Notes on Intuitionistic Fuzzy Sets
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Multi-dimensional modal topological structures, extended intuitionistic fuzzy index matrices and multi-person multi-criteria decision making procedures

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Abstract: The concept of multi-dimensional modal topological structures was introduced in the middle of 2025. One of the special forms of these structures are the intuitionistic fuzzy multi-dimensional modal topological structures. Here, we give examples of intuitionistic fuzzy multi-dimensional modal topological structures, related to extended intuitionistic fuzzy index matrices and to multi-person multi-criteria decision making procedures with intuitionistic fuzzy evaluations of the experts, who have intuitionistic fuzzy scores and use criteria having intuitionistic fuzzy evaluations.

Keywords: Intuitionistic fuzzy extended index matrix, Intuitionistic fuzzy set, Intuitionistic fuzzy multi-dimensional modal topological structure, Multi-person multi-criteria decision making procedure.

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1 Introduction

The paper is a continuation of [8]. In it, examples of Modal Topological Structures (MTSs, see, e.g., [5]) with elements from the area of intuitionistic fuzziness. Here, new examples related to the extension of the MTSs called Multi-Dimensional MTSs (MDMTSs, see, [7]).

All notations, related to Intuitionsitc Fuzzy Sets (IFSs) can be find e.g. in [1]) and to Index Matrices (IMs) can be seen in [3].

In Section 2 we give short remarks on MDMTSs, Intuitionistic Fuzzy MDMTSs (IFMDMTSs) and Extended Intuitionistic Fuzzy IMs (EIFIMs). In Section 3 we give 16 IFMDMTSs based on EIFIMs. Finally, in the Conclusion we discuss some other possible applications of the IFMDMTSs.

2 Preliminaries

Below, we will give some definitions related to MDMTSs and EIFIMs, but the definitions related to the standard IFSs and operations and relations over them will be omitted.

2.1 Definition of the Multi-Dimensional Modal Topological Structures

In this section, following and modifying [7], we will define the concept of a MDMTS, or, when the number of the dimensions is determined, e.g., as $s \geq 1$ for some natural number s – as s-dimensional MTS (s-DMTS).

Let for the arbitrary set Z we define

$$\mathcal{P}(Z) = \{Y | Y \subseteq Z\}.$$

Let us have the fixed sets $X_1, \ldots, X_s$, where the natural number $s \geq 1$. Let

$$\langle \mathcal{P}(X_1), \mathcal{E}, \zeta, *, \eta \rangle, \quad \ldots, \quad \langle \mathcal{P}(X_s), \mathcal{E}, \zeta, *, \eta \rangle$$

be given, where for each r $(1 \leq r \leq s)$, the object $\langle \mathcal{P}(X_r), \mathcal{E}, \zeta, *, \eta \rangle$ is a MTS.

Now, following the definition of a topological structure from [10] and the definition for a MTS from [5], we construct the object

$$\langle \mathcal{P}(X_1 \times \cdots \times X_s), \mathcal{E}, \zeta_s, *, \eta_s \rangle$$

that will denote as an s-Dimensional(χ, η_s)-Modal(φ, ζ_s)-Topological Structure (s-D(χ, η_s)-M(φ, ζ_s)-TS) or (more general) multi-Dimensional(χ, η_s)-Modal(φ, ζ_s)-Topological Structure (mD(χ, η_s)-M(φ, ζ_s)-TS), where for $A, B \in \mathcal{P}(X_1 \times \cdots \times X_s)$:

- $\zeta_s : (X_1 \times \cdots \times X_s) \times (X_1 \times \cdots \times X_s) \to X_1 \times \cdots \times X_s$ is an associative operation, being a generation of the function ζ in the following, for example, sense: if

$$\zeta_2(a_1, a_2) = \zeta(a_1, a_2) = a_1 a_2 = \prod_{i=1}^{2} a_i,$$

then

$$\zeta_s(a_1, \ldots, a_s) = \prod_{i=1}^{s} a_i;$$

- $\mathcal{E}$ is a topological operator and if it is from closure (cl) type, then it must satisfy the conditions

  Ct1 $\mathcal{E}(A\zeta_s B) = \mathcal{E}(A)\zeta_s\mathcal{E}(B)$,

  Ct2 $A \subseteq \mathcal{E}(A)$,

  Ct3 $\mathcal{E}(\mathcal{E}(A)) = \mathcal{E}(A)$,

  Ct4 $\mathcal{E}(O_1 \times \cdots \times O_s) = O_1 \times \cdots \times O_s$,

  where O_r is the minimal element of $\mathcal{P}(X_r)$ for $1 \leq r \leq s$, $O = O_1 \times \cdots \times O_s$ is the minimal element of the set $\mathcal{P}(X_1 \times \cdots \times X_s)$, and if it is from interior (in) type, then the topological operator $\mathcal{E}$ must satisfy the conditions

  It1 $\mathcal{E}(A\zeta_s B) = \mathcal{E}(A)\zeta_s\mathcal{E}(B)$,

  It2 $\mathcal{E}(A) \subseteq A$,

  It3 $\mathcal{E}(\mathcal{E}(A)) = \mathcal{E}(A)$,

  It4 $\mathcal{E}(X_1 \times \cdots \times X_s) = X_1 \times \cdots \times X_s$;

- $\eta_s : (X_1 \times \cdots \times X_s) \times (X_1 \times \cdots \times X_s) \to X_1 \times \cdots \times X_s$ is an associative operation being a generation of the function η in the above, for example, sense: if

  $$\eta_2(a_1, a_2) = \eta(a_1, a_2) = \min(a_1, a_2) = \min_{1 \leq i \leq 2} a_i,$$

  then

  $$\eta_s(a_1, \ldots, a_s) = \min_{1 \leq i \leq s} a_i;$$

- * is a modal operator and if it is from closure (cl) type, then it must satisfy the conditions

  Cm1 $*(A\eta_s B) = *A\eta_s * B$,

  Cm2 $A \subseteq *A$,

  Cm3 $* * A = *A$,

  and if it is from interior (in) type, then it must satisfy the conditions

  Im1 $*(A\eta_s B) = *A\eta_s * B$,

  Im2 $*A \subseteq A$,

  Im3 $* * A = *A$;

- $\chi, \varphi \in \{cl, in\}$,

- both operators (topological and modal) must satisfy the condition

  $$*\mathcal{E}(A) = \mathcal{E}(*A). \qquad (*)$$

We can see immediately that when $s = 1$, $\zeta_2 = \zeta, \eta_2 = \eta$, we obtain the definition of MTS from [5].

If we add as an additional conditions the operations ζ_s and η_s to be commutative and for every i, j $(1 \leq i < j \leq s)$ and for each $A_1 \times \cdots \times A_s \in \mathcal{P}(X_1 \times \cdots \times X_s)$:

$$\mathcal{E}(A_1 \times \cdots A_i \times \cdots \times A_j \times \cdots \times A_s) = \mathcal{E}(A_1 \times \cdots A_j \times \cdots \times A_i \times \cdots \times A_s), \qquad (**)$$

then the MDMTS will be called a commutative MDMTS.

When $E = E_1 \times \cdots \times E_s$, where $E_1, \ldots, E_s$ are fixed universes, and for each its subset A we can construct the IFS (more exactly - multi- (or s-)dimensional IFS, see [1])

$$A^* = \{\langle x, \mu_A(x_1, \ldots, x_s), \nu_A(x_1, \ldots, x_s)\rangle \mid \langle x_1, \ldots, x_s\rangle \in E\},$$

then

$$\langle \mathcal{P}(E_1 \times \cdots \times E_s), \mathcal{E}, \zeta_s, *, \eta_s\rangle$$

is an IFMDMTS.

2.2 Short remarks on EIFIMs

Let I be a fixed set. By EIFIM with index sets K and L $(K, L \subset I)$, we denote the object:

$$[K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}]$$

$$\equiv \begin{array}{c|ccccc} & l_1, \langle \alpha_1^L, \beta_1^L\rangle & \cdots & l_j, \langle \alpha_j^L, \beta_j^L\rangle & \cdots & l_n, \langle \alpha_n^L, \beta_n^L\rangle \\ \hline k_1, \langle \alpha_1^K, \beta_1^K\rangle & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1}\rangle & \cdots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j}\rangle & \cdots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_i, \langle \alpha_i^K, \beta_i^K\rangle & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1}\rangle & \cdots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle & \cdots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_m, \langle \alpha_m^K, \beta_m^K\rangle & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1}\rangle & \cdots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j}\rangle & \cdots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n}\rangle \end{array},$$

where

$$\begin{aligned} K &= \{k_1, k_2, ..., k_m\}, \\ L &= \{l_1, l_2, ..., l_n\}; \\ K^* &= \{\langle k_i, \alpha_i^K, \beta_i^K\rangle | k_i \in K\} = \{\langle k_i, \alpha_i^K, \beta_i^K\rangle | 1 \leq i \leq m\}, \\ L^* &= \{\langle l_j, \alpha_j^L, \beta_j^L\rangle | l_j \in L\} = \{\langle l_j, \alpha_j^L, \beta_j^L\rangle | 1 \leq j \leq n\} \end{aligned}$$

are IFSs and for every $1 \leq i \leq m, 1 \leq j \leq n$:

$$\begin{aligned} \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} &\in [0, 1], \\ \alpha_i^K, \beta_i^K, \alpha_i^K + \beta_i^K &\in [0, 1], \\ \alpha_j^L, \beta_j^L, \alpha_j^L + \beta_j^L &\in [0, 1]. \end{aligned}$$

For the EIFIMs

$$A = [K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\},$$
$$B = [P^*, Q^*, \{\langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle\}],$$

we will introduce only these operations, relations and operators that are necessary for the current research. The remaining ones can be found in [3].

Addition-(max,min)

$$A \oplus_{(\max,\min)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$\begin{aligned} T^* &= K^* \cup P^* = \{\langle t_u, \langle \alpha_u^K, \beta_u^K \rangle \vee \langle \alpha_u^P, \beta_u^P \rangle \rangle | t_u \in K \cup P\}, \\ &= \{\langle t_u, \alpha_u^T, \beta_u^T \rangle | t_u \in K \cup P\} \\ V^* &= L^* \cup Q^* = \{\langle v_w, \langle \alpha_w^L, \beta_w^L \rangle \vee \langle \alpha_w^Q, \beta_w^Q \rangle \rangle | v_w \in L \cup Q\}, \\ &= \{\langle v_w, \alpha_w^V, \beta_w^V \rangle | v_w \in L \cup Q\}, \end{aligned}$$

where

$$\alpha_u^T = \begin{cases} \alpha_i^K, & \text{if } t_u \in K - P \\ \alpha_r^P, & \text{if } t_u \in P - K \\ \max(\alpha_i^K, \alpha_r^P), & \text{if } t_u \in K \cap P \end{cases}, \qquad \beta_w^V = \begin{cases} \beta_j^L, & \text{if } v_w \in L - Q \\ \beta_s^Q, & \text{if } t_w \in Q - L \\ \min(\beta_j^L, \beta_s^Q), & \text{if } t_w \in L \cap Q \end{cases},$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \quad \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \quad \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle \max(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \quad \min(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \quad \text{and } v_w = l_j = q_s \in L \cap Q \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

Addition-(min,max)

$$A \oplus_{(\min,\max)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where $T^*, V^*, \alpha_u^T, \beta_w^V$, have the above forms and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \quad \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \quad \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle \min(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \quad \max(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \quad \text{and } v_w = l_j = q_s \in L \cap Q \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

Termwise multiplication-(max,min)

$$A \otimes_{(\max,\min)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$\begin{aligned} T^* &= K^* \cap P^* = \{\langle t_u, \langle \alpha_u^K, \beta_u^K \rangle \& \langle \alpha_u^P, \beta_u^P \rangle \rangle | t_u \in K \cup P\}, \\ &= \{\langle t_u, \alpha_u^T, \beta_u^T \rangle | t_u \in K \cup P\} \\ V^* &= L^* \cap Q^* = \{\langle v_w, \langle \alpha_w^L, \beta_w^L \rangle \& \langle \alpha_w^Q, \beta_w^Q \rangle \rangle | v_w \in L \cup Q\}, \\ &= \{\langle v_w, \alpha_w^V, \beta_w^V \rangle | v_w \in L \cup Q\}, \end{aligned}$$

$$\begin{aligned} \alpha_u^T &= \min(\alpha_i^K, \alpha_r^P), \text{ for } t_u = k_i = p_r \in K \cap P, \\ \beta_w^V &= \min(\beta_j^L, \beta_s^Q), \text{ for } v_w = l_j = q_s \in L \cap Q \end{aligned}$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \max(\mu_{k_i,l_j}, \rho_{p_r,q_s}), \min(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle.$$

Termwise multiplication-(min,max)

$$A \otimes_{(\min,\max)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where $T^*, V^*, \alpha_u^T, \beta_w^V$, have the above forms and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \min(\mu_{k_i,l_j}, \rho_{p_r,q_s}), \max(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle.$$

Let

$$\begin{aligned} &K^* \subset P^* \text{ iff } (K \subset P) \ \& \ (\forall k_i = p_i \in K)((\alpha_i^K < \alpha_i^P) \ \& \ (\beta_i^K > \beta_i^P)), \\ &K^* \subseteq P^* \text{ iff } (K \subseteq P) \ \& \ (\forall k_i = p_i \in K)((\alpha_i^K \le \alpha_i^P) \ \& \ (\beta_i^K \ge \beta_i^P)). \end{aligned}$$

Let the two EIFIMs A and B be given. We shall introduce the following definitions for relations over EIFIMs, where $\subset$ and $\subseteq$ denote the relations *"strong inclusion"* and *"weak inclusion"*, respectively. We must mention that in the present form, they are new and extend existing definitions given in [3].

The strict relation "inclusion about dimension" is

$$\begin{aligned} A \subset_d B \quad &\text{iff } (((K^* \subset P^*) \ \& \ (L^* \subset Q^*)) \ \vee \ ((K^* \subseteq P^*) \ \& \ (L^* \subset Q^*)) \\ &\vee((K^* \subset P^*) \ \& \ (L^* \subseteq Q^*))) \ \& \ (\forall k \in K)(\forall l \in L)(((\langle \alpha_k^K, \beta_k^K \rangle = \langle \alpha_k^P, \beta_k^P \rangle) \\ &\&(\langle \alpha_l^L, \beta_l^L \rangle = \langle \alpha_l^Q, \beta_l^Q \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle = \langle \rho_{k,l}, \sigma_{k,l} \rangle). \end{aligned}$$

The non-strict relation "inclusion about dimension" is

$$\begin{aligned} A \subseteq_d B \quad &\text{iff } (K^* \subseteq P^*) \ \& \ (L^* \subseteq Q^*) \ \& \ (\forall k \in K)(\forall l \in L) \\ &(((\langle \alpha_k^K, \beta_k^K \rangle = \langle \alpha_k^P, \beta_k^P \rangle) \ \&(\langle \alpha_l^L, \beta_l^L \rangle = \langle \alpha_l^Q, \beta_l^Q \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle = \langle \rho_{k,l}, \sigma_{k,l} \rangle))). \end{aligned}$$

The strict relation "inclusion about value" is

$$\begin{aligned} A \subset_v B \quad &\text{iff } (K^* = P^*) \ \& \ (L^* = Q^*) \ \& \ (\forall k \in K)(\forall l \in L) \\ &(((\langle \alpha_k^K, \beta_k^K \rangle = \langle \alpha_k^P, \beta_k^P \rangle) \ \&(\langle \alpha_l^L, \beta_l^L \rangle = \langle \alpha_l^Q, \beta_l^Q \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle < \langle \rho_{k,l}, \sigma_{k,l} \rangle))). \end{aligned}$$

The non-strict relation "inclusion about value" is

$$A \subseteq_v B \quad \text{iff } (K^* = P^*) \ \& \ (L^* = Q^*) \ \& \ (\forall k \in K)(\forall l \in L)$$
$$((\langle \alpha_k^K, \beta_k^K \rangle = \langle \alpha_k^P, \beta_k^P \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle \leq \langle \rho_{k,l}, \sigma_{k,l} \rangle)).$$

The strict relation "inclusion about indices" is

$$A \subset_i B \quad \text{iff } (((K^* \subset P^*) \ \& \ (L^* \subset Q^*)) \ \vee \ ((K^* \subseteq P^*) \ \& \ (L^* \subset Q^*))$$
$$\vee((K^* \subset P^*) \ \& \ (L^* \subseteq Q^*))) \ \& \ (\forall k \in K)(((\langle \alpha_k^K, \beta_k^K \rangle < \langle \alpha_k^P, \beta_k^P \rangle)$$
$$\&(\langle \alpha_l^L, \beta_l^L \rangle < \langle \alpha_l^Q, \beta_l^Q \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle = \langle \rho_{k,l}, \sigma_{k,l} \rangle)).$$

The non-strict relation "inclusion about indices" is

$$A \subseteq_i B \quad \text{iff } (K^* \subseteq P^*) \ \& \ (L^* \subseteq Q^*) \ \& \ (\forall k \in K)(\forall l \in L)$$
$$((\langle \alpha_k^K, \beta_k^K \rangle \leq \langle \alpha_k^P, \beta_k^P \rangle) \ \&(\langle \alpha_l^L, \beta_l^L \rangle \leq \langle \alpha_l^Q, \beta_l^Q \rangle) \ \& \ (\langle \mu_{k,l}, \nu_{k,l} \rangle = \langle \rho_{k,l}, \sigma_{k,l} \rangle)).$$

Now, we can introduce combinations of these relations in the forms

$$A \subset_{d,v} B \quad \text{iff } (A \subset_d B) \ \& \ (A \subset_v B),$$
$$A \subseteq_{d,v} B \quad \text{iff } (A \subseteq_d B) \ \& \ (A \subseteq_v B),$$
$$A \subset_{d,i} B \quad \text{iff } (A \subset_d B) \ \& \ (A \subset_i B),$$
$$A \subseteq_{d,i} B \quad \text{iff } (A \subseteq_d B) \ \& \ (A \subseteq_i B),$$
$$A \subset_{v,i} B \quad \text{iff } (A \subset_v B) \ \& \ (A \subseteq_i B),$$
$$A \subseteq_{v,i} B \quad \text{iff } (A \subseteq_v B) \ \& \ (A \subseteq_i B),$$
$$A \subset B \quad \text{iff } (A \subset_d B) \ \& \ (A \subset_v B) \ \& \ (A \subset_i B),$$
$$A \subseteq B \quad \text{iff } (A \subseteq_d B) \ \& \ (A \subseteq_v B) \ \& \ (A \subseteq_i B).$$

3 Extended intuitionistic fuzzy index matrices as multi-dimensional modal topological structures

Let us have two sets of indices I_1 and I_2 and a universe E. Let

$$I_1^* = \{\langle i_1, 1, 0 \rangle | i_1 \in I_1\},$$
$$I_2^* = \{\langle i_2, 1, 0 \rangle | i_2 \in I_2\},$$
$$E^* = \{I_1^*, I_2^*, \langle x, 1, 0 \rangle | x \in E\},$$
$$O^* = \{I_1^*, I_2^*, \langle x, 0, 1 \rangle | x \in E\}.$$

These sets are IFSs and for them, similarly to [8], we can construct the IFMTSs

$$\langle \mathcal{P}(I_1^*), \ \mathcal{E}, \zeta, *, \eta \rangle, \quad \langle \mathcal{P}(I_2^*), \ \mathcal{E}, \zeta, *, \eta \rangle, \quad \langle \mathcal{P}(E^*), \ \mathcal{E}, \zeta, *, \eta \rangle,$$

where $\mathcal{E} \in \{\mathcal{C}, \mathcal{I}\}$, $* \in \{\Box, \Diamond\}$, ζ and η are functions and $\mathcal{C}, \mathcal{I}, \Box, \Diamond$ are the standard intuitionistic fuzzy topological (from closure and interior type) and modal (from necessity and possibility type) operators, respectively (see, e.g., [1]).

Therefore, we can construct the object

$$\langle \mathcal{P}(I_1^* \times I_2^2 \times E^*), \mathcal{E}, \zeta_3, *, \eta_3 \rangle.$$

The elements of the set $\mathcal{P}(I_1^* \times I_2^2 \times E^*)$ are IFSs with the form

$$A^* = \{\langle\langle\langle k_i, \alpha_i^K, \beta_i^K\rangle, \langle l_i, \alpha_i^L, \beta_i^L\rangle\rangle, \mu(k_i, l_i), \nu(k_i, l_i)\rangle \mid \langle k_i, l_i\rangle \in K \times L\},$$

where $K^* \subseteq \mathcal{I}_1^*$ and $L^* \subseteq \mathcal{I}_2^*$.

Now, we can construct the EIFIM

$$A = [K^*, L^*, \{\langle \mu(k_i, l_i), \nu(k_i, l_i)\rangle \mid k_i \in K, l_i \in L\}].$$

It is interesting to mention that in [8] we perceived the set $I_1 \times I_2$ as one set, while below we will perceive the object $I_1^* \times I_2^2 \times E^*$ as a Cartesian product of three sets.

Obviously, the EIFIM A is another form of the IFS A^*. Therefore, on it we can define topological and modal operators as follows:

$$\begin{aligned}
\mathcal{C}(A) = [&\{\langle k_i, \sup_{k\in K} \alpha_k^K, \inf_{k\in K} \beta_k^K\rangle | k_i \in K\}, \{\langle l_j, \sup_{l\in L} \alpha_l^L, \inf_{l\in L} \beta_l^L\rangle | l_j \in L\},\\
&\{\langle \sup_{\langle k,l\rangle\in K\times L} \mu(k,l), \inf_{\langle k,l\rangle\in K\times L} \nu(k,l)\rangle \mid k_i \in K, l_j \in L\}],\\
\mathcal{I}(A) = [&\{\langle k_i, \inf_{k\in K} \alpha_k^K, \sup_{k\in K} \beta_k^K\rangle | k_i \in K\}, \{\langle l_j, \inf_{l\in L} \alpha_l^L, \sup_{l\in L} \beta_l^L\rangle | l_j \in L\},\\
&\{\langle \inf_{\langle k,l\rangle\in K\times L} \mu(k,l), \sup_{\langle k,l\rangle\in K\times L} \nu(k,l)\rangle \mid k_i \in K, l_j \in L\}],\\
\Box A = [&\{\langle k, \alpha_k^K, 1-\alpha_k^K\rangle | k \in K\}, \{\langle l, \alpha_l^L, 1-\alpha_l^L\rangle | l \in L\},\\
&\{\langle \mu(k,l), 1-\mu(k,l)\rangle\} \mid k \in K, l \in L\}],\\
\Diamond A = [&\{\langle k, 1-\beta_k^K, \beta_k^K\rangle | k \in K\}, \{\langle l, 1-\beta_l^L, \beta_l^L\rangle | l \in L\},\\
&\{\langle \mu(k,l), 1-\mu(k,l)\rangle\} \mid k \in K, l \in L\}].
\end{aligned}$$

It is important to mention that in [8] we perceived the set $I_1 \times I_2$ as one set, while below we will perceive the object $I_1^* \times I_2^2 \times E^*$ as a Cartesian product of three sets. In addition, these sets are such that just as a topological or modal operator can be defined over each of them separately, the same can be done over their product. Therefore, in the sense of the definition from Section 2.1, the structure will be 3-dimensional one.

Now, we can prove the following assertion.

Theorem 1. *For every two sets Y and Z:*

(1) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \oplus_\vee, \Diamond, \oplus_\vee \rangle$ *is an IF3DMTS,*
(2) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \oplus_\vee, \Diamond, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(3) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \oplus_\vee, \Box, \oplus_\vee \rangle$ *is an IF3DMTS,*
(4) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \oplus_\vee, \Box, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(5) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \otimes_\vee, \Diamond, \oplus_\vee \rangle$ *is an IF3DMTS,*
(6) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \otimes_\vee, \Diamond, \oplus_\wedge \rangle$ *is an IF3DMTS,*

(7) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \otimes_\vee, \Box, \oplus_\vee \rangle$ *is an IF3DMTS,*
(8) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{C}, \otimes_\vee, \Box, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(9) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \oplus_\wedge, \diamondsuit, \oplus_\vee \rangle$ *is an IF3DMTS,*
(10) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \oplus_\wedge, \diamondsuit, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(11) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \oplus_\wedge, \Box, \oplus_\vee \rangle$ *is an IF3DMTS,*
(12) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \oplus_\wedge, \Box, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(13) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \otimes_\wedge, \diamondsuit, \oplus_\vee \rangle$ *is an IF3DMTS,*
(14) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \otimes_\wedge, \diamondsuit, \oplus_\wedge \rangle$ *is an IF3DMTS,*
(15) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \otimes_\wedge, \Box, \oplus_\vee \rangle$ *is an IF3DMTS,*
(16) $\langle \mathcal{P}(I_1^* \times I_2^* \times E^*), \mathcal{I}, \otimes_\wedge, \Box, \oplus_\wedge \rangle$ *is an IF3DMTS.*

Proof. Let the sets I_1^*, I_2^2 and E^* be given and let

$$A = [K^*, L^*, \{\langle \mu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}] \in \mathcal{P}(I_1^* \times I_2^2 \times E^*),$$
$$B = [P^*, Q^*, \{\langle \mu(p,q), \nu(p,q)\rangle \mid p \in P, q \in Q\}] \in \mathcal{P}(I_1^* \times I_2^2 \times E^*).$$

We must mention that objects $\{\langle \mu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}$ and $\{\langle \mu(p,q), \nu(p,q)\rangle \mid p \in P, q \in Q\}$ that are the elements of EIFIMs can be interpreted as IFSs. Then for example for (1) we can check the conditions that the object is an EIFMTS as follows:

Ct1 As it is proved in [1], for every two IFSs X and Y:

$$\mathcal{C}(X \cup Y) = \mathcal{C}(X) \cup \mathcal{C}(Y)$$

and from this equality it follows:

$$\begin{aligned}
&\mathcal{C}(A \oplus_\vee B)\\
&= \mathcal{C}([K^* \cup P^*, L^* \cup Q^*, \{\langle \mu(k,l), \nu(k,l) \vee \langle \mu(p,q), \nu(p,q)\rangle \mid \\
&\qquad\qquad k, p \in K \cup P; l, q \in L \cup Q\}])\\
&= [\mathcal{C}(K^* \cup P^*), \mathcal{C}(L^* \cup Q^*), \mathcal{C}(\{\langle \max(\mu(k,l), \mu(p,q)), \min(\nu(k,l), \nu(p,q))\rangle \mid \\
&\qquad\qquad k, p \in K \cup P; l, q \in L \cup Q\})]\\
&= [\mathcal{C}(K^* \cup P^*), \mathcal{C}(L^* \cup Q^*),\\
&\qquad \{\langle \sup_{k,p \in K\cup P, l,q \in L \cup Q} \max(\mu(k,l), \mu(p,q)), \inf_{k,p \in K\cup P, l,q \in L \cup Q} \min(\nu(k,l), \nu(p,q))\rangle \mid \\
&\qquad\qquad k, p \in K \cup P; l, q \in L \cup Q\}]\\
&= [\mathcal{C}(K^*) \cup \mathcal{C}(P^*), \mathcal{C}(L^*) \cup \mathcal{C}(Q^*),\\
&\qquad \{\langle \max(\sup_{\langle k,l\rangle \in K \times L} \mu(k,l), \sup_{\langle p,q\rangle \in P \times Q} \mu(p,q)), \min(\inf_{\langle k,l\rangle \in K \times L} \nu(k,l), \inf_{\langle p,q\rangle \in P \times Q} \nu(p,q))\rangle\\
&\qquad\qquad \mid k \in K, l \in L\}]\\
&= \quad [\mathcal{C}(K^*), \mathcal{C}(L^*), \{\langle \sup_{\langle k,l\rangle \in K \times L} \mu(k,l), \inf_{\langle k,l\rangle \in K \times L} \nu(k,l)\rangle \mid k \in K, l \in L\}]\\
&\quad \oplus_\vee [\mathcal{C}(P^*), \mathcal{C}(Q^*), \{\langle \sup_{\langle p,q\rangle \in P \times Q} \mu(p,q), \inf_{\langle p,q\rangle \in P \times Q} \nu(p,q))\rangle \mid p \in P, q \in Q\}]\\
&= \mathcal{C}(A) \oplus_\vee \mathcal{C}(B);
\end{aligned}$$

Ct2 As it is proved in [1], for each IFS X:

$$\mathcal{C}(\mathcal{C}(X)) = \mathcal{C}(X)$$

and from this equality it follows:

$$\begin{aligned}\mathcal{C}(\mathcal{C}(A)) &= \mathcal{C}([\mathcal{C}(K^*), \mathcal{C}(L^*), \{\langle \sup_{\langle k,l\rangle \in K\times L} \mu(k,l), \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}])\\
&= [\mathcal{C}(\mathcal{C}(K^*)), \mathcal{C}(\mathcal{C}(L^*)), \{\langle \sup_{\langle p,q\rangle \in K\times L} \sup_{\langle k,l\rangle \in K\times L} \mu(k,l),\\
&\qquad \inf_{\langle p,q\rangle \in K\times L} \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}]\\
&= [\mathcal{C}(K^*), \mathcal{C}(L^*), \{\langle \sup_{\langle k,l\rangle \in K\times L} \mu(k,l), \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}]\\
&= \mathcal{C}(A);\end{aligned}$$

Ct3

$$\begin{aligned}A &= [K^*, L^*, \{\langle \mu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}]\\
&\subseteq_{v,i} [\mathcal{C}(K^*), \mathcal{C}(L^*), \{\langle \sup_{\langle k,l\rangle \in K\times L} \mu(k,l), \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}])\\
&= \mathcal{C}(A);\end{aligned}$$

Ct4

$$\begin{aligned}\mathcal{C}(O^*) &= [\mathcal{C}(I_1^*), \mathcal{C}(I_2^*), \{\langle \sup_{\langle k,l\rangle \in K\times L} 0, \inf_{\langle k,l\rangle \in K\times L} 1\rangle \mid k \in K, l \in L\}]\\
&= [I_1^*, I_2^*, \{\langle 0, 1\rangle \mid k \in K, l \in L\}]\\
&= O^*;\end{aligned}$$

Cm1 As it is proved in [1], for every two IFSs X and Y:

$$\Diamond(X \cup Y) = \Diamond X \cup \Diamond Y$$

and from this equality it follows:

$$\begin{aligned}&\Diamond(A \oplus_{\vee} B)\\
&= [\Diamond(K^* \cup P^*), \Diamond(L^* \cup Q^*), \Diamond\{\langle\langle \max(\mu(k,l), \mu(p,q)), \min(\nu(k,l), \nu(p,q))\rangle\rangle \mid\\
&\qquad k, p \in K \cup P; l, q \in L \cup Q\}]\\
&= [\Diamond K^* \cup \Diamond P^*, \Diamond L^* \cup \Diamond Q^*, \{\langle 1 - \min(\nu(k,l), \nu(p,q)), \min(\nu(k,l), \nu(p,q))\rangle \mid\\
&\qquad k, p \in K \cup P; l, q \in L \cup Q\}]\\
&= [\Diamond K^* \cup \Diamond P^*, \Diamond L^* \cup \Diamond Q^*, \{\langle \max(1 - \nu(k,l), 1 - \nu(p,q)), \min(\nu(k,l), \nu(p,q))\rangle \mid\\
&\qquad k, p \in K \cup P; l, q \in L \cup Q\}]\\
&= \quad [K^*, L^*, \{\langle 1 - \nu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}]\\
&\quad \oplus_{\vee} [P^*, Q^*, \{\langle 1 - \nu(p,q), \nu(p,q))\rangle \mid p \in P, q \in Q\}]\\
&= \Diamond A \oplus_{\vee} \Diamond B;\end{aligned}$$

Cm2 As it is proved in [1], for each IFS X:

$$\Diamond\Diamond X = \Diamond X$$

and from this equality it follows:

$$\begin{aligned}\Diamond\Diamond A &= \Diamond[\Diamond K^*, \Diamond L^*, \Diamond\{\langle \mu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= [\Diamond\Diamond K^*, \Diamond\Diamond^* L, \Diamond\{\langle 1-\nu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= [\Diamond K^*, \Diamond^* L, \{\langle 1-\nu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= \Diamond A;\end{aligned}$$

Cm3

$$\begin{aligned}A &= [K^*, L^*, \{\langle \mu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}]\\ &\subseteq_{v,i} [\Diamond K^*, \Diamond L^*, \{\langle 1-\nu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= \Diamond A.\end{aligned}$$

Cm4

$$\begin{aligned}\Diamond E^* &= \Diamond[I_1^*, I_2^*, \Diamond\{\langle 1, 0\rangle \mid k \in K, l \in L\}]\\ &= [I_1^*, I_2^*, \{\langle 1, 0\rangle \mid k \in K, l \in L\}]\\ &= E^*.\end{aligned}$$

$(*)$ As it is proved in [1], for each IFS X:

$$\mathcal{C}(\Diamond X) = \Diamond\mathcal{C}(X)$$

and from this equality it follows:

$$\begin{aligned}\mathcal{C}(\Diamond A)) &= \mathcal{C}([\Diamond K^*, \Diamond L^*, \{\langle 1-\nu(k,l), \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= [\mathcal{C}(\Diamond K^*), \mathcal{C}(\Diamond L^*), \{\langle \sup_{\langle k,l\rangle \in K\times L} (1-\nu(k,l)), \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= [\Diamond\mathcal{C}(K^*), \Diamond\mathcal{C}(L^*), \{\langle 1 - \inf_{\langle k,l\rangle \in K\times L} (1-\nu(k,l)), \inf_{\langle k,l\rangle \in K\times L} \nu(k,l)\rangle \mid k \in K, l \in L\}])\\ &= \Diamond\mathcal{C}(A).\end{aligned}$$

The remaining 15 assertions are proved in the same manner. □

Obviously, if $\mathcal{I}^* = \mathcal{I}_1^* \cup \mathcal{I}_2^*$, the above theorem can be modified to the form for the case of IM2DMTS.

By analogy with [8], we can see that each intuitionistic fuzzy graph (IFG, see [2, 3]) can be represented as an IFIM in which the set of indices is changed with a set of vertices V and each vertex $v \in V$ had degrees of existing ($\mu(v)$) and of non-existing ($\nu(v)$). Therefore, this set generates the IFS

$$V^* = \{\langle v, \mu_V(v), \nu_V(v)\rangle | v \in V\}.$$

As it is mentioned in [3], set V can be represented as

$$V = V_I \cup V^* \cup V_O,$$

where $V_I, \overline{V}, V_O$ are, respectively, the sets of input, inside and output vertices. Therefore, the graph will obtain the following IFIM-representation

$$\begin{aligned} G &= \{\langle\langle\langle v_1, \mu_V(v_1), \nu_V(v_1)\rangle, \langle v_2, \mu_V(v_2), \nu_V(v_2)\rangle\rangle, \mu_G(v_1, v_2), \nu_G(v_1, v_2)\rangle \mid \\ &\qquad v_1 \in V_I \cup \overline{V}, v_2 \in \overline{V} \cup V_O\}. \end{aligned}$$

Now, we can formulate some criteria for the correctness of the IFG. For example, it can be: *the IFG is correct if for every arc $\langle v_1, v_2\rangle$ with vertices v_1 and v_2 it is valid that*

$$\langle \mu_G(v_1, v_2), \nu_G(v_1, v_2)\rangle \leq \langle \mu_V(v_1), \nu_V(v_1)\rangle \wedge \langle \mu_V(v_2), \nu_V(v_2)\rangle$$

with motivation that the degree of existence (non-existence) of the arc must be less than (greater than) the degrees of existence (non-existence) of its vertices.

4 Intuitionistic fuzzy interpretation of multi-person multi-criteria decision making procedures

In [4, 9], a very general multi-person multi-criteria decision making procedure is described. We will use it as a basis of the next research in which elements of this procedure will be used.

Let us have e experts $X_1, X_2, \ldots, X_e$ who must evaluate (on first step) only one object, using c criteria $C_1, C_2, \ldots, C_c$.

Let each expert have his/her own (current) reliability score in the form of an IFP $\langle \delta_i, \varepsilon_i\rangle$ and his/her own (current) number of participations in expert investigations γ_i (these two values correspond to his last expert estimation). Expert's reliabiliy scores can be interpreted, e.g., as

$$\langle \delta_i, \varepsilon_i\rangle = \left\langle \frac{\sum_{j=1}^{e} \delta_{i,j}}{e}, \frac{\sum_{j=1}^{e} \varepsilon_{i,j}}{e} \right\rangle,$$

where $\langle \delta_{i,j}, \varepsilon_{i,j}\rangle$ are elements of the IM

$$T = \begin{array}{c|cccc} & X_1 & X_2 & \cdots & X_e \\ \hline C_1 & & & & \\ C_2 & & \langle \delta_{i,j}, \varepsilon_{i,j}\rangle & & \\ \vdots & & (1 \leq i \leq c, 1 \leq j \leq e) & & \\ C_c & & & & \end{array}$$

and $\langle \delta_{i,j}, \varepsilon_{i,j}\rangle$ is the score of the i-th expert with respect to the j-th criterion (we assume that i-th expert's knowledge reliability may differ over different criteria; the case when the expert is equally good a specialist with respect to the different criteria is a special one).

Now, we can construct an EIFIM

$$
\begin{array}{c|ccccc}
 & X_1, \langle \delta_1, \varepsilon_1 \rangle & \dots & X_j, \langle \delta_j, \varepsilon_j \rangle & \dots & X_e, \langle \delta_e, \varepsilon_e \rangle \\
\hline
C_1, \langle \alpha_1, \beta_1 \rangle & \langle \mu_{C_1,X_1}, \nu_{C_1,X_1} \rangle & \dots & \langle \mu_{C_1,X_j}, \nu_{C_1,X_j} \rangle & \dots & \langle \mu_{C_1,X_e}, \nu_{C_1,X_e} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_i, \langle \alpha_i, \beta_i \rangle & \langle \mu_{C_i,X_1}, \nu_{C_i,X_1} \rangle & \dots & \langle \mu_{C_i,X_j}, \nu_{C_i,X_j} \rangle & \dots & \langle \mu_{C_i,X_e}, \nu_{C_i,X_e} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_c, \langle \alpha_c, \beta_c \rangle & \langle \mu_{C_c,X_1}, \nu_{C_c,X_1} \rangle & \dots & \langle \mu_{C_c,X_j}, \nu_{C_c,X_j} \rangle & \dots & \langle \mu_{C_c,X_e}, \nu_{C_c,X_e} \rangle
\end{array}.
$$

Having in mind the results from Section 3, we see that object $\langle \mathcal{P}(C^* \times X^* \times E^*), \mathcal{E}, \zeta, *, \eta \rangle$ is an IF3DMTS, where $\mathcal{E} \in \{\mathcal{C}, \mathcal{I}\}$ is one of the topological operators, $* \in \{\Box, \Diamond\}$ is one of the modal operators, $\zeta, \eta \in \{\vee, \wedge\}$ and

$$
\begin{aligned}
C^* &= \{\langle C_i, \alpha_i, \beta_i \rangle | 1 \leq i \leq c\}, \\
X^* &= \{\langle X_j, \delta_j, \varepsilon_j \rangle | 1 \leq j \leq e\}, \\
E^* &= \{\langle \langle C_i, X_j \rangle, \mu_{C_i,X_j}, \nu_{C_i,X_j} \rangle | 1 \leq i \leq c, 1 \leq j \leq e\},
\end{aligned}
$$

are IFSs.

Now, we can take the next step – the experts will evaluate many objects, e.g., s in number – $S = \{S_1, S_2, \dots, S_s\}\}$. In this case the EIFIM will be 3-dimensional (see Figure 1).

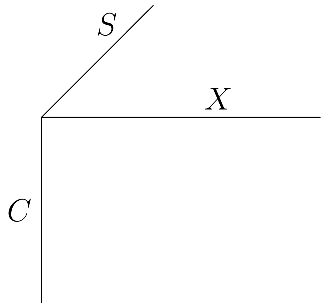

Figure 1. A 3-dimensional EIFIM

The q-th level of this parallelepiped, where $1 \leq q \leq s$ will be related to the experts' evaluations of the object S_q and will have the form

$$
\begin{array}{c|ccccc}
S_q & X_1, \langle \delta_1, \varepsilon_1 \rangle & \dots & X_j, \langle \delta_j, \varepsilon_j \rangle & \dots & X_e, \langle \delta_e, \varepsilon_e \rangle \\
\hline
C_1, \langle \alpha_1, \beta_1 \rangle & \langle \mu_{C_1,X_1,S_q}, \nu_{C_1,X_1,S_q} \rangle & \dots & \langle \mu_{C_1,X_j,S_q}, \nu_{C_1,X_j,S_q} \rangle & \dots & \langle \mu_{C_1,X_e,S_q}, \nu_{C_1,X_e,S_q} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_i, \langle \alpha_i, \beta_i \rangle & \langle \mu_{C_i,X_1,S_q}, \nu_{C_i,X_1,S_q} \rangle & \dots & \langle \mu_{C_i,X_j,S_q}, \nu_{C_i,X_j,S_q} \rangle & \dots & \langle \mu_{C_i,X_e,S_q}, \nu_{C_i,X_e,S_q} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_c, \langle \alpha_c, \beta_c \rangle & \langle \mu_{C_c,X_1,S_q}, \nu_{C_c,X_1,S_q} \rangle & \dots & \langle \mu_{C_c,X_j,S_q}, \nu_{C_c,X_j,S_q} \rangle & \dots & \langle \mu_{C_c,X_e,S_q}, \nu_{C_c,X_e,S_q} \rangle
\end{array}.
$$

In this EIFIM the set

$$S^* = \{\langle S_q, \varphi_q, \psi_q \rangle | 1 \leq q \leq s\}$$

is an IFS and the object $\langle \mathcal{P}(C^* \times X^* \times S^* \times E^*), \mathcal{E}, \zeta, *, \eta \rangle$ is an IF4DMTS.

5 Conclusion

We will finish the present paper with some short remarks for a future research.

First, we can further sophisticate our construction by introducing a temporal component. As a result, we will obtain the IF4DMTS $\mathcal{P}(C^* \times X^* \times T \times E^*)$ in which only the time-scale T will be not intuitionistically fuzzified.

Second, over the set $\mathcal{P}(C^* \times X^* \times E^*)$ or $\mathcal{P}(C^* \times X^* \times S^* \times E^*)$ we can apply the level operators introduced over IFSs, IFIMs and EIFIMs. In this case, we can construct IFMDM_level_TS, that will be an extension of the intuitionistic fuzzy level topological structures (IFMLTS) (see [6]).

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