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On one type of intuitionistic fuzzy modal operators

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1 Introduction

Intuitionistic Fuzzy Sets (IFSs, see [2]) were defined as extensions of the ordinary fuzzy sets. All results which are valid for the fuzzy sets can be transformed here, too. Also, all researches, for which the apparatus of the fuzzy sets can be used, can be described in the terms of the IFSs.

On the other hand, over the IFSs there have been defined not only operations and relations similar to the ordinary fuzzy set ones, but also operators that cannot be defined in case of ordinary fuzzy sets.

In the present paper we shall discuss two modal-like type of operators.

2 Basic concepts

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in E$ and these sets have the form:

$$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}.$$

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [2]), the most important of which are:

$$\begin{array}{lcl} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\ A \supset B & \text{iff} & B \subset A; \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\ \overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x) \rangle | x \in E\}; \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)) \rangle | x \in E\}; \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)) \rangle | x \in E\}; \end{array}$$

We shall define the following operators (see, e.g., [2]):

$$\begin{array}{l} \Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E\}; \\ \Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x) \rangle | x \in E\}; \\ D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1 - \alpha).\pi_A(x) \rangle | x \in E\}; \\ F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x) \rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1; \\ G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x) \rangle | x \in E\}; \\ H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x) \rangle | x \in E\}; \\ H^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x)) \rangle | x \in E\}; \\ J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x) \rangle | x \in E\}; \\ J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x) \rangle | x \in E\}; \\ P_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x)) \rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1; \\ Q_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x)) \rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1; \end{array}$$

We will describe four types of other modal-like operators, following [1, 2, 3].

Following [1, 2], we will introduce the operators of modal type, which are similar to the operators $\Box$ and $\Diamond$. They are the following (A is an IFS):

$$\boxplus A = \{\langle x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x) + 1}{2} \rangle | x \in E\},$$

$$\boxtimes A = \{\langle x, \frac{\mu_A(x) + 1}{2}, \frac{\nu_A(x)}{2} \rangle | x \in E\}.$$

Following [1], we will generalize the two operators introduced above.

Let $\alpha \in [0, 1]$ and let A be an IFS. Then we can define:

$$\boxplus_\alpha A = \{\langle x, \alpha.\mu_A(x), \alpha.\nu_A(x) + 1 - \alpha \rangle | x \in E\},$$

$$\boxtimes_\alpha A = \{\langle x, \alpha.\mu_A(x) + 1 - \alpha, \alpha.\nu_A(x) \rangle | x \in E\}.$$

Obviously,

$$0 \leq \alpha.\mu_A(x) + \alpha.\nu_A(x) + 1 - \alpha = 1 - \alpha.(1 - \mu_A(x) - \nu_A(x)) \leq 1.$$

For every IFS A:

$$\boxplus_{0.5} A = \boxplus A,$$

$$\boxtimes_{0.5} A = \boxtimes A.$$

Therefore, the new operators "$\boxplus_\alpha$" and "$\boxtimes_\alpha$" are generalizations of the first ones.

The second extension of operators $\boxplus$ and $\boxtimes$ is introduced in [3] by Katerina Dencheva. She extended the last two operators to the forms:

$$\boxplus_{\alpha,\beta} A = \{\langle x, \alpha.\mu_A(x), \alpha.\nu_A(x) + \beta\rangle | x \in E\},$$

$$\boxtimes_{\alpha,\beta} A = \{\langle x, \alpha.\mu_A(x) + \beta, \alpha.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \alpha + \beta \in [0, 1]$.

Obviously, for every IFS A:

$$\boxplus A = \boxplus A_{0.5,0.5},$$

$$\boxtimes A = \boxtimes A_{0.5,0.5},$$

$$\boxplus A_\alpha = \boxplus A_{\alpha,1-\alpha},$$

$$\boxtimes A_\alpha = \boxtimes A_{\alpha,1-\alpha}.$$

3 Main results

Now, we shall introduce third extensions of the above operators. They will have the forms:

$$\boxplus_{\alpha,\beta,\gamma} A = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x) + \gamma\rangle | x \in E\},$$

$$\boxtimes_{\alpha,\beta,\gamma} A = \{\langle x, \alpha.\mu_A(x) + \gamma, \beta.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\max(\alpha, \beta) + \gamma \leq 1$.

Obviously, for every IFS A:

$$\boxplus A = \boxplus A_{0.5,0.5,0.5},$$

$$\boxtimes A = \boxtimes A_{0.5,0.5,0.5},$$

$$\boxplus A_\alpha = \boxplus A_{\alpha,\alpha,1-\alpha},$$

$$\boxtimes A_\alpha = \boxtimes A_{\alpha,1-\alpha},$$

$$\boxplus A_{\alpha,\beta} = \boxplus A_{\alpha,\alpha,\beta},$$

$$\boxtimes A_{\alpha,\beta} = \boxtimes A_{\alpha,\alpha,\beta}.$$

The following assertions hold for the new operators.

Theorem 1: For every IFS A and for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$:

(a) $\neg\boxplus_{\alpha,\beta,\gamma}\neg A = \boxtimes_{\beta,\alpha,\gamma}A$,

(b) $\neg\boxtimes_{\alpha,\beta,\gamma}\neg A = \boxplus_{\beta,\alpha,\gamma}A$.

Theorem 2: For every IFS A and for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$,

(a) $\boxplus_{\alpha,\beta,\gamma}\boxplus_{\alpha,\beta,\gamma}A \subset \boxplus_{\alpha,\beta,\gamma}A$ is valid if and only if $\beta + \gamma = 1$,

(b) $\boxtimes_{\alpha,\beta,\gamma}\boxtimes_{\alpha,\beta,\gamma}A \supset \boxtimes_{\alpha,\beta,\gamma}A$ is valid if and only if $\alpha + \gamma = 1$.

Theorem 3: For every IFS A and for every $\alpha, \beta, \alpha + \beta \in [0, 1]$

$$\boxplus_{\alpha,\beta,\gamma}\boxtimes_{\alpha,\beta,\gamma}A = \boxtimes_{\alpha,\beta,\gamma}\boxplus_{\alpha,\beta,\gamma}A \text{ iff } \gamma = 0.$$

Theorem 4: For every IFS A and for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$ each one of the assertions:

(a) $\boxplus_{\alpha,\beta,\gamma} \square A = \square \boxplus_{\alpha,\beta,\gamma}A$,

(b) $\boxtimes_{\alpha,\beta,\gamma} \square A = \square \boxtimes_{\alpha,\beta,\gamma}A$,

(c) $\boxplus_{\alpha,\beta,\gamma}\Diamond A = \Diamond\boxplus_{\alpha,\beta,\gamma}A$,

(d) $\boxtimes_{\alpha,\beta,\gamma}\Diamond A = \Diamond\boxtimes_{\alpha,\beta,\gamma}A$,

are valid if and only if $\alpha = \beta$ and $\alpha + \gamma = 1$.
Theorem 5: For every two IFSs A and B and for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$:

(a) $\boxplus_{\alpha,\beta,\gamma}(A \cap B) = \boxplus_{\alpha,\beta,\gamma}A \cap \boxplus_{\alpha,\beta,\gamma}B$,

(b) $\boxtimes_{\alpha,\beta,\gamma}(A \cap B) = \boxtimes_{\alpha,\beta,\gamma}A \cap \boxtimes_{\alpha,\beta,\gamma}B$,

(c) $\boxplus_{\alpha,\beta,\gamma}(A \cup B) = \boxplus_{\alpha,\beta,\gamma}A \cup \boxplus_{\alpha,\beta,\gamma}B$,

(d) $\boxtimes_{\alpha,\beta,\gamma}(A \cup B) = \boxtimes_{\alpha,\beta,\gamma}A \cup \boxtimes_{\alpha,\beta,\gamma}B$,

(e) $\boxplus_{\alpha,\beta,\gamma}(A@B) = \boxplus_{\alpha,\beta,\gamma}A@\boxplus_{\alpha,\beta,\gamma}B$,

(f) $\boxtimes_{\alpha,\beta,\gamma}(A@B) = \boxtimes_{\alpha,\beta,\gamma}A@\boxtimes_{\alpha,\beta,\gamma}B$,

Theorem 6: For every IFS A and for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$:

(a) $\boxplus_{\alpha,\beta,\gamma}C(A) = C(\boxplus_{\alpha,\beta,\gamma}A)$,

(b) $\boxtimes_{\alpha,\beta,\gamma}C(A) = C(\boxtimes_{\alpha,\beta,\gamma}A)$,

(c) $\boxplus_{\alpha,\beta,\gamma}I(A) = I(\boxplus_{\alpha,\beta,\gamma}A)$,

(d) $\boxtimes_{\alpha,\beta,\gamma}I(A) = I(\boxtimes_{\alpha,\beta,\gamma}A)$.

Theorem 7: For every IFS A, for every $\alpha, \beta, \gamma \in [0, 1]$ for which $\max(\alpha, \beta) + \gamma \leq 1$ and for every δ, ε for which $\delta + \varepsilon \leq 1$:

(a) $\boxplus_{\alpha,\beta,\gamma} P_{\delta,\varepsilon}(A) = P_{\alpha\delta,\beta\varepsilon+\gamma}(\boxplus_{\alpha,\beta,\gamma} A)$,

(b) $\boxplus_{\alpha,\beta,\gamma} Q_{\delta,\varepsilon}(A) = Q_{\alpha\delta,\beta\varepsilon+\gamma}(\boxplus_{\alpha,\beta,\gamma} A)$,

(c) $\boxtimes_{\alpha,\beta,\gamma} P_{\delta,\varepsilon}(A) = P_{\alpha\delta+\gamma,\beta\varepsilon}(\boxplus_{\alpha,\beta,\gamma} A)$,

(d) $\boxtimes_{\alpha,\beta,\gamma} Q_{\delta,\varepsilon}(A) = Q_{\alpha\delta+\gamma,\beta\varepsilon}(\boxplus_{\alpha,\beta,\gamma} A)$.

4 Conclusion

The new operators can also be extended and the definition and the properties of the new operator extending them will be an object of our future research. It is interesting to note that in the step-by-step process of extending operators $\boxplus$ and $\boxtimes$, the new operators gradually lose some of their properties. In the next research we shall discuss this situation.

References

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