

A mathematical model using temporal intuitionistic fuzzy sets

S. P. Geetha^{1,*} and R. Parvathi²

¹ Department of Mathematics, Vellalar College for Women,
Erode-638 012, Tamilnadu, India
e-mail: geetha_sams@rediffmail.com

² Department of Mathematics, Vellalar College for Women,
Erode-638 012, Tamilnadu, India
e-mail: paarvathis@rediffmail.com

Received: 9 November 2022

Revised: 1 December 2022

Accepted: 2 December 2022

Abstract: Krassimir T. Atanassov's intuitionistic fuzzy sets (IFS), one of the extensions of fuzzy sets, have shown to be one of the most effective ways to handle ambiguity. John N. Mordeson and Davender S. Malik developed the idea of a fuzzy finite state machine. Intuitionistic fuzzy finite state machines were created by Jun as a generalisation of fuzzy finite state machines. In order to increase the uncertainty and lower the periodic functions in intuitionistic fuzzy finite state automata, new membership and non-membership functions based on transitions were introduced in this study. Also, temporal intuitionistic fuzzy automata (TIFA) were defined and used to model a pattern.

Keywords: Intuitionistic fuzzy finite state automata, Temporal intuitionistic fuzzy finite state automata, Modeling a pattern.

2020 Mathematics Subject Classification: 03D05.

1 Introduction

Automata theory is the study of abstract computing devices or machines. From the early 1930's, computers have been here in this world. Before that a researcher by name A. Turing found an

* Paper presented at the International Workshop on Intuitionistic Fuzzy Sets, founded by Prof. Beloslav Riečan, 2 December 2022, Banská Bystrica, Slovakia.

abstract machine that had all the capabilities of today's computers. The evolution of the above mentioned machine took place. In 1940's and 1950's which was christened as *finite automata*. It was studied by a number of researchers. This is so called "automata", was originally proposed on the model of brain function. But it turned out to be an extremely useful one for a variety of other purposes [4]. "Finite automata" played a crucial role in the theory of programming languages, compiler constructions, switching circuit designing, computer controller, neuron net, text editor and lexical analyzer [1].

Among the various classical changes in science and mathematics in the previous century, one important change concerns the concepts of uncertainty. Uncertainty is viewed in the modern world as important to science; it is not just an inescapable scourge but also has significant benefits. Zadeh in 1965 introduced the concept of *fuzzy set (FS)* to describe the vagueness mathematically in its abstract form and tried to solve problems by giving a grade of membership to each member of a given set [15]. Fuzzy set was defined as a generalization of the characteristic function of a crisp set.

A fuzzy set A in E , the universe of discourse under discussion is identified by a membership function $\mu_A : E \rightarrow [0, 1]$ defined such that for any element x in E , $\mu_A(x)$ is a real number in the closed interval $[0, 1]$ indicating *the degree of membership*. Since this single number does not tell us the uncertainty/impreciseness completely, it is further necessary to generalize the membership function.

In 1983, Krassimir T. Atanassov put forth a generalization fuzzy sets known as *intuitionistic fuzzy sets*, [2]. He introduced a new component *degree of non-membership* in addition to the degree of membership in fuzzy sets provided that their sum be less than or equal to unity. The complement of the two degrees to one is regarded as a degree of indeterminacy. An *intuitionistic fuzzy set (IFS)* A in E is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E \}$, where the functions: $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ define the degrees of membership and non-membership of the element $x \in E$, respectively, and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Obviously, each ordinary fuzzy set may be written as $\{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in E \}$.

Since the introduction of fuzzy sets as a method for representing uncertainty, this idea has been applied to a wide range of scientific areas. One such area is the automata theory and the language theory first introduced by Wee [13]. There is an important reason to study fuzzy automata: several languages are fuzzy by nature [3]. The basic idea in the formulation of a fuzzy automata is that, unlike the classical case, the fuzzy automata can switch from state to another to a certain (truth) degree.

Fuzzy automata are machines accepting fuzzy regular language [6]. This language is a feature of fuzzy language [5]. A fuzzy language is generated by a fuzzy grammar, the natural generalization of formal grammar which is introduced to reduce the gap between formal language and natural language. Fuzzy grammars have been found to be useful in the analysis of X-rays [8]. A fuzzy language \tilde{L} , in the set of finite alphabet Σ is a class of string $\Sigma^* \subseteq \Sigma$ along with a grade of membership in $[0, 1]$. This single value combines the evidence for $x \in \Sigma^*$ and the evidence against $x \in \Sigma^*$, without indicating how much of each is there. This single number is silent nothing about its accuracy. To getover this difficulty, it is necessary to generalize the grade of membership function $\mu_{\tilde{L}}$ of fuzzy languages.

The generalization of membership function of fuzzy language [14] has been achieved using intuitionistic fuzzy sets which leads to further development of intuitionistic fuzzy automata (IFA). Admissible relation and admissible partition for the Intuitionistic general fuzzy automata (IGFA), the quotient IGFA and language for an IGFA are discussed in [11].

Many of the signals which are captured from the real world are composed of characteristic temporal patterns, e.g., the electrocardiogram (ECG) waveform, the accelerations captured at the hip during the human gait cycle, etc., Time influences a signal in the order of the different events that compose this signal and in the duration of these events. One of the most advanced work using fuzzy finite automata for pattern recognition appeared in [12]. It is common for these patterns to present high variability in their amplitude or duration, and sometimes these patterns are corrupted by noise. Hence a concept can be developed to recognize these temporal patterns which take into account the aspects of time to be robust against the signal variability and noise. For this, intuitionistic fuzzy automata (IFA) will be applied for pattern recognition. It helps in expressing the imprecision of our knowledge of a given set by means of describing the set using smooth boundaries instead of sharp ones. IFA can deal with signals that have variability in amplitude and duration. It allows the modeling of restrictions over the duration of the states.

The present study contains four sections. In Section 2, some basic definitions are given which could be used to develop the concepts. An example of new membership and non-membership functions is introduced as part of a discussion on general intuitionistic fuzzy automata (GIFA). In Section 3, the concept of temporal intuitionistic fuzzy automata (TIFA) is introduced and how it is used to model a pattern. Section 4 concludes the result.

2 Preliminaries

In this section, the basic definitions are given. It also presents the basic concepts of the current literature of the intuitionistic fuzzy automata.

Definition 2.1 (Automata, [9]). A non-deterministic finite automata is a triple $A = (Q, \Sigma, \delta)$ where Q is a finite set (the set of states), Σ is an alphabet and δ is a subset of $Q \times \Sigma \times Q$, called the set of transitions. Two transitions (p, a, q) and (p', a', q') are consecutive if $p = q'$.

Consider a word a_0, a_1, \dots, a_{n-1} with $a_i \in \Sigma$. A run α in A is a sequence of states

$$q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2, \dots, q_{n-1} \xrightarrow{a_{n-1}} q_n.$$

Definition 2.2 (Fuzzy finite-state automata, [10]). A fuzzy finite-state automata (FFSA) is a quintuple $M = (Q, \Sigma, \mu, i, f)$ where:

- (i) Q is a set of non-empty, finite states;
- (ii) Σ is a finite non-empty set of input symbols;
- (iii) the fuzzy subset $\mu : Q \times \Sigma \times Q \rightarrow [0, 1]$ is a function, called the fuzzy transition function;
- (iv) i is a fuzzy subset of Q . i.e., $i : Q \rightarrow [0, 1]$ called the fuzzy subset of initial states; and
- (v) f is a fuzzy subset of Q , i.e., $f : Q \rightarrow [0, 1]$ called the fuzzy subset of final states.

Definition 2.3 (Intuitionistic fuzzy finite automata, [14]). An intuitionistic fuzzy finite automata (IFFA) is a five-tuple $M = (Q, \Sigma, A, B, C)$ where:

- (i) Q is a set of non-empty, finite states;
- (ii) Σ is a finite non-empty set of input symbols;
- (iii) $A = (\mu_A, \nu_A)$ is an IFS in $Q \times \Sigma \times Q$. i.e., $\mu : Q \times \Sigma \times Q \rightarrow [0, 1]$ and $\nu : Q \times \Sigma \times Q \rightarrow [0, 1]$;
- (iv) $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_B : Q \rightarrow [0, 1]$ and $\nu_B : Q \rightarrow [0, 1]$, is called the initial state of IFFA; and
- (v) $C = (\mu_C, \nu_C)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_C : Q \rightarrow [0, 1]$ and $\nu_C : Q \rightarrow [0, 1]$, is called the intuitionistic fuzzy subset of final states.

2.1 General intuitionistic fuzzy automata

In the previous section, problems involving the assigning of $\langle mv, nmv \rangle$ to the next state through transition has been discussed. Now some new concepts are introduced to deal with these problems. First the concept of membership and nonmembership assignment are elaborated. Then based on this, a general definition of intuitionistic fuzzy automata is put forth with an example.

2.2 Membership and nonmembership assignment

There is a generally approved approach for assigning $\langle mv, nmv \rangle$ to a next state (whether it is final or not), where the weight of the transition is used and the $\langle mv, nmv \rangle$ of the current state is ignored [4]. Thus the weight of the transition will be considered as the $\langle mv, nmv \rangle$ of the next state. This approach is called *transition-based membership and non-membership* of intuitionistic fuzzy automata.

Example 2.1. Let in a particular IFFA the $\langle mv, nmv \rangle$ of the state q_1 (current state) at time t is $(0.5, 0.1)$ and the weight of the transition upon input symbol a to the next state q_2 is $(1.0, 0.0)$. The IFFA for this is partially shown in Figure 1.

Using transition-based membership and non-membership and assuming the input symbol upon time t is a it can be represented as shown below: $\mu_A(q_1, a, q_2) = 1.0$ and $\nu_A(q_1, a, q_2) = 0.0$ $\Rightarrow (\mu^{t+1}(q_2), \nu^{t+1}(q_2)) = (1.0, 0.0)$.

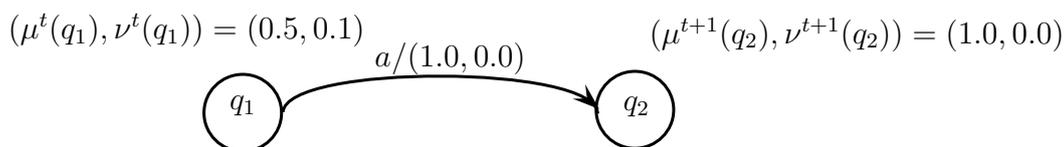


Figure 1. A full activation caused by a weak activation

This means that a state which is active to an extent of $(0.5, 0.1)$, $[(\mu^t(q_1), \nu^t(q_1)) = (0.5, 0.1)]$ causes its successor to be fully activated $[(\mu^{t+1}(q_2), \nu^{t+1}(q_2)) = (1.0, 0.0)]$. Obviously, such an

extension without considering the level of activation of the predecessor is not always reasonable. Even, in a specific application, the $\langle mv, nmv \rangle$ of q_2 becomes $(1.0, 0.0)$. In such a situation, it should be assured that $\langle mv, nmv \rangle$ assignment has been done considering the level of activation of the predecessor (for example the maximum of membership and minimum of the non-membership of the predecessor $\langle mv, nmv \rangle$ and the weight of the transition may have been assigned to the successor). The aim of this simple example, is to show the insufficiency of transition-based membership and non-membership as a general membership and non-membership assignment process.

2.2.1 Augmented-transition function

(A general method to assign membership and non-membership values)

To establish a general method to assign mv 's and nmv 's to next states, first the definition of transition functions in intuitionistic fuzzy finite automata may be generalized (Definition 2.3). This generalization enables it to incorporate both the level of activation of the current state and the weight of the transition.

In Definition 2.3, μ_A and ν_A were defined as $\mu_A : Q \times \Sigma \times Q \rightarrow [0, 1]$ and $\nu_A : Q \times \Sigma \times Q \rightarrow [0, 1]$ and the weight of the transition from q_i to q_j upon input a_k was denoted as $\langle \mu_A(q_i, a_k, q_j), \nu_A(q_i, a_k, q_j) \rangle$.

Now, define new transition functions $\tilde{\mu}_A, \tilde{\nu}_A$ which are called augmented transition functions, can be represented as:

$$\begin{aligned}\tilde{\mu}_A &: (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\mu, \mu_A)} [0, 1] \\ \tilde{\nu}_A &: (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\nu, \nu_A)} [0, 1]\end{aligned}$$

Here $\tilde{\mu}_A, \tilde{\nu}_A$ show the active state (reached from its predecessor), to the intuitionistic fuzzy interval $[0, 1]$ via function F_1 . This function is termed as the membership and non-membership assignment function.

Definition 2.4. (Membership and non-membership assignment function). In an IFFA, the membership and non-membership assignment function is a mapping function which is applied via augmented transition function $\tilde{\mu}_A$ and $\tilde{\nu}_A$ to assign $\langle mv, nmv \rangle$ respectively to the active states. $F_1 : [0, 1] \times [0, 1] \rightarrow [0, 1]$. This function F_1 is guided by two parameters: (μ, ν) : the $\langle mv, nmv \rangle$ of a predecessor and $\langle \mu_A, \nu_A \rangle$: the weight of a transition.

In this new definition, the process that takes place upon the transition from state q_i and q_j an input a_k is represented as:

$$\begin{aligned}\mu^{t+1}(q_j) &= \tilde{\mu}_A((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \mu(q_i, a_k, q_j)) \\ \nu^{t+1}(q_j) &= \tilde{\nu}_A((q_i, \nu^t(q_i)), a_k, q_j) = F_1(\nu^t(q_i), \nu(q_i, a_k, q_j))\end{aligned}$$

which actually means that the $\langle mv, nmv \rangle$ of the state q_j at time $t + 1$ is computed by function F_1 , using both $\langle mv, nmv \rangle$ of q_i at time t and the weight of the transition.

There are various methods to use function F_1 . The best way, however, depends on the application available. It can be either Max, Min, Mean or any other applicable mathematical function which incorporates time into its evaluation. Its exact form depends on the application it is put into. However it should be with in the following axioms:

Axiom 1. $0 \leq F_1 \leq 1$ where as $0 \leq \mu + \mu_A \leq 1$ and $0 \leq \nu + \nu_A \leq 1$.

Axiom 2. $F_1(0, 0) = 0$ and $F_1(1, 1) = 1$.

Axiom 2 guarantees the boundary conditions.

2.3 Multi-membership and non-membership resolution

Definition 2.5. (Multi-membership and non-membership resolution function). In an IFFA, the multi-membership and non membership resolution function, is something that specifies the strategy, and resolves the multi-membership and non-membership active state by assigning a single $\langle mv, nmv \rangle$ to them. It can be represented as $F_2 : ([0, 1])^* \rightarrow ([0, 1])$. Then, the combination of the operation of functions F_1 and F_2 on a multi-membership and non-membership state q_m will result in the multi-membership and non-membership resolution algorithm.

Algorithm (Multi-membership and non-membership).

If there are several parallel transitions to the active state q_m at time $t + 1$, the following algorithm will assign a unified mv and nmv to that:

- (1) Each transition weight $\langle \mu_A(q_i, a_k, q_m), \nu_A(q_i, a_k, q_m) \rangle$, together with the $\langle mv, nmv \rangle$ of the corresponding predecessor q_i , will be processed by the membership and non-membership assignment function F_1 (via augmented transition function $\tilde{\mu}_A, \tilde{\nu}_A$), and will produce a $\langle mv, nmv \rangle$. Denote these values by u_i and v_i :

$$\begin{aligned} u_i &= \tilde{\mu}_A((q_i, \mu^t(q_i)), a_k, q_m) = F_1(\mu^t(q_i), \mu_A(q_i, a_k, q_m)) \\ v_i &= \tilde{\nu}_A((q_i, \nu^t(q_i)), a_k, q_m) = F_1(\nu^t(q_i), \nu_A(q_i, a_k, q_m)) \end{aligned}$$

- (2) These mv's and nmv's (u_i 's and v_i 's) are not always equal. Hence, they will be processed by another function F_2 , called the multi-membership and non-membership resolution function.
- (3) The result produced by F_2 will be assigned as the instantaneous $\langle mv, nmv \rangle$ of the active state q_m .

$$\begin{aligned} [\mu^{t+1}(q_m), \nu^{t+1}(q_m)] &= F_{2_{i=1}^n}[u_i, v_i] \\ &= F_{2_{i=1}^n}[F_1\{(\mu^t(q_i), \nu^t(q_i)), (\mu_A(q_i, a_k, q_m), \nu_A(q_i, a_k, q_m))\}] \end{aligned}$$

where:

- * n is the number of simultaneous transitions from states q_i 's to state q_m prior to time $t + 1$ and $q_i \in Q_{pred}(q_m, a_k)$, i.e., n is the cardinality of the set $(\mu_{q_m}^{t+1}, \nu_{q_m}^{t+1})$.
- * $\langle \mu_A(q_i, a_k, q_m), \nu_A(q_i, a_k, q_m) \rangle$ is the weight of the transition from q_i to q_m upon input a_k .
- * $\langle \mu^t(q_i), \nu^t(q_i) \rangle$ is the membership and non-membership value of q_i at time t (possibly resolved, i.e., unified).
- * $\langle \mu^{t+1}(q_m), \nu^{t+1}(q_m) \rangle$ is the final $\langle mv, nmv \rangle$ of q_m at time $t + 1$.

Similar to F_1 , there are many options that could be applicable to F_2 . The best option should be selected on the basis of application in hand. However, the following axioms are the minimum requirements to be satisfied by F_2 :

Axiom 3. $0 \leq F_{2_{i=1}^n}(u_i, v_i) \leq 1$. Also $0 \leq u_i + v_i \leq 1$.

Axiom 4. $F_2(\emptyset, \emptyset) = 0$.

This axiom, essentially, allows the way for the ϵ -transition to be incorporated into the operation of intuitionistic fuzzy automata.

Axiom 5. $F_{2_{i=1}^n}(u_i, v_i) = (a, b)$ if $\forall i(u_i = a, v_i = b)$.

Whenever all predecessors of a multi-membership and non-membership state produce the same $\langle mv, nmv \rangle$, it is probable that the active state assumes this $\langle mv, nmv \rangle$. An immediate outcome of this axiom is:

$$\begin{aligned} F_{2_{i=1}^n}(u_i, v_i) &= (0, 0) \text{ if } \forall i(u_i = 0, v_i = 0); \\ F_{2_{i=1}^n}(u_i, v_i) &= (0, 1) \text{ if } \forall i(u_i = 0, v_i = 1); \text{ and} \\ F_{2_{i=1}^n}(u_i, v_i) &= (1, 0) \text{ if } \forall i(u_i = 1, v_i = 0). \end{aligned}$$

Another corollary of this axiom is $F_{2_{i=1}^n}(u_i, v_i) = (u_i, v_i)$. It enables F_2 to be seen as a general process on all active states, even if there are multi-membership and non-membership.

There are many possibilities that could be used for the function F_2 . But, the best strategy for any application should be selected based on the requirements of that application.

* Maximum multi-membership and minimum multi-nonmembership resolution

$$\begin{aligned} &(\mu^{t+1}(q_m), \nu^{t+1}(q_m)) \\ &= \left\{ \max_{i=1 \text{ to } n} [\tilde{\mu}_A((q_i, \mu^t(q_i)), a_k, q_m)], \min_{i=1 \text{ to } n} [\tilde{\nu}_A((q_i, \mu^t(q_i)), a_k, q_m)] \right\} \\ &= \left\{ \max_{i=1 \text{ to } n} [F_1(\mu^t(q_i), \mu_A(q_i)), a_k, q_m)], \min_{i=1 \text{ to } n} [F_1(\nu^t(q_i), \nu_A(q_i)), a_k, q_m)] \right\}, \end{aligned}$$

if

$$0 < \max_{i=1 \text{ to } n} [F_1(\mu^t(q_i), \mu_A(q_i)), a_k, q_m)] + \max_{i=1 \text{ to } n} [F_1(\nu^t(q_i), \nu_A(q_i)), a_k, q_m)] \leq 1,$$

otherwise

$$\begin{aligned} &(\mu^{t+1}(q_m), \nu^{t+1}(q_m)) \\ &= \left\{ 1 - \max_{i=1 \text{ to } n} [F_1(\mu^t(q_i), \mu_A(q_i)), a_k, q_m)], 1 - \min_{i=1 \text{ to } n} [F_1(\nu^t(q_i), \nu_A(q_i)), a_k, q_m)] \right\}. \end{aligned}$$

* Arithmetic mean multi-membership and multi-nonmembership resolution

$$\begin{aligned} &(\mu^{t+1}(q_m), \nu^{t+1}(q_m)) \\ &= \left\{ \frac{1}{n} \sum_{i=1}^n \tilde{\mu}_A((q_i, \mu^t(q_i)), a_k, q_m), \frac{1}{n} \sum_{i=1}^n \tilde{\nu}_A((q_i, \nu^t(q_i)), a_k, q_m) \right\} \\ &= \left\{ \frac{1}{n} \sum_{i=1}^n F_1(\mu^t(q_i), \mu_A(q_i), a_k, q_m), \frac{1}{n} \sum_{i=1}^n F_1(\nu^t(q_i), \nu_A(q_i), a_k, q_m), \right\} \end{aligned}$$

where n is the number of simultaneous transitions from q_i 's to q_m at time $t + 1$, and $q_i \in Q_{pred}(q_m, a_k)$.

2.4 Computational generality of intuitionistic fuzzy automata

In order to make Computational generality of intuitionistic fuzzy automata and its generalization capability more systematic and application-friendly, a more general definition is needed. In the sequel, by bringing in F_1 and F_2 , a new definition for IFFA is given that is much more general compared to the current ones.

Definition 2.6. (General intuitionistic fuzzy automaton). A General Intuitionistic Fuzzy Automaton (GIFA) \widetilde{M} is a 7-tuple denoted as

$$\widetilde{M} = (Q, \Sigma, \widetilde{A}, B, C, F_1, F_2),$$

where:

- Q is a finite non-empty set of states;
- Σ is a finite non-empty set of input symbols;
- $\widetilde{A} = (\widetilde{\mu}_A, \widetilde{\nu}_A)$ is the augmented transition function where:

$$\widetilde{\mu}_A : (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\mu, \mu_A)} [0, 1]$$

$$\widetilde{\nu}_A : (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\nu, \nu_A)} [0, 1]$$

- $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_B : Q \rightarrow [0, 1]$ and $\nu_B : Q \rightarrow [0, 1]$, called the intuitionistic initial fuzzy state;
- $C = (\mu_C, \nu_C)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_C : Q \rightarrow [0, 1]$ and $\nu_C : Q \rightarrow [0, 1]$, called the intuitionistic initial fuzzy subset of final states;
- $F_1 : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a mapping function, which is applied via \widetilde{A} to assign mv 's and nmv 's to the active states, thus called membership and non-membership assignment function.
- $F_2 : ([0, 1])^* \rightarrow [0, 1]$ is a multi-membership and non-membership resolution strategy that resolves the multi-membership and non-membership active states and thereafter assigns a single $\langle \text{mv}, \text{nmv} \rangle$ to them, it can thus be called multi-membership and non-membership resolution function.

Example 2.2. The example aims to show various implications of acceptance and the capabilities achievable by the augmented transition function \widetilde{A} (applied through function F_1).

A General Intuitionistic Fuzzy Automaton (GIFA) \widetilde{M} is defined as $\widetilde{M} = (Q, \Sigma, \widetilde{A}, B, C, F_1, F_2)$, where:

- $Q = \{q_1, q_2, q_3, q_4, q_5\}$ is a set of states;
- $\Sigma = \{0, 1\} \cup \epsilon$ is a set of input symbols;
- $\widetilde{A} = (\widetilde{\mu}_A, \widetilde{\nu}_A)$ is the augmented transition function where

$$\widetilde{\mu}_A : (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\mu, \mu_A)} [0, 1]$$

$$\widetilde{\nu}_A : (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\nu, \nu_A)} [0, 1];$$

- $B = \{(q_1, (\mu^{t_0}(q_1), \nu^{t_0}(q_1)))\} = \{q_1, (1, 0)\}$: start intuitionistic fuzzy set;

- $C = \{accept\}$: set of final labels;
- F_1 is defined in different ways as will be seen;
- F_2 is not applicable. There is no multi-membership.

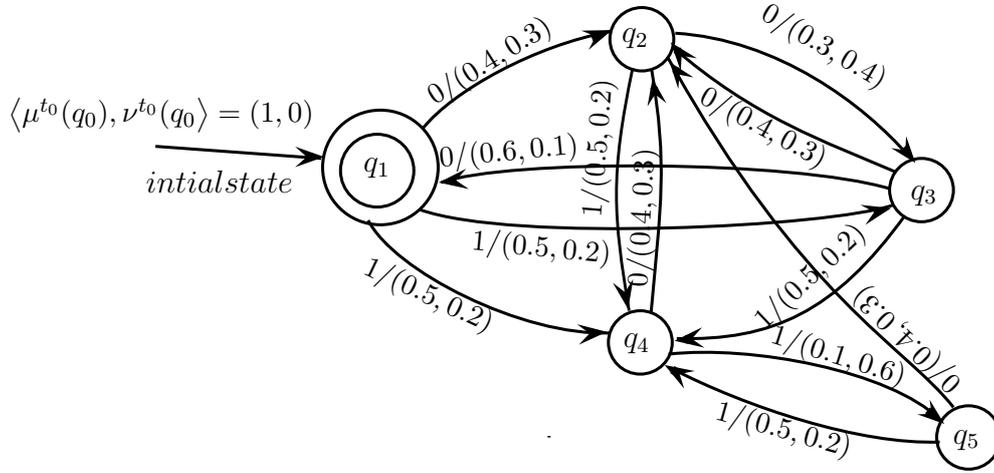


Figure 2. The IFFA of Example 2.2

Consider the transition based membership and non-membership, use \tilde{A} newly introduced function:

$$\begin{aligned} \mu^{t+1}(q_j) &= \tilde{\mu}_A((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \mu_A(q_i, a_k, q_j)) = \mu^t(q_i) + \mu_A(q_i, a_k, q_j) \\ \nu^{t+1}(q_j) &= \tilde{\nu}_A((q_i, \nu^t(q_i)), a_k, q_j) = F_1(\nu^t(q_i), \nu_A(q_i, a_k, q_j)) = \nu^t(q_i) + \nu_A(q_i, a_k, q_j) \\ \mu^{t+1} &= \begin{cases} \mu^t + \mu_A, & \text{if } 0 < \mu^t + \mu_A \leq 1 - C \\ (\mu^t + \mu_A) - (1 - C), & \text{if } \mu^t + \mu_A > 1 - C \end{cases} \\ \nu^{t+1} &= \begin{cases} \nu^t + \nu_A, & \text{if } 0 < \nu^t + \nu_A \leq 1 - C \\ (1 - C) - \nu^t, & \text{if } \nu^t + \nu_A > 1 - C \end{cases} \end{aligned}$$

where C be the uncertainty.

That is to say the function F_1 calculate the sum of the $\langle mv, nmv \rangle$ of the predecessor and the weight of the transition and bounds it in the interval $[0, 1]$. These values are assigned to the successor.

F_1 makes \tilde{M} a periodic intuitionistic fuzzy automaton. Upon periodic input strings, the successor will be repeated with periods that are the same as the length of the string period. This period is called state period (T_q) is shown in Table 1. But, the $\langle mv, nmv \rangle$ affiliated with each state, will vary from period to period. The $\langle mv, nmv \rangle$ will be periodically denoted as T_μ and T_ν respectively. The detailed operation of the automation upon input $(01)^m (m \geq 1)$ is also shown in Table 2.

Table 1. The state period of different strings

Input string	State period	T_q
$(01)^s$	$q_2q_4 \dots$	2
$(0^21^2)^s$	$q_2q_3q_4q_5 \dots$	4
$(1^20)^s$	$q_2q_4q_5 \dots$	3
$(0^21)^s$	$q_4q_2q_3 \dots$	3
$(10^301^20^4)^s$	$q_2q_4q_5 \dots$	3

Table 2. The $\langle mv, nmv \rangle$ of the string $(01)^m (m \geq 1)$. Choose $C = 0.3, T_q = 2, \langle T_\mu, T_\nu \rangle = 14$

Input	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
State	q_2	q_4														
mv	0.4	0.2	0.6	0.4	0.1	0.6	0.3	0.1	0.5	0.3	0.7	0.5	0.2	0.7	0.4	0.2
nmv	0.3	0.5	0.1	0.3	0.6	0.1	0.4	0.6	0.2	0.4	0.7	0.2	0.5	0.7	0.3	0.5

In an IFS, uncertainty (Π_A) is one of its part, i.e., $\Pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Let $\Pi_A(x) = C$. In the above example, the string $(01)^m, (m \geq 1)$ will have different periodic $\langle mv, nmv \rangle$ depending upon the value of C between 0 and 1. If $C = 0.1$, the string have high period. That is, the string have different $\langle mv, nmv \rangle$ up to the 18-th cycle, after that the same set of $\langle mv, nmv \rangle$ is repeated. The $\langle mv, nmv \rangle$ of a string $(01)^m$ for different values of C are listed below:

$C = 0.1$, the $\langle mv, nmv \rangle$ periodic may be 18.

$C = 0.2$, the $\langle mv, nmv \rangle$ periodic may be 16.

$C = 0.3$, the $\langle mv, nmv \rangle$ periodic may be 14. (as shown in Table 2)

$C = 0.4$, the $\langle mv, nmv \rangle$ periodic may be 12.

$C = 0.5$, the $\langle mv, nmv \rangle$ periodic may be 10.

$C = 0.6$, the $\langle mv, nmv \rangle$ periodic may be 08.

$C = 0.7$, the $\langle mv, nmv \rangle$ periodic may be 06.

$C = 0.8$, the $\langle mv, nmv \rangle$ periodic may be 04.

The increase of uncertainty gives less membership and non-membership periodic.

3 Temporal intuitionistic fuzzy automaton

Based on the definition of general intuitionistic fuzzy automaton, a Temporal Intuitionistic Fuzzy Automaton (TIFA) as an eight-tuple can be defined as $M_1 = \{Q, \Sigma, D, A, B, C, F_1, F_2\}$, where:

- Q is a finite non-empty set of states;
- Σ is a finite non-empty set of input symbols;
- D is a finite non-empty set of the duration of the states;

- $A = (\mu_A, \nu_A) : Q \times D \times \Sigma \times Q \rightarrow [0, 1]$ is the state transition function;
- $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_B : Q \rightarrow [0, 1]$ and $\nu_B : Q \rightarrow [0, 1]$, called the initial state of TIFA;
- $C = (\mu_C, \nu_C)$ is an intuitionistic fuzzy subset of Q , i.e., $\mu_C : Q \rightarrow [0, 1]$ and $\nu_C : Q \rightarrow [0, 1]$, called the initial state of TIFA;
- $F_1 : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is the augmented transition function;
- $F_2 : ([0, 1])^* \rightarrow [0, 1]$ is the multi-membership and nonmembership resolution.

In the following, the above notation are explained in detail. Some related conventions and definitions are also given.

State Q : This set is comprised of several states of the TIFA. But, IFFA can only be in one state at every instant, whereas TIFA can be in several states with different degrees.

Input set Σ : This set which is usually composed of several input streams x_j contains the samples of the signal that the TIFA has to recognize.

Convention 1 : The value of all the input at time t is expressed as x^t and particularly at time t and x_j can be expressed as x_j^t , where t is a natural number that indicates each sample of the input.

Definition 3.1. Consider a state q_i which is active at time t when $\mu_{q_i}^t > \theta$ and $\nu_{q_i}^t < \theta$ where θ is a constant defined by an expert depending on the application.

Duration of states D : This set gives information about the duration of the different states and it is used for providing time restriction in the TIFA.

Definition 3.2. Each element of the set D represents the time in which the state q_i has been continuously active. The duration of the state q_i at time t can be found using the following formula:

$$d_i^t = \begin{cases} 0, & \mu_{q_i}^t < \theta \quad \text{and} \quad \nu_{q_i}^t \geq \theta \\ d_i^{(t-1)}, & \mu_{q_i}^t \geq \theta \quad \text{and} \quad \nu_{q_i}^t < \theta \end{cases}$$

The state transition function \tilde{A} : This function gives the possible transitions of the TIFA.

Definition 3.3. A transition $T_{i,j}$ from the state q_i to the state q_j is possible if there exist x, d such that $\tilde{\mu}_A(q_i, x, d, q_j) > 0$ and $\tilde{\nu}_A(q_i, x, d, q_j) < 1$.

The TIFA follows a transition $T_{i,j}$ at time t . If this transition is possible and the state q_i belongs to the set of initial states B then:

$$\begin{aligned} \tilde{\mu}_A(q_i, x^t, d^t, q_j) &> 0 \wedge (\mu_{q_i}^t \geq \theta \wedge \nu_{q_i}^t < \theta \vee q_i \in B), \\ \tilde{\nu}_A(q_i, x^t, d^t, q_j) &< 1 \wedge (\mu_{q_i}^t \geq \theta \wedge \nu_{q_i}^t < \theta \vee q_i \in B). \end{aligned}$$

This $\tilde{\mu}_A$ and $\tilde{\nu}_A$ can be interpreted like the weight of the transition. These conditions are specific for each transition and they are functional of the inputs and the duration of the origin state. Section 3.1 deals with conditions for each transition using linguistic labels.

The state membership and nonmembership assignment function F_1 : This function can be used to update the $\langle mv, nmv \rangle$ of the next successor when the TIFA follows a transition $T_{i,j}$. To obtain $\langle mv, nmv \rangle$ of the state q_j after a transition from the state q_i the following equation is used:

$$\mu_{q_j}^t = F_1(\mu_{q_i}^t, \tilde{\mu}_A(q_i, x^t, d^t, q_j)) \text{ and } \nu_{q_j}^t = F_1(\nu_{q_i}^t, \tilde{\nu}_A(q_i, x^t, d^t, q_j)).$$

The multi-membership and nonmembership resolution function F_2 : Sometimes several transitions end in same state and then several mv's and nmv's will correspond with the same state. By using the function F_2 called multi-membership and non-membership resolution to provide only one $\langle mv, nmv \rangle$ to summarize all the mv's and nmv's of those transitions that end in the same state. The following equations can be used to calculate the $\langle mv, nmv \rangle$ of a state:

$$\mu_{q_j}^{t+1} = F_2(u_1, u_2, \dots, u_n) \text{ and } \nu_{q_j}^{t+1} = F_2(v_1, v_2, \dots, v_n),$$

where n is the total number of transitions that end in the state q_j , and each u_i and v_i are the $\langle mv, nmv \rangle$ provided by each transition respectively.

$$F_2(\emptyset, \emptyset) = 0,$$

$$F_2(u_1, u_2, \dots, u_n) = a \text{ if } \forall i(u_i = a),$$

$$F_2(v_1, v_2, \dots, v_n) = b \text{ if } \forall i(v_i = b).$$

Initial states B and final state C : The sets B and C are the nonempty sets of initial states and final states respectively of the TIFA. The process of TIFA begins from its initial states, after which it moves between different states according to its state transition function \tilde{A} until it reaches a final state when the pattern can be recognized.

3.1 Modeling a pattern using TIFA

This section deals with the pattern in which an expert can model TIFA following a simple methodology.

3.1.1 Defining the states

To model a pattern it is important to decide what inputs Σ are going to be used to describe it. Every state q_i represents a time during which the inputs satisfy some conditions. Although not necessary, the conditions are defined using the idea linguistic interval [7]. When modelling more complex patterns, the linguistic intervals make it possible to use different levels of granularity by combining continuous membership and nonmembership functions to obtain wider membership and nonmemberships. A linguistic interval $X_j^{[a,b]}$ is defined over the domain of the input x_j using a set of intuitionistic fuzzy linguistic labels X_j^k . The two limits a and b indicate the membership and nonmembership functions where the interval begins and ends respectively. The degree of membership and nonmembership functions for each value in the interval is found by using the following equations:

$$\mu_{X_j^{[a,b]}}(x_j) = \sum_{K=a}^b \mu_{X_j^k}(x_j) \text{ and } \nu_{X_j^{[a,b]}}(x_j) = \sum_{K=a}^b \nu_{X_j^k}(x_j).$$

To define the linguistic labels. There are several membership and nonmembership functions that compose a strong intuitionistic fuzzy partition of the input defined in because this restriction guarantees a membership degree of one and nonmembership degree of zero, except for in the boundaries where membership degree decreases and nonmembership degree increases.

In particular, the triangular functions are used, not only because of their simplicity, but also because, under some hypothetical conditions, they can build intuitionistic fuzzy partitions whose membership and nonmembership functions are uniformly activated.

In an ordinary instance, the rules of a state (rul_{q_i}) can affect several input streams. For example, in the following all the input streams are combined by using conjunction and disjunction:

$$rul_{q_j}(x_1, \dots, x_r) = \{(\mu_{X_1^{[a_1^i, b_1^i]}}(x_1) \wedge \dots \wedge \mu_{X_r^{[a_r^i, b_r^i]}}(x_r)), (\nu_{X_1^{[a_1^i, b_1^i]}}(x_1) \vee \dots \vee \nu_{X_r^{[a_r^i, b_r^i]}}(x_r))\}.$$

But the definition of TIFA does not deal with the structure, model nor its pattern of using left-right structure because the resulting TIFA is much more understandable. A pattern is described with several states and a last state, which actually detect the end of the pattern. During the execution of a pattern the TIFA will move from the first state to the final state, and as a result the set B contains only the first state and C contains only the final state.

3.1.2 Defining the transitions

The evolution of the TIFA is based only on the state transition function \tilde{A} . Therefore, the above said conditions are to be included in the different transitions of the TIFA. Using a left-right structure, the TIFA can move from the state q_i (feedback transition $T_{i,i}$) to the next state q_{i+1} (changing transition $T_{i,i+1}$.) All the transitions are defined by two rules: one is defined upon the inputs and the other upon the duration of the predecessor which makes it possible to include some time restrictions. For any transition $T_{i,j}$, the rule over the inputs is the rule for being one among the successors (rul_{q_i}). But the time rule differs depending on whether it is a feedback transition or a changing transition.

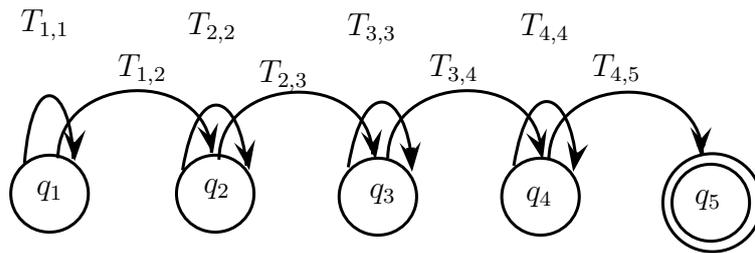


Figure 3. Structure of a pattern using TFA

In a feedback transition $T_{i,i}$

$$rul_{T_{i,i}}(x, d_i) = \{rul_{q_i}(x) \wedge (\mu_{TimeToStay_{i,i}}(d_i), \nu_{TimeToStay_{i,i}}(d_i))\}$$

The time rule is expressed with the linguistic label $TimeToStay_{i,i}$; it restricts the maximum time that the TIFA can stay in the State q_i . The membership and non-membership functions associated

with $TimeToStay_{i,i}$ are defined over the domain of the duration of the state (q_i). These functions start with the maximum membership value 1 and minimum non-membership value 0, and they decrease and increase respectively when the maximum duration is reached. The goal of this rule is to reject those signals which cause the TIFA to remain in a state for more time span than expected. The TIFA stays in the same state until the rules over the inputs are false or the duration of the state is longer than the time defined by the membership and non-membership functions of $TimeToStay_{i,i}$.

In a changing transition $T_{i,i+1}$

$$rul_{T_{i,i+1}}(x, d_i) = \{rul_{q_{i+1}}(x) \wedge (\mu_{TimeToMove_{i,i+1}}(d_i), \nu_{TimeToMove_{i,i+1}}(d_i))\}$$

The time rule is $TimeToMove_{i,i+1}$ which represents the minimum time that the TIFA has to be in a state q_i before moving to a state q_{i+1} . Therefore, the membership and non-membership functions associated with the linguistic label $TimeToMove_{i,i+1}$ starts with the lowest membership value zero and highest non-membership value one and they increase and decrease respectively when the expected minimum duration is reached. This restriction allows the TIFA to move from one state to another when the rules over the inputs for being in the state q_{i+1} is true and the TIFA has sufficient time in the successor (origin state) q_i .

But the last state is a special case because its goal is only to detect the end of the pattern. This last state has no feedback transition because the TIFA is not supposed to stay in this state. The transitions that end in this state have rules over time only to confirm that the previous state was active for a while:

$$rul_{T_{N-1,N}}(x, d_{N-1}) = \{\mu_{TimeToMove_{N-1,N}}(d_{N-1}), \nu_{TimeToMove_{N-1,N}}(d_{N-1})\}.$$

By using the above said model TIFA can deal with signals that have variability in amplitude and duration. It allows the modelling to be more robust against the variability in the state durations.

4 Conclusion

The most general formulation of intuitionistic fuzzy automata introduced here is atleast in the realm of discrete spaces. It degenerates other types of automata under various restrictions. Also, GIFA encompasses not only other types of automata, but also other computational paradigms. The generality of GIFA is so motivating and challenging that it has enough for further research.

By using the definition of GIFA, a definition of temporal intuitionistic fuzzy automata is introduced. To use this as a tool of the pattern recognition field. The new method uses intuitionistic fuzzy sets for defining the conditions imposed on the inputs. The result is an imprecise but robust model of the temporal pattern of the signal. The main contribution of this model is its capacity to include intuitionistic fuzzy conditions not only in the signal amplitude but also in the description of the signal temporal dimensions. The obtained models are easily understandable and therefore can be checked easily by an expert. Use of this model in detecting a pattern in signal will be done and the same will be reported in future work.

Acknowledgements

Sincerely acknowledge University Grants Commission's(UGC, Hyderabad, India) support in enabling the effective completion of this research project (MRP – 3813 (MRP/UGC-SERO) Link NO.: 3813, Dated: August 2012. Completed on March 2014).

References

- [1] Aho, A. V., & Ullman, J. D. (1994). *Foundations of Computer Science*. Computer Science Press, New York.
- [2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.
- [3] Honda, N. (1975). Applications of fuzzy set theory to automata and linguistics. *Journal of Japanese Automation and Automatic Control Engineers*, 19, 249–254.
- [4] Hopcroft, J. E., Motwani, R., & Ullman, J. D. (2011). *Introduction to Automata Theory, Languages, and Computation*, Third edition. Pearson.
- [5] Lee, E. T., & Zadeh, L. A. (1969). Note on fuzzy languages. *Information Sciences*, 1(4), 421–434.
- [6] Mateescu, A., Salomaa, A., Salomaa, K., & Yu, S. (1995). Lexical analysis with a simple finite-fuzzy-automaton model. *Journal of Universal Computer Science*. 1(5), 292–311.
- [7] Moreno-Garcia, J., Linares, L. J., Castro-Schez, J. J., & Benitez, L. R. (2004). A direct linguistic induction method for systems, *Fuzzy Sets and Systems*, 146(1), 79–96.
- [8] Pathak, A., & Pal, S. K. (1986). Fuzzy Grammars in Syntactic recognition of skeletal maturity from X-rays. *IEEE Transactions on Systems Man and Cybernetics*, 16(5), 657–667.
- [9] Perrin, D., & Pin, J.-E. (1995) Semigroup and automata on infinite words. In: *J. Fountain (Ed.), NATO Advanced Study Institute Semigroups, Formal Languages and Groups*. Kluwer Academic Publishers, Dordrecht, 466, 49–72.
- [10] Rajaretnam, T., & Ayyaswamy, S. K. (2011). Fuzzy monoids in a Fuzzy finite state automaton with unique membership transition on an input symbol. *International Journal of Mathematics and Scientific Computing*, 1(1), 48–51.
- [11] Shamsizadeh, M., & Zahedi, M. M. (2015). Intuitionistic general fuzzy automata. *Soft Computing*, 20(9), 3505–3519.
- [12] Tumer, M. B., Belfore, L. A., & Ropella, K. M. (2003) A syntactic methodology for automatic diagnosis by analysis of continuous time measurements using hierarchical signal representation. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 33(6), 951–965.

- [13] Wee, W. G., & Fu, K. S. (1969). A formulation of fuzzy automata and its applications as a model of learning Systems, *IEEE Transactions on Solid State Circuits, SSC-5*, 3, 215–223.
- [14] Zhang, X., & Li, Y. (2009). Intuitionistic fuzzy recognizers and intuitionistic fuzzy finite automata. *Soft Computing*, 13, 611–616.
- [15] Zadeh, L. A. (1965). Fuzzy sets, *Information and Control*, 8, 338–353.