\min_{t\in T'} \overline{\nu}_{A(T')}, 1)\rangle : (x,t) \in E \times T\} \\
&\qquad\qquad\qquad\qquad\qquad \text{if } t \in T' \\
&= \{\langle (x,t), 0, 1\rangle : (x,t) \in E \times T\} \text{ if } t \in T'
\end{aligned}
$$

It is analogous to prove that $C^*(A(T') \cap B(T'')) = C^*(A(T')) \cap C^*(B(T'')$ if $t \in T''$. Similarly, (2.2) is proved.

Definition 2.4 *Given TIFS $A(T)$ and for every $\alpha, \beta \in [0,1]$, the operators $P_{\alpha,\beta}(A(T))$ and $Q_{\alpha,\beta}(A(T))$ are defined by*

$$P_{\alpha,\beta}(A(T)) = \{\langle (x,t), \max(\alpha, \mu_A), \min(\beta, \nu_A)\rangle : (x,t) \in E \times T\},$$
$$Q_{\alpha,\beta}(A(T)) = \{\langle (x,t), \min(\alpha, \mu_A), \max(\beta, \nu_A)\rangle : (x,t) \in E \times T\}.$$

3 Properties of temporal intuitionistic fuzzy sets

Following the definitions of an IFS and TIFS, some properties of TIFS are defined in this section.

3.1 Max-min implication opeartor

Definition 3.1 *Let $A(T')$ and $B(T'')$ be any two TIFSs, The "Max-min implication", denoted by $A(T') \mapsto B(T'')$, is defined by*

$$A(T') \mapsto B(T'') = \{\langle (x,t), \max(\overline{\nu}_A, \overline{\mu}_B), \min(\overline{\mu}_A, \overline{\nu}_B)\rangle : (x,t) \in E \times (T' \cup T'')\}$$

where $\overline{\mu}_A, \overline{\nu}_A, \overline{\mu}_B$ and $\overline{\nu}_B$ are defined as in Definition 2.2 .

Proposition 3.1 *For any three TIFSs $A(T')$, $B(T'')$ and $C(T''')$, the following relations hold.*

(i) $((A(T') \cap B(T'')) \mapsto C(T''')) \supset ((A(T') \mapsto C(T''')) \cap (B(T'') \mapsto C(T'''))$

(ii) $((A(T') \cup B(T'')) \mapsto C(T''')) \subset ((A(T') \mapsto C(T''')) \cup (B(T'') \mapsto C(T'''))$

(iii) $((A(T') \cap B(T'')) \mapsto C(T''')) = ((A(T') \mapsto C(T''')) \cup (B(T'') \mapsto C(T'''))$

(iv) $((A(T') \cup B(T'')) \mapsto C(T''')) = ((A(T') \mapsto C(T''')) \cap (B(T'') \mapsto C(T'''))$

Proof : The proof is obivious from Theorem 2.1 and the Definition 3.1 .

Definition 3.2 *A TIFS A is said to be Temporal Intuitionistic Fuzzy Tautological Set (TIFTS) iff $\mu_A \geq \nu_A$ holds for every $(x,t) \in E \times T$.*

Definition 3.3 *A TIFS A is said to be Temporal Tautological Set (TTS) iff $\mu_A = 1, \nu_A = 0$ holds for every $(x,t) \in E \times T$.*

3.2 Temporal intuitionistic fuzzy sets of certain level

Definition 3.4 *(α,β)- level sets, generated by a TIFS $A(T)$, where $\alpha,\beta \in [0,1]$ are fixed numbers such that $\alpha+\beta \leq 1$, are defined as*

$$N_{\alpha,\beta}(A(T)) = \{\langle (x,t), \mu_A, \nu_A\rangle : (x,t) \in E \times T, \mu_A \geq \alpha, \nu_A \leq \beta\}.$$

Two other sets are also introduced, related to TIFS. Let $\alpha \in [0,1]$ is a fixed number.

Definition 3.5 *The set $N_\alpha(A(T)) = \{\langle (x,t), \mu_A, \nu_A\rangle : (x,t) \in E \times T, \mu_A \geq \alpha\}$ is called a level set of membership α generated by $A(T)$.*

Definition 3.6 *The set $N^\beta(A(T)) = \{\langle (x,t), \mu_A, \nu_A\rangle : (x,t) \in E \times T, \nu_A \leq \beta\}$ is called a level set of non-membership β generated by $A(T)$.*

Theorem 3.2 *For every TIFS $A(T)$ and for every $\alpha,\beta \in [0,1]$, such that $\alpha+\beta \leq 1$, there holds $N_{\alpha,\beta}(A(T)) \subset \left\{ \begin{matrix} N^\beta(A(T)) \\ N_\alpha(A(T)) \end{matrix} \right\} \subset A(T)$, where the $\subset$ relation is in the set-theoretical sense.*

3.3 Operator $G^*_{\alpha,\beta}$

Definition 3.8 Given an TIFS $A(T)$ and for every $\alpha,\beta \in [0,1]$, the operator $G^*_{\alpha,\beta}$ is defined by $G^*_{\alpha,\beta}(A(T)) = \{\langle (x,t), \alpha.\mu_A, \beta.\nu_A\rangle : (x,t) \in E \times T\}$.

Obviously, $G^*_{1,1}(A(T)) = A(T)$ and $G^*_{0,0}(A(T)) = \{\langle (x,t), 0, 0\rangle : (x,t) \in E \times T\}$

Theorem 3.3 *For every TIFS $A(T)$ and for every $\alpha,\beta,\gamma \in [0,1]$,*

(i) $G^*_{\alpha,\beta}(A(T))$ *is a TIFS*

(ii) *If* $\alpha \leq \gamma$, *then* $G^*_{\alpha,\beta}(A(T)) \subset G^*_{\gamma,\beta}(A(T))$

(iii) *If* $\beta \leq \gamma$, *then* $G^*_{\alpha,\beta}(A(T)) \supset G^*_{\alpha,\gamma}(A(T))$

(iv) *If* $\gamma,\lambda \in [0,1]$, *then*
$G^*_{\alpha,\beta}(G^*_{\gamma,\lambda}(A(T))) = G^*_{\alpha.\gamma,\beta.\lambda}(A(T)) = G^*_{\gamma,\lambda}(G^*_{\alpha,\beta}(A(T)))$

(v) $G^*_{\alpha,\beta}(C^*(A(T))) = C^*(G^*_{\alpha,\beta}(A(T)))$

(vi) $G^*_{\alpha,\beta}(I^*(A(T))) = I^*(G^*_{\alpha,\beta}(A(T)))$

(vii) $Comp\,(G^*_{\alpha,\beta}(Comp\,(A(T)))) = G^*_{\alpha,\beta}(A(T))$

Proof :

$$\text{Consider,}\quad G^*_{\alpha,\beta}(G^*_{\gamma,\lambda}(A(T))) = \{\langle (x,t), \alpha.\gamma.\mu_A, \beta.\lambda.\nu_A\rangle : (x,t) \in E \times T\} = G^*_{\alpha.\gamma,\beta.\lambda}(A(T))$$

Similarly, $G^*_{\alpha.\gamma,\beta.\lambda}(A(T)) = G^*_{\gamma,\lambda}(G^*_{\alpha,\beta}(A(T)))$.

$$\text{Consider,}\quad G^*_{\alpha,\beta}(C^*(A(T))) = \{\langle (x,t), \alpha.\max_{t\in T}\mu_{A(T)}, \beta.\min_{t\in T}\nu_{A(T)}\rangle : (x,t) \in E \times T\} = C^*(G^*_{\alpha,\beta}(A(T)))$$

Similarly, (i), (ii), (iii), (vi) and (vii) are proved.

Theorem 3.4 *For every two TIFSs $A(T')$ and $B(T'')$ and for every* $\alpha, \beta, \gamma \in [0,1]$, *(i)* $G^*_{\alpha,\beta}(A(T') \cap B(T'')) = G^*_{\alpha,\beta}(A(T')) \cap G^*_{\alpha,\beta}(B(T''))$,
(ii) $G^*_{\alpha,\beta}(A(T') \cup B(T'')) = G^*_{\alpha,\beta}(A(T')) \cup G^*_{\alpha,\beta}(B(T''))$

Proof : Consider,

$$\begin{aligned} G^*_{\alpha,\beta}(A(T') \cap B(T'')) &= \{\langle (x,t), \alpha.\min(\overline{\mu}_A, \overline{\mu}_B), \beta.\max(\overline{\nu}_A, \overline{\nu}_B)\rangle : (x,t) \in E \times (T' \cup T'')\} \\ &= G^*_{\alpha,\beta}(A(T')) \cap G^*_{\alpha,\beta}(B(T'')) \end{aligned}$$

Similarly, (ii) is proved.

3.4 Relation between Operators defined over TIFSs

Theorem 3.5 *For every two TIFSs $A(T')$ and $B(T'')$ and for every* $\alpha, \beta, \gamma \in [0,1]$,
(i) $G^*_{\alpha,\beta}(A(T') \mapsto B(T'')) = G^*_{\beta,\alpha}(A(T')) \mapsto G^*_{\alpha,\beta}(B(T''))$
(ii) $N_{\alpha,\beta}(A(T') \mapsto B(T'')) = N_{\alpha,\beta}(A(T')) \mapsto N_{\alpha,\beta}(B(T''))$
(iii) $N_{\alpha}(A(T') \mapsto B(T'')) = N_{\alpha}(A(T')) \mapsto N_{\alpha}(B(T''))$
(iv) $N^{\alpha}(A(T') \mapsto B(T'')) = N^{\alpha}(A(T')) \mapsto N^{\alpha}(B(T''))$

Proof : Consider,

$$\begin{aligned} & G^*_{\alpha,\beta}(A(T') \mapsto B(T'')) \\ &= \{\langle (x,t), \alpha.\max(\overline{\nu}_A, \overline{\mu}_B), \beta.\min(\overline{\mu}_A, \overline{\nu}_B)\rangle : (x,t) \in E \times (T' \cup T'')\} \\ &= \{\langle (x,t), \max(\alpha.\overline{\nu}_A, \alpha.\overline{\mu}_B), \min(\beta.\overline{\mu}_A, \beta.\overline{\nu}_B)\rangle : (x,t) \in E \times (T' \cup T'')\} \\ &= G^*_{\beta,\alpha}(A(T')) \mapsto G^*_{\alpha,\beta}(B(T'')) \end{aligned}$$

Similarly, (ii), (iii) and (iv) are proved.

Theorem 3.6 *For every two TIFSs $A(T')$ and $B(T'')$ and for every* $\alpha, \beta \in [0,1]$, *such that* $\alpha + \beta \leq 1$,

$$\begin{aligned} N_{\alpha,\beta}(P_{\alpha,\beta}(A(T))) &= P_{\alpha,\beta}(N_{\alpha,\beta}(A(T))) \\ N_{\alpha}(P_{\alpha,\beta}(A(T))) &= P_{\alpha,\beta}(N_{\alpha}(A(T))) \\ N^{\alpha}(P_{\alpha,\beta}(A(T))) &= P_{\alpha,\beta}(N^{\alpha}(A(T))) \end{aligned}$$

4 Conclusion

In this paper, some basic properties of the operators like max-min implication operator, level set operator, some other operators and their relationship are analyzed. Further, the authors propose to extend the concepts of Intuitionistic Fuzzy Sets into Temporal Intuitionistic Fuzzy Sets and to study the applications of these extensions in real world problems involving time-based mathematical modeling.

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