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On two new intuitionistic fuzzy topological operators and four new intuitionistic fuzzy feeble modal topological structures

Krassimir T. Atanassov [1,2]

[1] Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria

[2] Intelligent Systems Laboratory,
Prof. Asen Zlatarov University, Burgas-8010, Bulgaria
e-mail: krat@bas.bg

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Abstract: In the intuitionistic fuzzy sets theory there are some intuitionistic fuzzy topological operators. Here, two new operators are defined, some of their properties are shown and on their basis, four new intuitionistic fuzzy feeble modal topological structures are introduced and some of their properties are discussed.
Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy operation, Intuitionistic fuzzy operator, Intuitionistic fuzzy topological structure.
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1 Introduction

In a series of papers of the author (see, [4–9]*), the concept of an Intuitionistic Fuzzy Modal Topological Structure (IFMTS) of closure, of interior, or of mixed type has been discussed and some examples given. The definition is based on Kuratowski's definitions for closure and interior topological structures [11] and on the Intuitionistic Fuzzy Sets (IFS, see, e.g., [3]) theory.

The IFMTS is of a closure or of an interior type in respect of the form of its topological operator (of closure ($\mathcal{C}$) or interior ($\mathcal{I}$) type), respectively. On the other hand, the same notation is used about the form of the modal operator, because at least conditionally we can accept that the modal operator (see, e.g., [10]) "possibility" ($\Diamond$) is related to operation "union" ($\cup$, as well as the topological operator closure), while modal operator "necessity" ($\Box$) is related to operation "intersection" ($\cap$, as well as the topological interior).

Let E be a fixed universe,

$$E^* = \{\langle x, 1, 0\rangle | x \in E\},$$
$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$\mathcal{O}, \mathcal{Q} : \mathcal{P}(E^*) \to \mathcal{P}(E^*)$ be operators of a closure and of an interior types related to operations $\Delta, \nabla : \mathcal{P}(E^*) \times \mathcal{P}(E^*) \to \mathcal{P}(E^*)$, respectively, $\circ, \bullet : \mathcal{P}(E^*) \to \mathcal{P}(E^*)$ be modal operators. Let for every two IFSs $A, B \in \mathcal{P}(E^*)$ and for the (standard) intuitionistic fuzzy negation of the IFS

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

(see [3]) that has the form:

$$\neg A = \{\langle x, \nu, \mu\rangle | x \in E\},$$

the following equalities hold:

$$\begin{aligned}
A\Delta B &= \neg(\neg A \nabla \neg B),\\
A\nabla B &= \neg(\neg A \Delta \neg B),\\
\mathcal{O}(A) &= \neg(\mathcal{Q}(\neg A)),\\
\mathcal{Q}(A) &= \neg(\mathcal{O}(\neg A)),\\
\circ A &= \neg \bullet \neg A,\\
\bullet A &= \neg \circ \neg A.
\end{aligned}$$

Now, we can define the following four structures: cl-cl-IFMTS, in-in-IFMTS, cl-in-IFMTS, in-cl-IFMTS, that for every two IFSs $A, B \in \mathcal{P}(E^*)$ the following nine conditions hold, respectively contained in Table 1.

* We must mention that in [8], in conditions CC5 and IC9 the relation "=" is changed with relation "$\subseteq$" and in conditions II5 and CI9 the relation "=" is changed with relation "$\supseteq$".

	cl-*cl*-IFMTS		*in*-*in*-IFMTS
CC1	$\mathcal{O}(A\Delta B) = \mathcal{O}(A)\Delta\mathcal{O}(B)$	II1	$\mathcal{Q}(A\nabla B) = \mathcal{Q}(A)\nabla\mathcal{Q}(B)$
CC2	$A \subseteq \mathcal{O}(A)$	II2	$\mathcal{Q}(A) \subseteq A$
CC3	$\mathcal{O}(O^*) = O^*$	II3	$\mathcal{Q}(E^*) = E^*$
CC4	$\mathcal{O}(\mathcal{O}(A)) = \mathcal{O}(A)$	II4	$\mathcal{Q}(\mathcal{Q}(A)) = \mathcal{Q}(A)$
CC5	$\circ(A\nabla B) = \circ A\nabla \circ B$	II5	$\bullet(A\nabla B) = \bullet A\nabla \bullet B$
CC6	$\circ A \subseteq A$	II6	$\bullet A \subseteq A$
CC7	$\circ E^* = E^*$	II7	$\bullet O^* = O^*$
CC8	$\circ \circ A = \circ A$	II8	$\bullet \bullet A = \bullet A$
CC9	$\circ\, \mathcal{O}(A) = \mathcal{O}(\circ A)$	II9	$\bullet\, \mathcal{Q}(A) = \mathcal{Q}(\bullet A)$

	cl-*in*-IFMTS		*in*-*cl*-IFMTS
CI1	$\mathcal{O}(A\Delta B) = \mathcal{O}(A)\Delta\mathcal{O}(B)$	IC1	$\mathcal{Q}(A\nabla B) = \mathcal{Q}(A)\nabla\mathcal{Q}(B)$
CI2	$A \subseteq \mathcal{O}(A)$	IC2	$\mathcal{Q}(A) \subseteq A$
CI3	$\mathcal{O}(O^*) = O^*$	IC3	$\mathcal{Q}(E^*) = E^*$
CI4	$\mathcal{O}(\mathcal{O}(A)) = \mathcal{O}(A)$	IC4	$\mathcal{Q}(\mathcal{Q}(A)) = \mathcal{Q}(A)$
CI5	$\bullet(A\nabla B) = \bullet A\nabla \bullet B$	IC5	$\circ(A\Delta B) = \circ A\Delta \circ B$
CI6	$\bullet A \subseteq A$	IC6	$A \subseteq \circ A$
CI7	$\bullet O^* = O^*$	IC7	$\circ E^* = E^*$
CI8	$\bullet \bullet A = \bullet A$	IC8	$\circ \circ A = \circ A$
CI9	$\bullet\, \mathcal{O}(A) = \mathcal{O}(\bullet A)$	IC9	$\circ\, \mathcal{Q}(A) = \mathcal{Q}(\circ A)$

As it was seen in [5], some conditions are not valid for some objects that look like IFMTSs. In some cases, some conditions do not exist, while in other cases the relations in the conditions are changed with weak (feeble) relations. Having in mind that in topology (see, e.g., [12]) the word "weak" is defined in another sense, in [5] we offered to use for these structures the word "feeble" and these structures were called there Intuitionistic Fuzzy Feeble Modal Topological Structures (IFFMTSs).

Below, we will describe four new IFFMTSs.

2 Preliminaries

In [1] the following pair of intuitionistic fuzzy operations union and intersection, twelfth in succession, are defined as:

$$A \cup_{12} B = \{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\},$$
$$A \cap_{12} B = \{\langle x, 1 - \max(\nu_A(x), \nu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}.$$

For these operations, we can check directly that for each IFS A:

$$\neg(\neg(A) \cap_{12} \neg B) = A \cup_{12} B,$$
$$\neg(\neg(A) \cup_{12} \neg B) = A \cap_{12} B.$$

Using the method for construction the first two intuitionistic fuzzy topological operators $\mathcal{C}$ and $\mathcal{I}$ on the bases of the standard operations $\cup$ and $\cap$ (see [2, 3]), below, we construct two new intuitionistic fuzzy topological operators on the basis of operations $\cup_{12}$ and $\cap_{12}$.

$$\begin{aligned} \mathcal{C}_{12}(A) &= \{\langle x, \sup_{y\in E} \mu_A(y), 1 - \sup_{y\in E} \mu_A(y)\rangle | x \in E\}; \\ \mathcal{I}_{12}(A) &= \{\langle x, 1 - \sup_{y\in E} \nu_A(y), \sup_{y\in E} \nu_A(y)\rangle | x \in E\}; \end{aligned}$$

For both operators again we see that for each IFS A:

$$\neg\mathcal{C}_{12}(\neg A) = \mathcal{I}_{12}(A),$$
$$\neg\mathcal{I}_{12}(\neg A) = \mathcal{C}_{12}(A).$$

Really, we check for example that

$$\begin{aligned} \neg\mathcal{I}_{12}(\neg A) &= \neg\mathcal{I}_{12}(\{\langle x, \nu_A(y), \mu_A(y)\rangle | x \in E\}); \\ &= \neg\{\langle x, \langle x, 1 - \sup_{y\in E} \mu_A(y), \sup_{y\in E} \mu_A(y)\rangle | x \in E\} \\ &= \{\langle x, \sup_{y\in E} \mu_A(y), 1 - \sup_{y\in E} \mu_A(y)\rangle | x \in E\} \\ &= \mathcal{C}_{12}(A). \end{aligned}$$

The geometrical interpretations of the two intuitionistic fuzzy topological operators are given on Figures 1 and 2.

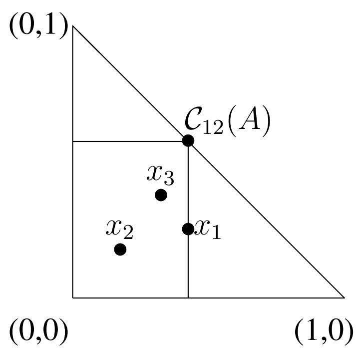

Figure 1. A geometrical interpretations of the topological operator $\mathcal{C}_{12}$ over an IFS A in universe $\{x_1, x_2, x_3\}$

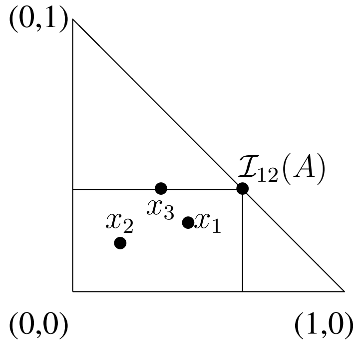

Figure 2. A geometrical interpretations of the topological operator $\mathcal{I}_{12}$ over an IFS A in universe $\{x_1, x_2, x_3\}$

3 Four new intuitionistic fuzzy feeble modal topological structures

In this Section, we will illustrate the IFFMTS with four examples using the above discussed operations and operators.

Theorem 1. $\langle \mathcal{P}(E^*), \mathcal{C}_{12}, \cup_{12}, \diamondsuit \rangle$ is a *cl*-*cl*-IFFMTS for which in the condition CC9 relation "=" is changed with relation $\subseteq$".

Proof. Let the IFSs $A, B \in \mathcal{P}(E^*)$ be given. Then, we check sequentially the validity of the conditions CC1 – CC5 and CC9, while the checks of conditions CC6 – CC8 are given in [4].

CC1.

$$\begin{aligned}
\mathcal{C}_{12}(A \cup_{12} B) &= \mathcal{C}_{12}(\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} \cup_{12} \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}) \\
&= \mathcal{C}_{12}(\{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\}) \\
&= \{\langle x, \sup_{y \in E} \max(\mu_A(x), \mu_B(x)), 1 - \sup_{y \in E}(\max(\mu_A(x), \mu_B(x)))\rangle | x \in E\} \\
&= \{\langle x, \max(\sup_{y \in E} \mu_A(x), \sup_{y \in E} \mu_B(x)), 1 - \max(\sup_{y \in E} \mu_A(x), \sup_{y \in E} \mu_B(x))\rangle | x \in E\} \\
&= \{\langle x, \sup_{y \in E} \mu_A(x), 1 - \sup_{y \in E} \mu_A(x)\rangle | x \in E\} \\
&\quad \cup_{12} \{\langle x, \sup_{y \in E} \mu_B(x), 1 - \sup_{y \in E} \mu_B(x)\rangle | x \in E\} \\
&= \mathcal{C}_{12}(A) \cup_{12} \mathcal{C}_{12}(B);
\end{aligned}$$

CC2.

$$\begin{aligned}
A &= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} \\
&\subseteq \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\} \\
&\subseteq \{\langle x, \sup_{y \in E} \mu_A(x), 1 - \sup_{y \in E} \mu_A(x)\rangle | x \in E\} \\
&= \mathcal{C}_{12}(A);
\end{aligned}$$

CC3.

$$\begin{aligned}\mathcal{C}_{12}(O^*) &= \mathcal{C}_{12}(\{\langle x,0,1\rangle|x\in E\})\\ &= \{\langle x, \sup_{y\in E} 0, 1-\sup_{y\in E} 0\rangle|x\in E\}\\ &= \{\langle x,0,1\rangle|x\in E\}\\ &= O^*;\end{aligned}$$

CC4. Having in mind that $\sup_{y\in E}\mu_A(y)$ is a constant, we obtain that:

$$\begin{aligned}\mathcal{C}_{12}(\mathcal{C}_{12}(A)) &= \mathcal{C}_{33}(\{\langle x, \sup_{y\in E}\mu_A(x), 1-\sup_{y\in E}\mu_A(x)\rangle|x\in E\})\\ &= \{\langle x, \sup_{z\in E}\sup_{y\in E}\mu_A(x), 1-\sup_{z\in E}\sup_{y\in E}\mu_A(x)\rangle|x\in E\}\\ &= \{\langle x, \sup_{y\in E}\mu_A(x), 1-\sup_{y\in E}\mu_A(x)\rangle|x\in E\}\\ &= \mathcal{C}_{12}(A);\end{aligned}$$

CC5.

$$\begin{aligned}\Diamond(A\cap_{12}B) &= \Diamond\{\langle x, 1-\max(\nu_A(x),\nu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle|x\in E\};\\ &= \{\langle x, 1-\max(\nu_A(x),\nu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle|x\in E\};\\ &= \{\langle x, 1-\nu_A(x),\nu_A(x)\rangle|x\in E\}\cup_{12}\{\langle x,1-\nu_B(x),\nu_B(x)\rangle|x\in E\}\\ &= \Diamond A\cap_{12}\Diamond B;\end{aligned}$$

CC9.

$$\begin{aligned}\Diamond\mathcal{C}_{12}(A) &= \Diamond\{\langle x, \sup_{y\in E}\mu_A(x), 1-\sup_{y\in E}\mu_A(x)\rangle|x\in E\}\\ &= \{\langle x, \sup_{y\in E}\mu_A(y), 1-\sup_{y\in E}\mu_A(y)\rangle|x\in E\})\\ &\subseteq \{\langle x, \sup_{y\in E}(1-\nu_A(y)), 1-\sup_{y\in E}(1-\nu_A(y))\rangle|x\in E\})\\ &= \mathcal{C}_{12}(\{\langle x,1-\nu_A(x),\nu_A(x)\rangle|x\in E\})\\ &= \mathcal{C}_{12}(\Diamond A).\end{aligned}$$

This completes the proof. □

Theorem 2. $\langle\mathcal{P}(E^*),\mathcal{C}_{12},\cup_{12},\Box\rangle$ *is a cl-in-IFFMTS for which in the condition CI5 relation "$=$" is changed with relation $\supseteq$".*

Proof. Let the IFSs $A,B\in\mathcal{P}(E^*)$ be given. Now, the checks of conditions CI1 – CI4 coincide with the checks of conditions CC1 – CC4 from Theorem 1 and the checks of conditions CC6 – CC8 are given in [4], i.e., we must check the validity only of two conditions, CI5 and CI9. They are as follows:

CI5.

$$\begin{aligned}
\Box\,(A \cap_{12} B) &= \Box\,\{\langle x, 1-\max(\nu_A(x),\nu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle | x \in E\};\\
&= \{\langle x, 1-\max(\nu_A(x),\nu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle | x \in E\};\\
&\supseteq \{\langle x, 1-\max(1-\mu_A(x), 1-\mu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle | x \in E\};\\
&= \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle | x \in E\} \cup_{12} \{\langle x, \mu_B(x), 1-\mu_B(x)\rangle | x \in E\}\\
&= \Box\,A \cap_{12} \Box\,B;
\end{aligned}$$

CI9.

$$\begin{aligned}
\Box\,\mathcal{C}_{12}(A) &= \Box\,\{\langle x, \sup_{y\in E}\mu_A(x), 1-\sup_{y\in E}\mu_A(x)\rangle | x \in E\}\\
&= \{\langle x, \sup_{y\in E}\mu_A(y), 1-\sup_{y\in E}\mu_A(y)\rangle | x \in E\})\\
&= \mathcal{C}_{12}(\{\langle x, \mu_A(x), 1-\mu_A(x)\rangle | x \in E\})\\
&= \mathcal{C}_{12}(\Box\,A).
\end{aligned}$$

This completes the proof. □

Theorem 3. *$\langle \mathcal{P}(E^*), \mathcal{I}_{12}, \cap_{12}, \diamondsuit\rangle$ is an in-cl-IFFMTS for which in the condition IC5 relation "$=$" is changed with relation $\supseteq$".*

Proof. Let the IFSs $A, B \in \mathcal{P}(E^*)$ be given. Then, we check sequentially the validity of the six conditions IC1 – IC5 and IC9, while the checks of conditions IC6 – IC8 are given in [4].

IC1.

$$\begin{aligned}
\mathcal{I}_{12}(A \cap_{12} B) &= \mathcal{I}_{12}(\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} \cap_{12} \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\})\\
&= \mathcal{I}_{12}(\{\langle x, 1-\max(\nu_A(x),\nu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle | x \in E\})\\
&= \{\langle x, 1-\sup_{y\in E}\max(\nu_A(x),\nu_B(x)), \sup_{y\in E}(\max(\nu_A(x),\nu_B(x)))\rangle | x \in E\}\\
&= \{\langle x, 1-\max(\sup_{y\in E}\nu_A(x), \sup_{y\in E}\nu_B(x)), \max(\sup_{y\in E}\nu_A(x), \sup_{y\in E}\nu_B(x))\rangle | x \in E\}\\
&= \{\langle x, 1-\sup_{y\in E}\nu_A(x), \sup_{y\in E}\nu_A(x)\rangle | x \in E\}\\
&\quad \cap_{12}\{\langle x, 1-\sup_{y\in E}\nu_B(x), \sup_{y\in E}\nu_B(x)\rangle | x \in E\}\\
&= \mathcal{I}_{12}(A) \cap_{12} \mathcal{I}_{12}(B);
\end{aligned}$$

IC2.

$$\begin{aligned}
\mathcal{I}_{12}(A) &= \{\langle x, 1-\sup_{y\in E}\nu_A(x), \sup_{y\in E}\nu_A(x)\rangle | x \in E\}\\
&\subseteq \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle | x \in E\}\\
&\subseteq \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}\\
&= A;
\end{aligned}$$

IC3.

$$\begin{aligned}\mathcal{I}_{12}(E^*) &= \mathcal{I}_{12}(\{\langle x, 1, 0\rangle | x \in E\})\\ &= \{\langle x, 1 - \sup_{y\in E} 0, \sup_{y\in E} 0\rangle | x \in E\}\\ &= \{\langle x, 1, 0\rangle | x \in E\}\\ &= E^*;\end{aligned}$$

IC4. Having in mind that $\sup_{y\in E} \nu_A(y)$ is a constant, we obtain that:

$$\begin{aligned}\mathcal{I}_{12}(\mathcal{I}_{12}(A)) &= \mathcal{I}_{12}(\{\langle x, 1 - \sup_{y\in E} \nu_A(x), \sup_{y\in E} \nu_A(x)\rangle | x \in E\})\\ &= \{\langle x, 1 - \sup_{z\in E}\sup_{y\in E} \nu_A(x), \sup_{z\in E}\sup_{y\in E} \nu_A(x)\rangle | x \in E\}\\ &= \{\langle x, 1 - \sup_{y\in E} \nu_A(x), 1 - \sup_{y\in E} \nu_A(x)\rangle | x \in E\}\\ &= \mathcal{I}_{12}(A);\end{aligned}$$

IC5.

$$\begin{aligned}\Diamond(A \cup_{12} B) &= \Diamond(\{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\});\\ &= \{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\};\\ &\subseteq \{\langle x, \max(1 - \nu_A(x), 1 - \nu_B(x)), 1 - \max(1 - \nu_A(x), 1 - \nu_B(x))\rangle | x \in E\};\\ &= \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\} \cup_{12} \{\langle x, 1 - \nu_B(x), \nu_B(x)\rangle | x \in E\}\\ &= \Diamond A \cup_{12} \Diamond B;\end{aligned}$$

IC9.

$$\begin{aligned}\Diamond\mathcal{I}_{12}(A) &= \Diamond\{\langle x, 1 - \sup_{y\in E} \nu_A(x), \sup_{y\in E} \nu_A(x)\rangle | x \in E\}\\ &= \{\langle x, 1 - \sup_{y\in E} \nu_A(x), \sup_{y\in E} \nu_A(x)\rangle | x \in E\}\\ &= \mathcal{I}_{12}(\{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\})\\ &= \mathcal{I}_{12}(\Diamond A).\end{aligned}$$

This completes the proof. □

Theorem 4. $\langle \mathcal{P}(E^*), \mathcal{I}_{12}, \cap_{12}, \Box \rangle$ *is an* in-in-IFFMTS *for which in the condition IC9 relation "$=$" is changed with relation $\supseteq$".*

Proof. Let the IFSs $A, B \in \mathcal{P}(E^*)$ be given. Now, the checks of conditions II1 – II4 coincide with the checks of conditions IC1 – IC4 from Theorem 3 and the checks of conditions II6 – II8 are given in [4], i.e., we must check the validity only of two conditions, II5 and II9. They are as follows:

II5.

$$\begin{aligned}
\square(A \cup_{12} B) &= \square(\{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\}); \\
&= \{\langle x, \max(\mu_A(x), \mu_B(x)), 1 - \max(\mu_A(x), \mu_B(x))\rangle | x \in E\}; \\
&= \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\} \cup_{12} \{\langle x, \mu_B(x), 1 - \mu_B(x)\rangle | x \in E\} \\
&= \square A \cup_{12} \square B;
\end{aligned}$$

II9.

$$\begin{aligned}
\square \mathcal{I}_{12}(A) &= \square \{\langle x, 1 - \sup_{y\in E} \nu_A(x), \sup_{y\in E} \nu_A(x)\rangle | x \in E\} \\
&= \{\langle x, 1 - \sup_{y\in E} \nu_A(x), \sup_{y\in E} \nu_A(x)\rangle | x \in E\} \\
&\supseteq \{\langle x, 1 - \sup_{y\in E}(1 - \mu_A(x)), \sup_{y\in E}(1 - \nu_A(x))\rangle | x \in E\} \\
&= \mathcal{I}_{12}(\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}) \\
&= \mathcal{I}_{12}(\square A).
\end{aligned}$$

This completes the proof. □

4 Conclusion

After the burst of the idea of modal topological structures – the IFMTSs are the first examples of them, a lot of examples have been constructed and all of them have been based on the IFSs. The largest share of these examples are from the IFFMTS-type – and with the above examples we extended their number. Now, there is a pressing problem of finding an appropriate notation for the separate IFMTSs and IFFMTSs, and that will be an object of discussion in the near future.

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